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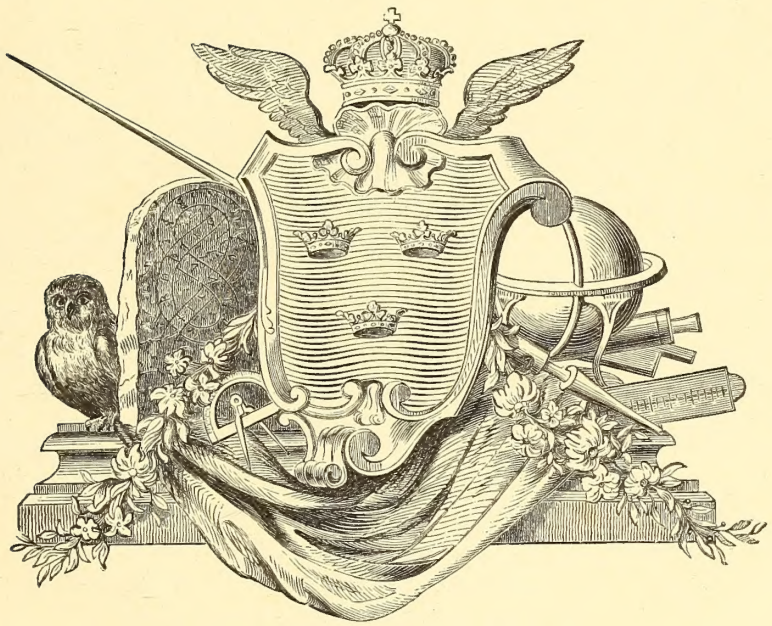

SERIEI TERTIÆ VOL. IX.

UPSALIÆ
EXCUDIT ED. BERLING REG. ACAD. TYPOGRAPHUS.
MDCCCLXXV.

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SCIENTIARUM

UPSALIENSIS.

SERIEI TERTIÆ.

VOL. IX.

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NOVA ACTA

REGIÆ SOCIETATIS

SCIENTIARUM

UPSALIENSIS.

SERIEI TERTIÆ VOL. IX.

FASCICULUS PRIOR.

UPSALIÆ
EXCUDIT ED. BERLING REG. ACAD. TYPOGRAPHUS.
MDCCCLXXIV.

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OBSERVATIONES PHYCOLOGICAE

AUCTORE

J. E. ARESCHOUG

BOT. PROF. UPSALIENS.

PARTICULA SECUNDA

DE UROSPORA MIRABILI Aresch ET. DE CHLOROZOOSPORARUM COPULATIONE.

(Recitatæ in Regia Societate Scientiar. Upsal. die xviii Oct. mdcccclxxiii.)

UPSALIÆ,
Excudit ed. berling reg. acad. typographus,
mdcccclxxiv.

In dissertatione phycologica, quæ his Actis inserta est[1]), duo algarum genera, *Hormisciam* Fr. et *Urosporam* Aresch., firmioribus characteribus circumscribere sumus conati. *Hormisciæ* adnumeratæ sunt duæ species, *H. zonata* et *H. penicilliformis*. *Urosporæ* autem sola *U. mirabilis*. Utriusque generis character nunc quoque firmus est et stabilis, ut in eodem non multa sint, quæ mutanda esse videantur, nisi terminos nonnullos cum aptioribus mutare velles. Sed cum e genere *Hormiscia* prorsus excludenda sit *Hormiscia penicilliformis*, quæ iterum iterumque examinata nobis videtur *Urosporæ mirabilis* forma, character generis *Hormisciæ* corrigendus est ita, ut quattuor cilia (nec duo quoque) macrozoosporis tribuantur. Ut vero terminis certis atque definitis utamur, nos dicere oportet, diversa zoosporarum nomina quid significent. Megazoosporæ sunt nobis zoosporæ neutræ, nulla præeunte fœcundatione germinantes. Microzoosporæ autem sunt zoosporæ sexuales, duplicis generis, masculinæ et femineæ, quæ copulatione in zygozoosporas transmutantur. Absoluto nostro de *Urospora mirabili* tractatu, de microzoosporarum in *Cladophora sericea* et in *Enteromorpha compressa* copulatione pauca afferre nobis concessum credimus.

1. UROSPORA mirabilis Aresch. Obs. phyc. p. 16 tab. III.

Synonym. Hormiscia penicilliformis Aresch. l. c. p. 12. tab. I.

Zoosporarum in *Urospora* evolutionem cum diligentius perquirere in animo esset, in Bahusiam nos contulimus, ubi per totum mensem Majum, non quotidie dicimus, sed quovis fere hebdomate, ex hujus plantæ articulis zoosporas egressas vidimus. Fuerunt autem omnes microzoosporæ, ut dicuntur, a nobis (l. cit. p. 15 tab. III fig. 8) jam antea descriptæ atque depictæ. Maji fine Junioque ineunte, nec antea, ut nos quidem observavimus, inceperunt aliæ aliusque generis zoosporæ, a nobis macrozoosporæ nominatæ (l. cit. p.

[1]) *Observationes phycologicæ;* particula prima, *de Confervaceis nonnullis.* Acta Reg. Soc. Scientiar. Ups. Ser. III. vol. VI. Upsaliæ MDCCCLXVI.

13 tab. III fig. 3—6), se ostendere, quarum numerus diem ex die auctus est eodem gradu, quo deminueretur microzoosporarum. Julio et Augusto mensibus Marstrandi quoque vidimus magna megazoosporarum, nec ulla microzoosporarum examina. Quæ cum ita sint, megazoosporæ æstivali, microzoosporæ vero præsertim vernali tempore procreari nobis videntur.

Megazoosporæ a nobis (l. cit. p. 13 tab. III fig. 3—6) ita sunt descriptæ et depictæ, ut nil fere novi addere possimus. Eas in filis validioribus evolvi diximus, quod hoc anno confirmatum invenimus. Fila, in quibus gignuntur, variant crassitie inter $70\frac{1}{3}$ Micro^{mm} et 60 Micro^{mm}; quæ tamen mensura nullam absolutam, sed tantummodo relativam dignitatem habere potest. Megazoosporæ in cellula matrice seu articulo ita sunt coordinatæ, ut earum rostra interiori cellulæ parieti inpositæ sint, cuspidibus posticis axin cellulæ aut centrum spectantibus. Cum ex articulo extruduntur, ut loco citato descripsimus, sunt congestæ in globum, indumento lubrico circumdatum. Globo extruso, indumentum resolvitur, quo facto megazoosporæ circumvagari incipiunt. Alio autem tempore accidere potest, ut in articulo resolvatur hoc indumentum, tum megazoosporæ in articulo se movere incipientes, longo agmine per ostiolum exeunt, nullo indumento impeditæ. Magnitudo earum, forma et prompta germinatio ut *l. c.* diximus.

Microzoosporæ, quantum nos comperimus, evolvuntur semper in aliis quam megazoosporæ filis. Utrarumque fila sunt itaque unisexualia, quod potest sæpenumero observari. Sæpe enim vidimus sub microscopio fila duo vicina, quorum utrumque per totam fere longitudinem ex articulis emittebat zoosporas, alterum macrozoosporas, alterum microzoosporas. Proverbium illud, "nulla regula sine exceptione," hac in re quid valeat, dicere non possumus. Fila microzoosporarum quam megazoosporarum plerumque multo angustiora, inter $16\frac{2}{3}$ Micro^{mm} et 40 Micro^{mm} crassitie variantia, quæ vero mensura non absoluta, sed modo approximativa est habenda. Microzoosporarum cellulæ matrices, seu articuli, longitudine valde variant, modo diametro triplo longiores, modo 2plo l. 3plo breviores. Quo ex articulis effundantur modo, in dissertatione supra citata (l. cit. p. 8 et p. 15) diximus, quibus plura addenda non habemus. Magnitudo microzoosporarum eo, quo natant tempore, in aliis alia. Maxima longitudo, quam hoc anno observavimus, $13\frac{1}{3}$ Micro^{mm}, minima $6\frac{2}{3}$ Micro^{mm}. Latitudo maxima $6\frac{2}{3}$, minima $1\frac{2}{3}$ Micro^{mm}. Ex articulo cum efferuntur, magnitudine et forma sunt magis æquales, ovatæ l. ovato-elongatæ, duobus ciliis longissimis granuloque fusco in media zoospora laterali ornatæ. Sed eo, quo celeriter natant tempore et ad latus vasis fenestræ proximum congregantur, forma earum cum longitudine et latitudine vario modo mutatur,

quæ autem mutatio in aliis major, in aliis minor esse potest (tab. I fig. 1 a, fig. 3 a, b). Hoc in cursu copulantur.

Microzoosporarum copulatio. Microzoosporas celeriter natantes cum intuemur, duas videmus sæpenumero microzoosporas, quarum altera alteram fugientem persequitur. Hæc est microzoospora feminea, quæ illam, seu microzoosporam masculinam, effugere tendit, quæ contentio, quamquam nonnunquam prospera, est tamen plerumque inanis. Tunc microzoospora masculina rostrum suum rostro microzoosporæ femineæ infigit, quo facto utraque rostris cohæret. Corpus huc et illuc jactans, microzoospora feminea se liberare conatur (tab. I. f. 1 b b., f 4 a.), spe autem fracta, utraque cursum celerrime iterat. Hoc in cursu lateribus earum adpositis, ab rostris versus extremitates earum inferiores sensim adglutinantur (tab. I. f. 1 b., f. 2 a, f. 4 b), ut denique, copulatione absoluta, adsit zoospora, e duabus microzoosporis sexualibus composita, quattuor ciliis ornata et utrinque lateraliter granulo fusco-rubro insignita (tab. I f. 2 b, f. 5 a. b. c), quam zygozoosporam nominare licet.

Zygozoosporæ quam microzoosporæ crassiores esse, naturæ consentaneum videtur, sæpe autem tam parvæ sunt, ut nisi cilia quattuor granulaque duo fusco-rubra observari possint, earum cum microzoosporis confusio perfacilis sit, quod non debet movere nostram mirationem, magnitudinis varietatem in microzoosporis cum supra attulerimus. Post absolutam copulationem zygozoosporæ, forma plerumque ovata l. elongato-ovata, natare pergunt; mox vero cilia obliterantur l dissolvuntur.

Zygozoosporarum germinatio. Hac de re nil dicere possumus, nam zygozoosporam solitariam usque in ejus germinationem persequi, nobis impossibile fuit. In vase autem, in quo coluimus hanc plantam, numerosissimas zygozoosporas beneque evolutas vidimus. Post 8 dies latera vasis et fundus vestita erant cellulis protococciformibus, quas zoosporas fuisse, dubitari non potest (tab. I f. 6 a). In nonnullis harum cellularum duo granula fusco-rubra observata sunt (tab. I f. 6 a.), quæ granula ipsius zygozoosporæ tam bene in memoriam revocant, ut illas saltim cellulas esse zygozoosporas quiescentes, membrana cellulari circumdatas (tab. I f. 6 b.), possis vix dubitare. Qua de re ulterius conferenda sunt ea, quæ de microzoosporis quiescentibus *Hormisciæ penicilliformis* antea diximus (l. cit. p. 9 tab. I fig. 9.). Cum nostra planta amphibia sit species, nunc in scopulo exsiccata, nunc aqua suffusa, zygozoosporis quiescentibus carere vix potest.

Hormiscia penicilliformis (Conf. speciosa) et *Urospora mirabilis* (Conf. hormoides), quas in dissertatione nostra species distinctas habuimus, sunt, ut supra diximus, ejusdem speciei formæ. Hæc nostra sententia digna est,

quæ argumentis ulterius confirmetur. In *Hormiscia penicilliformi* descripsimus duplex zoosporarum genus, mega- et microzoosporarum (l. cit. p. 6 et 8 tab. I), quarum utræque duobus ciliis coronatæ sunt. Inter eas, quæ a nobis depictæ sunt megazoosporæ (l. cit. tab. I fig. 5—6), observantur duæ (l. c. fig. 5 c et c.), quarum utraque zoosporam minorem in rostro portat. Ut vero nostra his de rebus nunc fert cognitio, hæ duæ zoosporæ sunt microzoosporæ seu zoosporæ sexuales, quæ copulantur [1]); major, a nobis megazoospora habita, est feminea, minor vero, illius rostro infixa, masculina microzoospora.

Quas megazoosporas l. c. diximus, eæ sunt igitur microzoosporæ femineæ. Veris macrozoosporis hac in forma tunc temporis nondum inventis, majore harum microzoosparum magnitudine decepti fuimus. In ea enim copulatione, quam figuris illis citatis proposuimus, microzoospora masculina quam microzoospora feminea 3plo est minor. Quas vero præterlapsa æstate copulatas vidimus, eæ fuerunt fere omnes ejusdem magnitudinis (cfr. tab. I.). Nostra *Hormiscia penicilliformis* est igitur forma microzoosporifera *Urosporæ mirabilis*, et utriusque earum specifica diversitas longe abhinc in dubitationem a nobis vocata (l. cit. pag. 16.).

Nomen genericum et triviale Urosporæ mirabilis. Detectis hujus plantæ megazoosporis in extremitate posteriore cuspidem portantibus, nobis concessum credidimus hac in nota novum condere genus (l. cit. p. 15), cujus nomen *Urosporæ* intactum relinquimus. An Kützingianum genus *Hormotrichum* nostræ *Urosporæ* sit anteferendum, postera docebit dies; continet quidem multas nostræ plantæ formas, sed sunt quoque ei adscriptæ species, ex. gr. *H. viridi-fuscum* (e Brasilia), *H. bangioides* et *H. Wormskjoldii*, quarum zoosporæ nobis sunt ignotæ. Quæ cum ita sint, generi Kützingiano tribuere characterem *Urosporæ* par esse non putamus. — Nomen autem triviale, quod dedimus, postquam cum *Confervam penicilliformem* Roth. tum *Confervam hormoidem* Lyngb. ad nostram speciem pertinere certe comperimus, postponendum est ei, quod ætate antecellit, quare nominanda est:

UROSPORA penicilliformis (Roth).

Conferva penicilliformis Roth Cat. Bot. III. p. 271.
— hormoides Lyngb. Hydr. p. 145 tab. 49. — Ag. Syst. p. 100. Aresch. Phyc. Scand. mar. p. 205. Alg. Sc. exs. n. 133 et 186.

[1]) A nobis observatum est hoc phænomenon die 23 Aprilis 1863, et copulatio seu fœcundatio in illo suspecta. Cum autem propter temporis angustias in ipsam rem accuratius inquirere nobis non licuerit, zoosporas copulatas depingere et in explicatione figurarum tantum memorare, quam conjectura augurari, satius duximus.

Conferva flacca Dillw. Lyngb. Hydr. p. 144 tab. 49. — Ag. Syst. p. 102. — Aresch. Phyc. Scand. mar. p. 205.

— speciosa (Carm.) Aresch. Phyc. Scand. mar. p. 204. — Alg. Scand. exs. n. 132,185.

Hormiscia penicilliformis (Roth) Aresch. Obs. phycolog. I. p. 12 tab. I. (planta microzoosporas gerens).

— flacca (Dillw.) Alg. Scand. exs. n. 342.

Urospora mirabilis Aresch. Obs. phycol. I. p. 15 tab. III (planta et mega- et microzoosporas gerens). — Alg. Scand. exs. n. 340.

Habitat. Ad oram Scandinaviæ cum occidentalem tum orientalem hæc alga vulgaris est. Rupibus lapidibusque adnata, crescit in ipso limite aquæ marinæ, præsertim in sinubus plus minusve a mari aperto seclusis, a mense Aprili usque in finem Septembris, aquam et salsissimam et subdulcem perferens; in illa autem robustior magisque explicata, in hac flaccida et tenuior. In mari orientali est forsan rarior, a nobis tamen ad Dalarö lecta. In mari Bahusiensi, ubi valde irregulariter accedit receditque aqua, cum in medio limite aquæ crescat, nunc aqua submersa est, nunc, recedente mari, in rupibus exsiccata. Specimina, octo per dies exsiccata aerique exposita, in aqua cum deposita fuissent, denuo revivixerunt et megazoosporas ediderunt. Hoc in cultura idemque in natura quoque. Est igitur hæc planta vere amphibia, quæ zoosporis, ut ajunt, quiescentibus vix carere possit.

Plures ab auctoribus filorum crassitie longitudineque articulorum distinctas formas in unam speciem cum conjunxerimus, ipsius speciei, quæ in natura percurrit, evolutionis stadia breviter attingere volumus. Quæ ut melius perciperentur, Majo mense filorum ex eodem fere puncto enatorum et implexorum fasciculum funiformem selegimus. Hujus fila diversissimæ fuerunt crassitiei, $\frac{3}{300}$, $\frac{5}{300}$, $\frac{8}{300}$, $\frac{9}{300}$, $\frac{12}{300}$, $\frac{15}{300}$, $\frac{16}{300}$, $\frac{18}{300}$, $\frac{20}{300}$, $\frac{21}{300}$, $\frac{23}{300}$ mm crassa. Quæ inter tenuissima et crassissima differentia! Horum filorum alia fuerunt sterilia, alia aut megazoosporis aut microzoosporis fere maturis referta, ætate nullo modo æqualia, nunc natu majora, nunc minora, totusque fasciculus fuit familia, quæ complectebatur plures generationes, eodem tempore minime enatas. Quod tamen non impedit, quominus alio loco alioque tempore inveniri possit fasciculus, cujus fila ætate æqualia sint omnia ejusdem crassitiei. Hoc in causa est, cur in speciminibus formarum, quæ ex exteris maribus allata sunt, quoad crassitiem fila inter se sint nunc simillima, nunc dissimilia. Articulorum longitudinem cum latitudine eorum si conferimus, sunt plerumque articuli filorum natu minorum longiores, majorum vero breviores. Longissimi articuli, quos vidimus, sunt latitudine us-

que triplo longiores, brevissimi autem eadem usque 6plo breviores. In tanta filorum et articulorum varietate characteres specificos non quærendos esse, omnibus patere credimus.

Megazoosporas in crassioribus, microzoosporas vero in tenuioribus filis evolvi jam diximus. In fasciculo illo, cujus fila supra mensi sumus, fila megazoosporifera inter $73\frac{1}{3}$ Micro^{mm} et 60 Micro^{mm}, microzoosporifera vero inter 40 Micro^{mm} et $16\frac{2}{3}$ Micro^{mm} crassitie variare vidimus. Fila inter 60 Micro^{mm} et 40 Micro^{mm} crassa sterilia fuerunt, megazoosporas futuro tempore forsan editura. Quæ tamen mensura est nullo modo absoluta, sed ut exemplum hoc loco memorata.

Vulgarem hanc speciem ad littora Galliæ Angliæque crescere ibidemque aliis nominibus notari, cum nobis persuasum sit, nonnullas his ex terris communicatas species quoad structuram cum nostra planta comparare lubet.

1. *Lyngbya speciosa* (Carm.) Harv. Phycol. Brit. t. 186 B filorum crassitie articulorumque longitudine convenit omnino cum filis nostræ plantæ crassioribus et sterilibus.

2. *Lyngbya Carmichaelii* Harv. l. c. t. 185 A est forma, quam in Alg. Sc. exs. n. 186 distribuimus, nam ipsi quidem nullam videmus differentiam.

3. *Lyngbya Cutleriæ* Harv. l. c. t. 336, Ulothrix Cutleriæ Thur. in Le Jolis Liste. Hæc forma, quam e speciminibus a Cel. Thuret communicatis judicamus, ab illis formis, quæ in Alg. Sc. exs. sub n. 132 et n. 133 sunt distributæ, nihilo differt.

4. *Lyngbya flacca* Harv. l. c. t. 300 ea videtur forma, quam in Obs. phycol. t. 3 fig. 9 depinximus.

5. *Conferva Youngana* Harv. l. c. t. 328 pertinet certe, ut suspicari videtur Harvey, ad formas supra allatas, quod vero, nullo specimine viso, dijudicare non possumus.

6. *Ulothrix tenerrima* Le Jolis Liste p. 56, a Kützingio Ulothrix tenerrima Kütz. determinata (cf. Le Jolis), hæc forma esse non potest. Planta Le Jolisii est forma Conf. flaccæ Auctorr.; Ulothrix tenerrima Kütz., aquæ dulcis incola, ad Hormisciam zonatam est approximanda.

Nonnullas alias formas prætermissas facimus. Omnes fere ab Kützingio generi *Hormothrici* adscriptas species, nullis speciminibus authenticis visis, nobis re vera ignotas esse valde dolemus. Quæ cum ita sint, ex autopsia judicare non possumus. Sed descriptionibus specierum generis *Hormotrichi* Kütz. diligenter perpensis, multæ apud Kützingium (Spec. Algar.) allatæ formæ cum nostra planta conferendæ sunt, ex. gr. *Hormotrichum affine* Kg., *H. albicans* Kg., *H. flaccum* Kg., *H. didymum* Kg., *H. fa-*

sciculare Kg., *H. vermiculare* Kg., *H. penicilliforme* (Roth), *H. globiferum* Kg. (?), *H. Carmichaelii* (Harv.), *H. Younganum* (Dillw.). De ceteris speciebus huic generi a Kützingio adnumeratis judicium nostrum supprimere cogimur. Nec quo in genere collocanda sit *Conferva bangioides* Harv. (Aresch. Phyc. sc. mar. p. 204 tab. III C) pro certo dicere possumus, quamquam *Urosporæ* species esse videtur.

In omnibus allatis formis structura et mensura nullas præbent notas, quæ stabiles sint. Sed plenior zoosporarum cognitio ipsiusque speciei evolutio dabunt non modo genus, verum etiam speciem; utraque cum sit obscura, de speciebus anglicis et Kützingianis, quas supra enumeravimus, ulterius disserere nolumus.

2. CLADOPHORA sericea (Huds.)

Hæc est planta vulgaris et polymorpha, cujus innumeræ, plus minusve a specie primaria recedentes formæ, per omnes fere Europæ, Americæ borealis et Maris rubri oras sunt dispersæ. Nomina earum trivialia sunt diversissima formarumque, quas significant, certa extricatio difficillima et vix speranda. In *Phyc. scand. mar.* insigniores formæ, quas ad nostras oras invenimus, sunt descriptæ nonnullæque earum in *Alg. scand. exs.* distributæ. Ex speciebus, quæ in aqua dulci crescunt, *Cladophoræ glomeratae* maxime affini est, formas ejus magis evolutas, virides ramulisque secundis instructas si respicimus. Quæ coluimus specimina, quo zoosporarum evolutionem observaremus, ea pertinent ad *Confervæ sericeæ* formas *a* et *b* in *Phyc. scand. mar. pag. 195* descriptas.

In *Cladophora lætevirente*, quæ nostræ speciei levis est forma, et in *Cladophora glomerata* zoosporas duobus ciliis ornatas (zoosporas sexuales seu microzoosporas) invenit Cel. *Thuret* (Recherches p. 11). Cum vero in *Cladoph. sericea* invenerimus non solum microzoosporas, verum etiam macrozoosporas, utrarumque diversam naturam breviter describere abs re haud alienum putamus.

Evolvuntur zoosporæ, macrozoosporæ non minus quam microzoosporæ, in articulis ramulorum supremis, in apicalibus l., ut res sæpissime se habet, in 3—4 infraapicalibus, ex quibus sæpe eodem temporis momento egrediuntur. Ostiolum poriforme, per quod emittuntur zoosporæ, situm est lateraliter ad extremitatem superiorem articuli seu cellulæ matricis, omnino ut in *Cladophora glomerata* (Thuret, Recherches tab. 16.)

Megazoosporæ. In ramulis supremis observantur 2—4 articuli vicini, in quibus megazoosporæ, nondum se moventes, æquale attigerunt evolutio-

nis stadium. In quovis articulo eo modo sunt ordinatæ, ut extremitas earum inferior crassiorque parieti articuli interiori imposita sit, extremitate anteriore seu ciliifera (= rostro) axin articuli aut centrum spectante. Cum penitus evolutæ sunt, per ostiolum simplici serie leniter effluunt, altera alteram sequens, extremitate infera in exitu antica, supera et ciliifera (= rostro) postica; ex articulo autem egressæ, in contrariam partem se vertunt et vivide natare incipiunt. Potest tamen accidere interdum, ut nonnullæ megazoosporæ in articulo se movere incipiant, exitum seu ostiolum quærentes; quod cum invenerint, extremitate ciliifera (seu rostro) antica et extremitate inferiore postica egrediuntur (tab. I f. 7.). Megazoosporarum longitudo maxima $21\frac{2}{3}$ Micro^{mm}, minima $16\frac{2}{3}$ Micro^{mm}; latitudo maxima $13\frac{1}{3}$ Micro^{mm}, minima 10 Micro^{mm}. Forma sunt, cum ex articulo egrediuntur, late ovato-acuminatæ, in motu sensim plus minusve elongatæ granulisque chlorophyllinis majoribus, numerosis et æqualiter usque in rostrum distributis intus vestitæ. Rostrum breve, subtrianguliforme et breviter quasi bicuspidatum. Infra rostrum mox observatur granulum laterale, rubro-fuscum et verticaliter elongatum, quod tamen interdum videtur desiderari. Videre cilia est hac in specie difficillimum. Quamquam enim adminicula nostra optica fuerunt optima, vidimus tamen in una modo megazoospora mortua tria cilia tenuissima et longissima circa basin rostri verticillata quartumque cilium sine ulla cum megazoospora cohærentia (tab. I. f. 8). Inde his in zoosporis quattuor cilia re vera adesse, dubitare non possumus. Finito motu mox, germinare incipiunt sextoque die ostendunt formas, quas depinximus (tab. I f. 9 a. b. c.).

Memoravimus et depinximus in nostra dissertatione (l. cit. p. 21 tab. IV. f 10—15) nonnullas *Confervæ Lini* zoosporas deformatas. Modo analogo deformantur quoque megazoosporæ *Cladophoræ sericeæ* (tab. II f. 22, amplific. 500, et f. 23 et 24, utraque amplific. 1000); omnes, cum vixerunt plus minus vivide se moventes. In f. 23 altera megazoospora alteri est adglutinata, utraque Amœbarum modo formam suam varians.

Microzoosporæ in aliis evolvuntur plantæ speciminibus nec, ut ipsi observavimus, in iisdem ac megazoosporæ, qua tamen in re dijudicanda perfacile est errare. Eodem modo, quo megazoosporæ, in articulo seu cellula matrice sunt ordinatæ effluuntque leniter ex eodem, ut illæ, extremitate ciliifera (seu rostro) postica, extremitate autem inferiore antica. Sed altero quoque modo, quo megazoosporæ, ex articulo exeunt (tab. I f. 10). Maximas microzoosporas cum maximis megazoosporis si comparamus, illas his duplo minores invenimus. Microzoosporarum longitudo maxima 10 Micro^{mm}, minima $6\frac{2}{3}$ Micro^{mm}, latitudo maxima 5 Micro^{mm}, minima $2\frac{1}{2}$ Micro^{mm}. Forma earum, cum ex articulo egrediuntur, est ovato-acuminata, quæ tamen in motu plus

minusve elongatur, ut ipsæ microzoosporæ angustiores evadant; intus cytioplasmate usque in rostrum vestitæ sunt, sed granula viridia minora et quam in megazoosporis duplo pauciora. Rostrum, ut in illis, subbicuspidatum. Cilia duo longissima, valde perspicua, basi rostri infixa. Granulum fusco-rubrum lateraliter in media microzoospora situm est nec, ut in megazoosporis, mox infra rostrum. Ex articulis egressæ, in aqua primum sparguntur, mox vero ad illud vasis latus, quod fenestræ proximum est. se conferunt, ut saltationem illam nuptialem incipiant (tab. I f. 11.).

Microzoosporarum copulatio. Si aquæ microzoosporis plenæ gutta sub microscopio deponitur, magnum, si fors ita tulerit, videbimus microzoosporarum, quæ copulantur, agmen, ex quibus aliæ rostris sunt connexæ (tab. II f. 12 a), aliæ ad mediam partem lateraliter adglutinatæ, aliæ per totam longitudinem coalitæ in zygozoosporam ovato-elongatam, ciliis quattuor coronatam granulumque fusco-rubrum utrinque portantem (tab. II f. 12 b. c.). Est itaque copulatio re vera eadem, quam in *Urospora mirabili* descripsimus. In ipsa copulatione accidit nonnunquam, ut microzoosporæ lateribus, nec rostris, cohæreant (tab. II fig. 13 a. b.). Microzoospora feminea et masculina nunc ejusdem magnitudinis, nunc masculina quam feminea minor. Zygozoospora a microzoosporis, quæ nondum copulatæ sunt, nisi ciliorum numerum et granula observare possis, difficillime discernuntur. Cum moriuntur, utrarumque cilia dissolvuntur l. abjiciuntur, quo facto zoosporas nunquam copulatas resolvi seu diffluere sub microscopio vidimus, zygozoosporas autem in formam sphæricam contrahi, unum alterumve diem persistentes nec ultra. Cujus rei causa in eo certe est quærenda, quod zoosporæ in vitrea lamina vix nisi rarissime germinant. Hujus plantæ microzoosporæ ex articulis egredientes ad latus vasis fenestræ proximum congregantur; facta autem copulatione, vasis fundum petunt et latera, quæ, motu finito, tinctu viridi obducunt. Post nonnullos dies omnes germinarunt, modo lateribus vasis affixæ, modo in aqua libere fluitantes (tab. II f. 14 a, a. et b. b.).

3. ENTEROMORPHA compressa (L.).

Innumeræ hujus speciei formæ, quæ ad oram nostram occidentalem crescunt, certis characteribus definiri non possunt, quo fit, ut eam, cujus zoosporas nunc proponere in animo est, describere inutile videatur. Invenitur forma, quam coluimus, in ipso limite maris, nunc saxis, nunc *Fuco vesiculoso* infixa.

Duplex in Ulvaceis zoosporarum genus invenit Cel. *Thuret* et solita sua sagacitate illustravit (Recherches p. 15.); utriusque in cellulis

matricibus evolutionem tam ornate exposuit, ut hac de re nil novi afferri possit. Quæ vidimus nova, ea breviter enarrabimus.

Megazoosporae cum e cellulis egrediuntur, forma sunt late ovata, quæ tamen in motu elongatur interdum. Intus usque in rostrum sunt viride tinctæ, inferne granula minora viridia includentes. Granulum fusco-rubrum lateraliter in media megazoospora. Cilia earum quattuor et longissima. Maxima longitudo 10 Micro^{mm}, minima $8\frac{1}{3}$ Micro^{mm}; latitudo maxima $6\frac{2}{3}$ Micro^{mm}, minima $5\frac{5}{6}$ Micro^{mm}. Earum germinatio tam cognita, ut eam omissam facere possimus (tab. II f. 15.).

Microzoosporæ et megazoosporæ, sub microscopio cum adsunt commixtæ, ab incauto facile commutari possunt, nam sunt sæpe fere ejusdem longitudinis, et apud utrasque granulum fusco-rubrum in media zoospora lateraliter est positum. Microzoosporæ vero cum e cellulis egrediuntur, sunt longe ovato-attenuatæ intusque usque in granulum fusco-rubrum, raro altius, viride tinctæ, granula minora viridia inferne includentes. Cilia duo longissima. Maxima earum longitudo $8\frac{2}{3}$ Micro^{mm}, minima $6\frac{1}{3}$ Micro^{mm}. Latitudo maxima $3\frac{1}{3}$ Micro^{mm}, minima $2\frac{1}{2}$ Micro^{mm}. In motu suam formam insigniter mutant, nam interdum elongantur ita, ut longitudine sint megazoosporis fere æquales. Sed eodem tempore tam angustæ fiunt, ut sint fere lineares et utrinque sæpe attenuatæ. Hujus formæ microzoosporæ nonnunquam immediatim e cellulis exeunt, tum vero in eodem fere loco remanent trepidantes et vibrantes. Hoc est phænomenon, quod in multis zoosporis antea observavimus (tab. II f. 16.).

Microzoosporarum copulatio, quo apud præcedentes species, eodem quoque apud hanc perficitur modo, ut persequentem microzoosporam, fugientem et utriusque fusionem describere sit inutile. Hoc phænomenon pluries vidimus, nostræque figuræ satis superque illustrant cum ipsam copulationem (tab. II f. 17 a. b.), tum zygozoosporas (tab. II f. 17. c. d.).

Num emoriuntur et dissolvuntur omnes microzoosporæ non copulatæ, quod in *Cladophora sericea* sub microscopio, ut supra memoravimus, nobis visi sumus observare, aut germinare possunt? Hæ sunt quæstiones, quas experientia ducti solvere non possumus, cum illas ab zygozoosporis sejungere et utrasque usque in completam germinationem distinctas observare res sit difficillima, quam nondum exsequi potuimus. Si vero, nulla præterita copulatione, microzoosporæ germinare possint, ipsa copulatio habenda est supervacanea et inutilis. Quas itaque memoravimus *Cladophorae sericeæ* et *Enteromorphæ compressæ*, post copiosam microzoosporarum copulationem, inchoatas germinationes, eæ ducunt omnes, ut opinamur, suam originem e zygozoosporis. Quod si verum sit, zygozoosperæ harum plantarum, motu finito.

forma sphærica indutæ, immediatim germinant nec, membrana cellulari circumdatæ, certi temporis spatium immutatæ jacent, ut alio tempore zoosporas procreent, quod de zygozoosporis *Pandorinæ Mori* tam ornate narravit cel. Pringsheim.

Sed in *Urospora penicilliformi* res se aliter habere videtur. Microzoosporarum in ea copulationem multoties vidimus; jam vero copulatæ, post 6—8 dierum spatium sphæricæ sunt membranaque cellulari circumdatæ nec germinantes (l. cit. p. 9 tab. I f. 9), quod hoc quoque anno vidimus, earum evolutionem observare quamquam nobis concessum non fuit (tab. I f. 6.)*). Sunt igitur zygozoosporæ hujus plantæ, ut ajunt, *quiescentes*, futuro quodam tempore zoosporas edituræ. Nec mirum, cum hæc species amphibia sit planta, ut supra diximus, cui, ut multis algis aquæ dulcis, sporis quiescentibus, opus est. *Cladophora sericea* et *Enteromorpha compressa*, semper aqua perfusæ, megazoosporis et zygozoosporis continue propagari possunt.

4. CLADOPHORA arcta (Dillw.)

Hæc ad oras nostras vulgaris est planta, quam a genere *Cladophorae* segregandam novoque in genere, cui nomen *Spongosiphoniae* (l. cit. p. 19) imposuimus, collocandam esse putavimus. Cujus opinionis causa posita erat in eo, quod hanc apud plantam nobis visi sumus videre duo propagationis genera, alterum antherozoidorum seu spermatozoidorum, alterum "oosporarum". Horum vero utroque in natura iterum iterumque a nobis perscrutato, antherozoida, quæ diximus, sunt microzoosporæ et "oosporæ" nil aliud quam cytioplamata articulorum matricum, quæ per ostiola extrusa, nunc post tempus quoddam solvuntur l. diffluunt, nunc in zoosporas resolvuntur. De utrisque pauca, quæ observavimus, afferre ab re non alienum putamus.

Microzoosporæ (l. cit. p. 18. 3tio. tab. IV fig. 6 et 7) cum fere maturæ sunt, articulus, in quo formantur, quoad formam vix mutatus, intus cytioplasmate fusco vestitus videtur, qui color e granulis fuscis microzoosporarum originem ducit. Microzoosporæ cum penitus sunt maturæ, per ostiolum egrediuntur, nullo indumento gelatinoso circumdatæ (tab. II fig. 19 a). Plerumque vero in articulo dissolvuntur l. paucæ tantummodo egrediuntur, statim morientes et dissoluti. Longe rarius plures l. omnes ex articulis egressæ vivide, ut solent zoosporæ, per campum microscopii natant.

*) In dissertatione nostra depinximus (l. cit. tab. I. f. 6. B.) microzoosporas (perperam, ut scimus, ibidem megazoosporas nominatas) germinantes. Sed hæ germinationes e veris megazoosporis, quas prætervidimus, suam originem certe ducunt et identicæ sunt cum harum germinationibus, quæ l. cit. in tab. III f. 6 b. b. b. depictæ sunt.

Harum forma ovata est, denique in motu ovato-elongata. Intus usque in mediam partem colore viridi tenuissime sunt tinctæ granulisque nonnullis minimis ornatæ. Granulum fuscum in media microzoospora lateralis. Longitudo earum maxima $6\frac{2}{3}$ Micro^{mm} l. 5 Micro^{mm}; latitudo circiter $2\frac{1}{2}$ Micro^{mm}. Cilia longissima plerumque horizontaliter porrecta. Nulla copulatio a nobis observata nec germinatio, ut diximus.

"*Oosporæ*", quas diximus (l. cit. p. 17), e cytioplasmate totius articuli in globum subsphæricum contracto, nunc granulis amylaceis ornato (l. cit. tab. IV fig. 3. b), nunc granulis minutissimis intus vestito, formantur (l. cit. tab. IV fig. 3. c. et fig. 4 b. c.). Aestate præterlapsa semel vidimus talem "oosporam", qualem in dissertatione nostra (l. cit. tab. IV fig. 4 b.) depinximus, per ostiolum dimidia ejus parte extrusa, in zoosporas dissolvi (tab. II fig. 19 b.). Sæpenumero autem observavimus "oosporas" modo ellipticas, modo fere cylindraceas et utrinque rotundatas, ex articulis earum matricibus extrusas, quæ vero denique sunt dissolutæ (tab. II fig. 21). Hanc ob causam "oosporæ", quas diximus, nondum maturarum zoosporarum globi sunt habendæ, quæ tempore non idoneo ex articulis ignoto modo protruduntur, quamquam hujus rei vestigium vidimus nullum apud ceteras Confervas. Zoosporae, in quas dissolvebatur illa "oospora", de qua diximus, veris microozoosporis (tab. II fig. 19 a.) immiscebantur ita, ut utrasque discernere non potuerimus. Quamquam nulla observata est copulatio, harum alterum genus sit forsan masculinum, alterum femineum, conjecturam si facere liceat.

Explicatio Figurarum.

Tab. I.

Fig. 1—6. *Urospora penicilliformis*: Fig. 1 *a.* microzoosporæ liberæ; *b.* microzoosporæ rostris copulatæ. — Fig. 2 *a.* microzoosporæ longitudinaliter copulatæ; *b.* zygozoospora, ciliis quattuor non visibilibus. — Fig. 3 *a.* et *b.* microzoosporæ minores, liberæ. — Fig. 4 *a.* et *b.* microzoosporæ in copulatione. — Fig. 5 *a. b.* et *c.* zygozoosporæ. — Fig. 6 *a.* zygozoosporæ in formam sphæricam contractæ; *b.* cædem in gelatina conservatæ, quo membranæ cellulares conspici possint.

Fig. 7—11 *Cladophora sericea*: Fig 7 particula ramuli, e cujus articulo megazoosporæ exeunt (amplif. 300). — Fig. 8 megazoospora. — Fig. 9 megazoosporæ germinantes: *a.* et *b.* adnatæ; *c.* in fundo vasis libere jacentes. — Fig. 10 particula ramuli, e cujus articulo microzoosporæ egrediuntur (amplif. 300). — Fig. 11 microzoosporæ liberæ.

Tab. II.

Fig. 12—14 *Cladophora sericea:* Fig. 12 *a.* et *b.* microzoosporæ in copulatione; *c.* zygozoospora, ciliis quattuor ornata. — Fig. 13 *a.* et *b.* microzoosporæ lateraliter, nec rostris, copulatæ (deformatio). — Fig. 14 zygozoosporæ (?) post germinationem: *a. a.* adnatæ, *b. b.* in fundo vasis jacentes.

Fig. 15—18 *Enteromorpha compressa* Fig. 15 megazoosporæ. — Fig. 16 microzoosporæ liberæ. Fig. 17 *a. b.* microzoosporæ in copulatione; *c. d.* Zygozoosporæ. Fig. 18 *a. b. c.* zygozoosporæ post germinationem.

Fig. 19—21 *Cladophora arcta:* Fig. 19 particula ramuli, e cujus articulis microzoosporæ egrediuntur (amplific. 75). Fig. 20 microzoosporæ liberæ; fig. 21 cytioplasma articuli cujusdam per ostiolum extrusum.

Fig. 22—24 *Cladophora sericea:* megazoosporarum deformatio. Fig. 22 *a. b. c. d. e.* solitariæ sunt megazoosporæ; in f. duæ, ut videtur, confusæ (500 amplific.). Fig. 23 duæ megazoosporæ, altera alteri adnata. — Fig. 24 megazoospora solitaria.

Obs. Omnes figuræ, quarum amplificatio intra parenthesin memorata non est, millies sunt amplificatæ. Millemicrometrum = 0,001^{mm}.

Auctor del.

Fig. 1–6 Urospora penicilliformis. Fig. 7–11 Cladophora sericea.

Auctor del.

Fig. 12–14 Cladophora sericea. Fig. 15–18 Enteromorpha compressa.
Fig. 19–21 Cladophora arcta. Fig. 22–24 Cladophora sericea.

SUR LA RÉFLEXION DE LA LUMIÈRE

A LA SURFACE DES CORPS ISOTROPES.

PAR

G. LUNDQUIST.

(Présenté a la Société Royale des Sciences d'Upsal le 6 Décembre 1873.)

UPSAL,
ED. BERLING, IMPRIMEUR DE L'UNIVERSITÉ.
1874.

Depuis les travaux de Fresnel sur la réflexion de la lumière, ce phénomène a été l'objet de nombreuses et importantes études tant mathématiques qu'expérimentales. Cependant, il s'en faut encore de beaucoup que la connaissance de ce phénomène soit complète, et que la théorie en soit parfaitement satisfaisante. Pour atteindre ce but, il sera nécessaire, en particulier, d'étudier avec soin les corps qui présentent le phénomène remarquable de la coloration superficielle. C'est pourquoi j'ai entrepris sur ce sujet quelques expériences, dont les premiers résultats se trouvent dans ce mémoire. Avant de les faire connaître, je me propose d'exposer les résultats principaux auxquels on est arrivé dans les recherches théoriques sur la réflexion.

I. Les théories de la réflexion sur des corps parfaitement transparents.

En 1832, M. Airy [1]) découvrit la propriété qu'a le diamant de transformer, par réflexion, en polarisation elliptique, la polarisation rectiligne d'un faisceau de lumière incidente, lorsque l'angle d'incidence diffère peu de l'angle de polarisation. Une explication de ce phénomène a été donnée dans les théories générales établies par Green [2]) et par Cauchy [3]), quelques années plus tard.

Selon la théorie de Green, la lumière plane polarisée se polarise elliptiquement par réflexion chez tous les corps, et cela d'autant plus que l'indice de réfraction est plus grand, cette constante étant la seule dont dépendent l'intensité et l'état de la lumière réfléchie et réfractée. Dans les

[1]) *Phil. Mag.* (3) II., p. 20, (1832).

[2]) *Mathematical Papers*, London 1871, p. 291, (1837).

[3]) *Compt. Rend.* VIII, p. 7 et suiv., p. 985, (1839); XXVIII, p. 2, 121 & (1849); *Exerc. d'Anal. et de Phys. mathém.* I, p. 133 &, (1840).

formules de Cauchy, au contraire, il entre, outre l'indice de réfraction, une autre constante désignée depuis sous le nom de coefficient d'ellipticité. Ainsi, d'après les vues de ce dernier, la lumière ne se polarise pas avec nécessité elliptiquement par la réflexion, parce qu'il suffit de réduire à zéro la nouvelle constante pour obtenir les formules de Fresnel. Les deux savants admettent que l'éther passe très-brusquement de l'état où il se trouve dans un milieu, à l'état où il se trouve dans un autre, et que les vibrations des rayons lumineux font naître des vibrations longitudinales à la surface réfléchissante; mais ils diffèrent en ce qui concerne les hypothèses qu'il faut faire relativement à la propagation de ces dernières vibrations.

Dale [1]) a trouvé que non-seulement le diamant, mais encore un certain nombre d'autres corps transparents, dont l'indice de réfraction est assez grand, possèdent la propriété de produire par réflexion la polarisation elliptique. Mais c'est M Jamin [2]), qui le premier a reconnu que ce genre de polarisation est généralement produit par la réflexion de la lumière à la surface des corps transparents. Il a aussi mesuré avec exactitude les effets de la réflexion, et en comparant ses résultats avec ceux déduits des formules de Cauchy [3]), il a trouvé un accord parfaite entre eux. Au seul égard de la relation qui, d'après cette théorie, doit exister entre les valeurs relatives des coefficients d'ellipticité, l'expérience n'a pas justifié les prédictions théoriques. Parmi les résultats les plus remarquables de ses expériences, il faut signaler ceux-ci: le coefficient d'ellipticité, toujours très-petit, augmente en général avec l'indice de réfraction, comme l'indique la théorie de Green, sans être uniquement dépendant de cette seule constante; et ce coefficient prend des valeurs même négatives pour certains corps, qui réfractent peu la lumière. Ces faits, on le remarquera, n'étaient pas prévus par la théorie de Cauchy.

A ces recherches sont venus depuis s'ajouter le travail de M. Haughton [4]) et celui de M. Quincke [5]). Le premier a fait voir que les formules

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[1]) *On Elliptic Polarization.* 16th Rep. of Brit. Assoc. 1846.

[2]) *Annal. de Chim. et de Phys.* (3) XXIX, p. 263, (1850); XXXI, p. 165, (1851).

[3]) La démonstration des formules dont s'est servi M. Jamin pour cette comparaison, n'a pas été donnée en detail par Cauchy, mais cette lacune a été comblée par d'autres savants. Beer, *Pogg. Ann.* XCI, p. 467, (1854); — v. Ettingshausen. *Wien. Sitz. Ber.* XVIII, p. 369, (1855); — Eisenlohr, *Pogg. Ann.* CIV, p. 346, (1858); — Briot, *Journ. de Liouv.* (2) XI, p. 305, (1866).

[4]) *Phil. Mag.* (4) VI, p. 81, (1853); (4) VIII, p. 507, (1854); — *Phil. Trans.* 1863, p. 81.

[5]) *Pogg. Ann.* CXXVIII, p. 355. (1866).

de Green ne peuvent reproduire les valeurs fournies par les expériences, l'ellipticité calculée étant toujours plus grande que celle observée, et il en a conclu qu'il faut nécessairement faire entrer deux constantes dans les formules théoriques. M. Quincke a soumis les formules de Cauchy à des épreuves variées, et il a trouvé, en général, comme l'a fait M. Jamin, une concordance très-satisfaisante, sauf en ce qui concerne la relation entre les coefficients d'ellipticité, mentionnée ci-dessus Partant de ce fait et d'autres observations plus directes [1]), il arrive à la conclusion, qu'il faut admettre que l'épaisseur de la couche intermédiaire, dans laquelle a lieu le passage de l'éther d'un corps à celui d'un autre, n'est pas une quantité qu'il soit permis de négliger.

Au reste, cette assertion avait déjà été énoncée par M. Lorenz [2]), qui veut expliquer par là, même dans sa totalité, l'ellipticité produite par la réflexion sur des corps transparents. Ce savant a fait, en outre, des objections bien fondées contre les suppositions de Cauchy par rapport aux vibrations longitudinales [3]). A ces objections M. Strutt [4]) en a ajouté encore d'autres. Il défend aussi la théorie de Green, dont il a modifié les formules de manière à reproduire les observations.

Récemment, M. Potier [5]) s'est rangé à l'opinion de M. Lorenz, et il a exposé plusieurs expériences, qui prouvent au moins que l'épaisseur de la couche de transition d'un corps à un autre ne peut pas être regardée comme une quantité infiniment petite.

Nous allons maintenant examiner plus en détail les différentes théories de la réflexion à la surface des corps transparents.

Remarquons d'abord que, dans ces théories, on admet ordinairement: en premier lieu, que les déplacements des molécules de l'éther sont des quantités infiniment petites du premier ordre, et que l'action réciproque de ces molécules n'est sensible qu'à de très-petites distances; ensuite, qu'on peut négliger la dispersion, et enfin, que la matière pondérable ne participe pas au mouvement de l'éther. Supposons d'abord, en outre, qu'il soit permis de regarder comme discontinu dans le calcul le passage de l'éther compris dans un milieu, à celui compris dans un autre.

[1]) *Pogg. Ann.* CXXIX, p. 177, (1866); CXXXII, p. 321 et 561, (1867); CXLII, p. 177 et 380, (1871).

[2]) *Pogg. Ann.* CXL, p. 460, (1860); CXVIII, p. 111, (1863).

[3]) *Pogg Ann.* CXIV, p. 249, (1861).

[4]) *Phil. Mag.* (4) XLII, p. 95, (1871).

[5]) *Compt. Rend.* LXXV, p. 617 et 674, (1872); — *Pogg. Ann.* CXLVIII, p. 650, (1873).

Le principe fondamental de la théorie de Green est celui qu'on appelle aujourd'hui le principe de *la conservation de l'énergie.* Il déduit de là, par des développements analytiques, tant les équations générales de mouvement que les conditions relatives à la surface, sans faire aucune hypothèse sur la nature de l'influence réciproque des différentes parties de l'éther. En effet, en opérant ainsi, il obtient, pour le premier milieu (celui d'où vient la lumière), les équations de mouvement suivantes:

$$\left.\begin{aligned}
\varrho\frac{d^2\xi}{dt^2} &= (L+M)\frac{d\upsilon}{dx}+M\left(\frac{d^2\xi}{dx^2}+\frac{d^2\xi}{dy^2}+\frac{d^2\xi}{dz^2}\right),\\
\varrho\frac{d^2\eta}{dt^2} &= (L+M)\frac{d\upsilon}{dy}+M\left(\frac{d^2\eta}{dx^2}+\frac{d^2\eta}{dy^2}+\frac{d^2\eta}{dz^2}\right),\\
\varrho\frac{d^2\zeta}{dt^2} &= (L+M)\frac{d\upsilon}{dz}+M\left(\frac{d^2\zeta}{dx^2}+\frac{d^2\zeta}{dy^2}+\frac{d^2\zeta}{dz^2}\right),
\end{aligned}\right\} \quad \ldots\ldots (1)$$

ξ, η, ζ étant les projections du déplacement sur les axes des coordonnées, ϱ la densité de l'éther, L et M ses deux coefficients d'élasticité, et υ sa dilatation cubique, donnée par la formule

$$\upsilon=\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}. \quad \ldots\ldots\ldots\ldots (2)$$

Pareillement, on obtient les équations correspondantes pour le second milieu, si l'on remplace ξ, η, ζ, ϱ etc. par ξ', η', ζ', ϱ' etc.

Prenons lex axes des y et des z dans le plan de séparation, et l'axe positif des x dans le second milieu. Alors les équations de condition relatives à la surface, où l'on a $x=0$, deviennent, dans cet ordre d'idées, de la forme

$$\left.\begin{aligned}
\xi=\xi' \quad \eta=\eta' \quad \zeta=\zeta'&\\
L\upsilon+2M\frac{d\xi}{dx} &= L'\upsilon'+2M'\frac{d\xi'}{dx},\\
M\left(\frac{d\xi}{dy}+\frac{d\eta}{dx}\right) &= M'\left(\frac{d\xi'}{dy}+\frac{d\eta'}{dx}\right),\\
M\left(\frac{d\zeta}{dx}+\frac{d\xi}{dz}\right) &= M'\left(\frac{d\zeta'}{dx}+\frac{d\xi'}{dz}\right),
\end{aligned}\right\} \quad \ldots\ldots (3)$$

C'est précisément le résultat auquel on arrive en partant de la théorie de l'élasticité [1]). En effet, supposons que l'éther des deux milieux soit légèrement déformé par l'action de forces extérieures, dont les composantes par unité de masse sont représentées par X, Y, Z; si l'on désigne par N_1, N_2, N_3 les forces élastiques normales et T_1, T_2, T_3 les forces élastiques tangentielles, les équations d'équilibre de l'éther du premier milieu peuvent s'écrire [2])

$$\left.\begin{aligned} \frac{dN_1}{dx} + \frac{dT_3}{dy} + \frac{dT_2}{dz} + \varrho X = 0, \\ \frac{dT_3}{dx} + \frac{dN_2}{dy} + \frac{dT_1}{dz} + \varrho Y = 0, \\ \frac{dT_2}{dx} + \frac{dT_1}{dy} + \frac{dN_3}{dz} + \varrho Z = 0, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4)$$

où l'on a

$$\left.\begin{aligned} N_1 = Lv + 2M\frac{d\xi}{dx}, \quad T_1 = M\left(\frac{d\eta}{dz} + \frac{d\zeta}{dy}\right), \\ N_2 = Lv + 2M\frac{d\eta}{dy}, \quad T_2 = M\left(\frac{d\zeta}{dx} + \frac{d\xi}{dz}\right), \\ N_3 = Lv + 2M\frac{d\zeta}{dz}, \quad T_3 = M\left(\frac{d\xi}{dy} + \frac{d\eta}{dx}\right). \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (5)$$

On obtient des équations analogues pour le second milieu. Les conditions relatives à la surface sont

$$\left.\begin{aligned} \Big/^{x=0}\ \xi = \xi', \quad \eta = \eta', \quad \zeta = \zeta', \\ \Big/^{x=0}\ N_1 = N_1', \ T_3 = T_3', \ T_2 = T_2', \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (6)$$

où les trois dernières équations expriment le principe de *l'égalité entre les pressions*.

La légimité de l'application de ce principe, quand il s'agit de l'éther, a été contestée par Cauchy [3]). En effet, supposons, pour fixer les idées, que le premier milieu soit de l'éther libre, le second de l'éther renfermé dans un corps. Alors les pressions supportées par un élément de la surface de

[1]) Comparez: v. Lang, *Wien. Sitz. Ber.* XLIV 2, p. 147, (1861).
[2]) Lamé, *Théor. math. de l'élast.* 2:e Ed., Paris 1866, p. 65.
[3]) *Compt. Rend.* XXVIII, p. 27 et 60, (1849).

séparation seraient, selon lui, exercées d'un côté par l'éther libre, de l'autre côté non-seulement par l'éther, mais aussi par la matière pondérable, et, par conséquent, il ne serait pas permis d'égaler entre elles les pressions exercées par les masses d'éther seules. Cependant, cette objection ne peut pas être d'une très-grande valeur. En effet, M. Ångström [1]) a montré, qu'il faut nécessairement traiter séparément les milieux qui se pénètrent mutuellement, lorsque ils sont aussi hétérogènes que l'éther et les parties solides d'un corps. M. de Saint-Venant [2]) a pareillement signalé l'impossibilité de concilier l'assertion de Cauchy avec le principe de la continuité du mouvement établi par ce dernier, dont nous parlerons plus bas. On peut aussi se convaincre de la vérité des équations (6) en observant que toute la masse d'éther assujettie à l'action de la matière pondérable d'un corps, doit être regardée comme appartenant à ce même corps, d'où il suit avec nécessité qu'il n'est pas correct de ce figurer une pression directe exercée par les molécules pondérables d'un corps, sur l'éther compris dans un autre.

Cela posé, supposons, qu'il n'agisse pas des forces extérieures, mais que l'éther soit agité par des mouvements vibratoires s'y propageant; il suffit, d'après le principe de d'Alembert, pour obtenir les équations de mouvement, de remplacer les

$$X,\ Y,\ Z,\ \text{par} -\frac{d^2\xi}{dt^2},\ -\frac{d^2\eta}{dt^2},\ -\frac{d^2\zeta}{dt^2},$$

par où l'on arrivera, en ayant égard aux expressions (2) et (5), aux équations (1). La forme des conditions relatives à la surface (6), qui sont identiques avec les équations (3), restera évidemment la même. Cette méthode laisse aussi indéterminée l'influence réciproque des différentes parties de l'éther [3]).

Pour simplifier le calcul, Green a fait la supposition, que l'on a

$$L = L' \text{ et } M = M',\ ^4) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

en s'appuyant sur des analogies présentées par les gaz. Maintenant, en différentiant les trois premières des équations (3) par rapport à y ou à z, opération évidemment permise, on réduit les conditions à la surface, en vertu des relations (7), à la forme

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[1]) *Nov. Act. Soc. Upsal.* (2) XIV, p. 312, Mém. sur . . . . . cristaux à trois ax. obl. p. 22, (1849).

[2]) *Ann. de Chim. et de Phys.* (4) XXV, p. 335, (1872).

[3]) Lamé, *Théor. math. de l'élast* p. 35.

[4]) M. Strutt a démontré, dans le mémoire déjà cité, qu'on peut obtenir les formules de Green sans poser $L = L'$.

$$\left.\begin{array}{l} \text{pour } x=0 \quad \xi=\xi' \quad \eta=\eta' \quad \zeta=\zeta' \\ \text{pour } x=0 \quad \dfrac{d\xi}{dx}=\dfrac{d\xi'}{dx}, \quad \dfrac{d\eta}{dx}=\dfrac{d\eta'}{dx}, \quad \dfrac{d\zeta}{dx}=\dfrac{d\zeta'}{dx}. \end{array}\right\} \quad \ldots\ldots\ldots \quad (8)$$

Cauchy, au contraire, considérait l'éther comme un milieu composé de molécules sollicitées par des forces d'attraction ou de répulsion mutuelle. Partant de cette hypothèse, on arrivera à des équations de mouvement, qui deviennent identiques avec les équations (1), pourvu que l'on remplace respectivement

$$\frac{L+M}{\varrho} \text{ et } \frac{M}{\varrho} \text{ par } 2h \text{ et } g+h,$$

et de même

$$\frac{L'+M'}{\varrho'} \text{ et } \frac{M'}{\varrho'} \text{ par } 2h' \text{ et } g'+h',$$

les g et les h étant des constantes, données par des expressions de la forme

$$g=\frac{1}{2.3}\sum mr^2 f(r), \; h=\frac{1}{2.3.5}\sum mr^3 f'(r), \quad \ldots\ldots \quad (9)$$

m étant la masse d'une molécule d'éther quelconque, r sa distance à la molécule, dont on considère le mouvement, et $rf(r)$ la loi des forces agissantes entre les molécules [1]).

Pour obtenir les conditions à la surface, Cauchy a établi son principe de *la continuité du mouvement* à l'aide d'une méthode analytique, dont il n'est pas facile de contester l'exactitude, [2]) et qui peut être ramenée, comme l'a indiqué M. v. Ettingshausen, [3]) à celle de la variation des constantes arbitraires. Suivant ce principe, les conditions relatives à la surface deviennent précisément les équations (8), d'où l'on peut conclure à la légitimité de la supposition de Green, exprimée par les relations (7). Maintenant, pour que les équations de Cauchy soient en accord avec les relations déduites de la théorie de l'élasticité et du principe de l'égalité des pressions dont nous avons montré la justesse, il faut que les g et les h vérifient les expressions

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[1]) Cauchy, *Compt. Rend.* VIII, p. 985, (1839); — Briot, *Théor. math. de la lum.* Paris 1864, p. 43.

[2]) *Compt. Rend.* VIII, p. 374, 432, 459, (1839); X, p. 347, (1840).

[3]) *Pogg. Ann.* L, p. 409, (1840).

$$\frac{g}{g'} = \frac{h}{h'} = \frac{\varrho'}{\varrho}, \quad \ldots \ldots \ldots \quad (10)$$

en vertu des relations (7).

Il importe d'observer qu'on arrivera à la première des équations (10), même en laissant à part le principe de l'égalité des pressions, comme le fait Cauchy, pourvu que l'on fasse la supposition, très-souvent admise, que la loi de l'action exercée entre deux molécules d'éther est la même dans tous les milieux et qu'elle s'exprime par l'équation

$$rf(r) = \frac{c}{r^n},$$

la constante c étant positive ou négative suivant que la force est attractive ou répulsive. En effet, les deux constantes g et h ayant alors les valeurs

$$g = \frac{1}{2.3} \sum \frac{mc}{r^{n-1}}, \quad h = -\frac{n+1}{2.3.5} \sum \frac{mc}{r^{n-1}},$$

$$\text{on a } h = -\frac{n+1}{5} g \quad \text{et de même} \quad h' = -\frac{n+1}{5} g', \quad \ldots \quad (11)$$

d'où l'on tire

$$\frac{g}{g'} = \frac{h}{h'}.$$

On peut satisfaire aux équations de mouvement (1) par les intégrales simples

$$\xi = Ae^{ux+vy+wz-st}, \quad \eta = Be^{ux+vy+wz-st}, \quad \zeta = Ce^{ux+vy+wz-st},$$

si A, B, C, u, v, w, s sont des constantes assujetties à vérifier les relations

$$\left.\begin{aligned} (\varrho s^2 - Mk^2) A - (L+M) u (uA + vB + wC) &= 0, \\ (\varrho s^2 - Mk^2) B - (L+M) v (uA + vB + wC) &= 0, \\ (\varrho s^2 - Mk^2) C - (L+M) w (uA + vB + wC) &= 0, \end{aligned}\right\}$$

où l'on a

$$k^2 = u^2 + v^2 + w^2. \quad \ldots \ldots \ldots \quad (12)$$

Ces équations peuvent être vérifiées des deux manières suivantes: on doit avoir

$$\varrho s^2 = Mk^2, \quad uA + vB + wC = 0, \quad \ldots \ldots \quad (13)$$

ou bien

$$\varrho s^2=(L+2M)k^2,\ \frac{A}{u}=\frac{B}{v}=\frac{C}{w},\ \ldots\ldots\ (14)$$

Le premier mode donne les vibrations transversales, le second les vibrations longitudinales. Pour le second milieu, nous obtenons des équations analogues.

En portant les ξ, η, ζ dans les équations relatives à la surface (8), qui peuvent s'écrire

$$\left.\begin{aligned} &\text{pour } x=0\ \ \Sigma\xi=\Sigma\xi',\ \Sigma\eta=\Sigma\eta',\ \Sigma\zeta=\Sigma\zeta',\\ &\text{pour } x=0\ \ \Sigma\frac{d\xi}{dx}=\Sigma\frac{d\xi'}{dx},\ \Sigma\frac{d\eta}{dx}=\Sigma\frac{d\eta'}{dx},\ \Sigma\frac{d\zeta}{dx}=\Sigma\frac{d\zeta'}{dx}, \end{aligned}\right\}\ \ldots\ldots\ (15)$$

ces relations deviennent toutes de la forme

$$\Sigma\, P e^{vy+wz-st}=0,$$

où les P sont des constantes. En remarquant que ces équations doivent être vérifiées par des valeurs quelconques de y, z, t, on trouvera facilement au moyen de différentiations successives par rapport à ces variables que v, w, s ont les mêmes valeurs dans tous les termes.

Supposons maintenant le plan xoy perpendiculaire à l'onde plane incidente; si l'on appelle α l'angle d'incidence, λ la longueur de l'onde, τ la durée de la vibration, les projections de la vibration incidente sur les axes des coordonnées peuvent être représentées par des expressions de la forme

$$\mathrm{P}\cos 2\pi\left(\frac{x\cos\alpha+y\sin\alpha}{\lambda}-\frac{t}{\tau}+\frac{\varpi}{2\pi}\right),$$

ou

$$\mathrm{P}\cos(\mathrm{u}x+\mathrm{v}y-\mathrm{s}t+\varpi),$$

P et ϖ étant des constantes réelles; par suite, ces projections sont les parties réelles des expressions

$$\xi=Ae^{i(ux+vy-st)},\ \eta=Be^{i(ux+vy-st)},\ \zeta=Ce^{i(ux+vy-st)},\ \ldots\ (16)$$

A, B, C étant des constantes complexes. En observant que le coefficient de x dans les expressions de la vibration du rayon réfléchi doit être négatif, parce que ce rayon se propage en s'éloignant de la surface de séparation, on obtiendra, de même, pour cette vibration transversale

$$\xi_1=A_1e^{i(-u_1x+vy-st)},\ \eta_1=B_1e^{i(-u_1x+vy-st)},\ \zeta_1=C_1e^{i(-u_1x+vy-st)},\ \ldots\ (17)$$

et pour la vibration transversale du rayon réfracté

$$\xi' = A' e^{i(u'x+vy-st)}, \quad \eta' = B' e^{i(u'x+vy-st)}, \quad \zeta' = C' e^{i(u'x+vy-st)} \quad \ldots \quad (18)$$

Si l'on substitue ces valeurs dans les équations (15) on trouvera facilement, en observant qu'il faut satisfaire aussi aux conditions (13) et (14), qu'il y a plus d'équations que d'inconnues. C'est cette circonstance qui a nécessité la supposition, faite tant par Green que par Cauchy, que les vibrations lumineuses font naître à la surface de séparation deux ondes à des vibrations longitudinales, l'une se propageant dans le premier milieu, l'autre dans le second. Posons pour la vibration longitudinale du premier milieu

$$\xi_{,,} = A_{,,} e^{i(-u_{,,}x+vy-st)}, \quad \eta_{,,} = B_{,,} e^{i(-u_{,,}x+vy-st)}, \quad \zeta_{,,} = 0, \quad \ldots \quad (19)$$

et pour la seconde

$$\xi'' = A'' e^{i(u''x+vy-st)}, \quad \eta'' = B'' e^{i(u''x+vy-st)}, \quad \zeta'' = 0 \ldots \quad (20)$$

Portons toutes ces valeurs, (16)....(20), dans les équations (15), nous obtenons, en remarquant qu'on a $u_, = u$ en vertu de la formule (12), les équations suivantes

$$\left.\begin{aligned} A + A_, + A_{,,} &= A' + A'', \quad (A - A_,)u - A_{,,}u_{,,} = A'u' + A''u'', \\ B + B_, + B_{,,} &= B' + B'', \quad (B - B_,)u - B_{,,}u_{,,} = B'u' + B''u'', \\ C + C_, &= C', \qquad\qquad (C - C_,)u = C'u', \end{aligned}\right\} \quad (21)$$

auxquelles on joindra les relations

$$\left.\begin{aligned} Au + Bv &= 0, \\ -A_,u + B_,v &= 0, \end{aligned}\right\}, \quad s^2 = \omega^2 k^2, \quad k^2 = u^2 + v^2, \quad \ldots \quad (22)$$

$$\frac{A_{,,}}{-u_{,,}} = \frac{B_{,,}}{v}, \quad s^2 = \omega_{,,}{}^2 k_{,,}{}^2, \quad k_{,,}{}^2 = u_{,,}{}^2 + v^2, \quad \ldots \quad (23)$$

$$A'u' + B'v = 0, \quad s^2 = \omega'^2 k'^2, \quad k'^2 = u'^2 + v^2, \quad \ldots \quad (24)$$

$$\frac{A''}{u''} = \frac{B''}{v}, \quad s^2 = \omega''^2 k''^2, \quad k''^2 = u''^2 + v^2, \quad \ldots \quad (25)$$

$$s = \frac{2\pi}{\tau}, \quad k = \frac{2\pi}{\lambda}, \quad k' = \frac{2\pi}{\lambda'} \quad \ldots \quad (26)$$

$$u = k\cos\alpha, \quad u' = k'\cos\alpha', \quad v = k\sin\alpha = k'\sin\alpha', \quad \ldots \quad (27)$$

$$u_{,,} = k\sqrt{\frac{\omega^2}{\omega_{,}^2} - \sin^2\alpha}\,,\quad u'' = k\sqrt{\frac{\omega^2}{\omega_{,}^2} - \sin^2\alpha}\,. \quad . \quad . \quad . \quad . \quad (28)$$

Des équations (26) et (27) on tire

$$\frac{\sin\alpha}{\sin\alpha'} = \frac{k'}{k} = \frac{\lambda}{\lambda'} = \mu = \text{l'indice de réfraction.}$$

Les quantités ω sont les vitesses de propagation respectives, car on a, par exemple,

$$\omega = \frac{s}{k} = \frac{\lambda}{\tau}.$$

Elles sont données par les relations

$$\omega^2 = \frac{M}{\varrho}\,,\quad \omega'^2 = \frac{M}{\varrho'}\,,\quad \omega_{,,}^{\,2} = \frac{L+2M}{\varrho}\,,\quad \omega''^2 = \frac{L+2M}{\varrho'} \quad . \quad . \quad . \quad (29)$$

d'où l'on déduit

$$\frac{\omega}{\omega'} = \frac{\omega_{,,}}{\omega''} = \frac{k'}{k} = \mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Les équations de Cauchy, d'autre part, donnent

$$\omega^2 = g + h\,,\quad \omega'^2 = g' + h'\,,\quad \omega_{,,}^{\,2} = g + 3h\,,\quad \omega''^2 = g' + 3h'\,, \quad . \quad . \quad (31)$$

d'où l'on peut tirer, de même, l'équation (30), en ayant égard à la relation

$$\frac{g}{g'} = \frac{h}{h'}.$$

Des équations (21) . . . . (25) on déduit les valeurs des inconnues:

$$C_{,} = C\frac{u-u'}{u+u'} = -C\frac{\sin(\alpha-\alpha')}{\sin(\alpha+\alpha')}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$B_{,} = B\frac{u-u'}{u+u'}\cdot\frac{(uu'-v^2)(u_{,,}u''+v^2)+v^2(u+u')(u_{,,}-u'')}{(uu'+v^2)(u_{,,}u''+v^2)+v^2(u-u')(u_{,,}-u'')}, \quad . \quad (33)$$

et, pareillement, les valeurs de $A_{,}$, A', B', C' etc.

Supposons que la vibration du rayon incident, polarisée dans l'azimut ϑ, soit représentée par

$$s = E\cos(ux + vy - st);$$

on peut décomposer cette vibration en deux: l'une perpendiculaire au plan d'incidence, l'autre située dans ce plan. L'amplitude de la première sera donc

$$E\cos\vartheta = C,$$

et de la seconde

$$E\sin\vartheta = \frac{B}{\cos\alpha}.$$

Si, maintenant, les quantités qui entrent dans l'équation (33) étaient réelles, les amplitudes correspondantes de la vibration du rayon réfléchi seraient respectivement:

$$C_1 \text{ et } \frac{B_1}{\cos\alpha},$$

et leurs équations

$$\zeta_1 = C_1\cos(-\mathrm{u}x + \mathrm{v}y - \mathrm{s}t),$$

$$\varphi_1 = -\frac{B_1}{\cos\alpha}\cos(-\mathrm{u}x + \mathrm{v}y - \mathrm{s}t);$$

par conséquent, il n'y aurait pas de différence de phase. Mais, ce résultat étant en contradiction avec l'expérience, il faut revenir aux expressions des composantes des vibrations longitudinales pour y introduire les changements nécessaires, c'est-à-dire pour y remplacer les quantités réelles $u_{,,}$ et u'' par des quantités complexes. Or il résulte immédiatement de la forme des équations (23') et (25'), que la partie réelle de ces quantités se réduit à zéro. Ainsi, on aura, au lieu des expressions (19) et (20), les formules

$$\xi_{,,} = A_{,,}e^{\mathrm{U}_{,,}x + i(\mathrm{v}y - \mathrm{s}t)},\quad \eta_{,,} = B_{,,}e^{\mathrm{U}_{,,}x + i(\mathrm{v}y - \mathrm{s}t)},\quad \zeta_{,,} = 0, \quad . \quad . \quad . \quad . \quad (34)$$

$$\xi'' = A''e^{-\mathrm{U}''x + i(\mathrm{v}y - \mathrm{s}t)},\quad \eta'' = B''e^{-\mathrm{U}''x + i(\mathrm{v}y - \mathrm{s}t)},\quad \zeta'' = 0, \quad . \quad . \quad . \quad (35)$$

auxquelles on joindra les conditions

$$\frac{A_{,,}}{-\mathrm{U}_{,,}i} = \frac{B_{,,}}{\mathrm{v}},\quad \frac{A''}{\mathrm{U}''i} = \frac{B''}{\mathrm{v}},\quad \mathrm{k}_{,,}^2 = -\mathrm{U}_{,,}^2 + \mathrm{v}^2,\quad \mathrm{k}''^2 = -\mathrm{U}''^2 + \mathrm{v}^2 \quad . \quad . \quad . \quad (36)$$

$\mathrm{U}_{,,}$ et U'' étant deux quantités réelles données par les formules

$$\mathrm{U}'' = \mathrm{k}\sqrt{\sin^2\alpha - \frac{\omega^2}{\omega_{,,}^2}}\quad \mathrm{U}'' = \mathrm{k}\sqrt{\sin^2\alpha - \frac{\omega^2}{\omega''^2}}$$

Ces nouvelles expressions des vibrations longitudinales doivent être introduites dans les conditions relatives à la surface (15), pour en déduire les valeurs nouvelles des inconnues. Mais on peut évidemment déduire ces

mêmes valeurs d'une manière plus simple en remplaçant $u_{,,}$ et u'' par $U_{,,}i$ et $U''i$ dans les expressions des inconnues déjà obtenues. Ainsi, en posant, pour abréger,

$$\varkappa = \frac{v(U_{,,} - U'')}{v^2 - U_{,,}U''}, \quad \ldots\ldots\ldots\ldots \quad (37)$$

on aura

$$B_{,} = B\,\frac{u - u'}{u + u'}\cdot\frac{(uu' - v^2) + i\varkappa v(u + u')}{(uu' + v^2) + i\varkappa v(u - u')}, \quad \ldots\ldots \quad (38)$$

et, pareillement, les valeurs de $A_{,}$, A', B', etc.

Si l'on pose

$$\operatorname{tang}\delta_1 = \varkappa \operatorname{tang}(\alpha + \alpha'),\ \operatorname{tang}\delta_2 = \varkappa \operatorname{tang}(\alpha - \alpha') \quad \ldots \quad (39)$$

et, pour abréger, $\delta = \delta_1 + \delta_2$,

la formule (33) devient $B_{,} = -B\dfrac{\sin\delta_2}{\sin\delta_1}\,e^{\delta i}$,

et, par suite, la vibration réfléchie a pour composantes les parties réelles des formules

$$\zeta_{,} = -E\cos\vartheta\,\frac{\sin(\alpha - \alpha')}{\sin(\alpha + \alpha')}\,e^{i(-ux + vy - st)},$$

$$\varphi_{,} = -E\sin\vartheta\,\frac{\sin\delta_2}{\sin\delta_1}\,e^{i(-ux + vy - st + \delta)},$$

c'est-à-dire

$$\zeta_{,} = -E\cos\vartheta\,\frac{\sin(\alpha - \alpha')}{\sin(\alpha + \alpha')}\cos(-ux + vy - st), \quad \ldots\ldots \quad (40)$$

$$\varphi_{,} = -E\sin\vartheta\,\frac{\sin\delta_2}{\sin\delta_1}\cos(-ux + vy - st + \delta)\ ^{1}). \quad \ldots\ldots \quad (41)$$

Ainsi, la différence de phase est δ, et pour déterminer le rapport m des amplitudes des composantes on a

$$m = \operatorname{tang}\vartheta_{,} = \operatorname{tang}\vartheta\cdot\frac{\sin(\alpha + \alpha')}{\sin(\alpha - \alpha')}\cdot\frac{\sin\delta_2}{\sin\delta_1} \quad \ldots\ldots \quad (42)$$

On observera que la formule (37) peut s'écrire

$$\varkappa = \frac{v(U_{,,}^2 - U''^2)}{(v^2 - U_{,,}^2)U'' + (v^2 - U''^2)U_{,,}} = \frac{\operatorname{Sin}\alpha\left(\dfrac{\omega^2}{\omega''^2} - \dfrac{\omega^2}{\omega_{,,}^2}\right)}{\dfrac{\omega^2}{\omega_{,,}^2}\sqrt{\sin^2\alpha - \dfrac{\omega^2}{\omega''^2}} + \dfrac{\omega^2}{\omega''^2}\sqrt{\sin^2\alpha - \dfrac{\omega^2}{\omega_{,,}^2}}} \quad \ldots \quad (43)$$

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[1]) Eisenlohr, *Pogg. Ann.* CIV, p. 355; — Briot, *Journ. de Liouv.* (2) XI, p. 321.

Jusqu'à ce point les formules déduites sont les mêmes dans les différentes théories; mais celles-ci diffèrent en ce qui concerne les suppositions à établir par rapport aux quantités $\omega_{,,}^{2}$ et ω''^{2}. En effet, les coefficients d'extinction $U_{,,}$ et U'' doivent être réels pour toutes les valeurs de α, à l'exception peut-être de valeurs très-petites, et cela peut se faire de deux manières: on peut supposer les quantités $\omega_{,,}^{2}$ et ω''^{2} ou négatives ou positives et très-grandes.

1:o) $\omega_{,,}^{2}$ et $\omega''^{2} < 0$; c'est la supposition de Cauchy. Si l'on pose

$$\frac{\omega^2}{\omega_{,,}^{2}} = -q_{,,}^{2} \text{ et } \frac{\omega^2}{\omega''^{2}} = -q''^{2},$$

on a

$$U_{,,} = k\sqrt{q_{,,}^{2}+\sin^2\alpha}, \quad U'' = k\sqrt{q''^{2}+\sin^2\alpha},$$

d'où l'on voit que $kq_{,,}$ et kq'' sont les coefficients d'extinction des vibrations longitudinales sous l'incidence normale. La formule (43) peut s'écrire

$$\varkappa = \sin\alpha\left(\frac{1}{q_{,,}} - \frac{1}{q''}\right)\cdot\frac{q_{,,}+q''}{q_{,,}\sqrt{1+\frac{\sin^2\alpha}{q''^{2}}}+q''\sqrt{1+\frac{\sin^2\alpha}{q_{,,}^{2}}}}.$$

Si $q_{,,}$ et q'' sont très-grandes, on a, en posant

$\frac{1}{q_{,,}} - \frac{1}{q''} = \varepsilon =$ le coefficient d'ellipticité (Jamin), sans erreur sensible

$$\varkappa = \varepsilon \sin\alpha. \quad \ldots\ldots\ldots\ldots \quad (44)$$

Trois milieux, M, M_1, M_2, étant donnés, on obtient

le coefficient d'ellipticité relative de M et $M_1 = \frac{1}{q_{,,}} - \frac{1}{q''_1} = \varepsilon_1$,

„ „ „ de M et $M_2 = \frac{1}{q_{,,}} - \frac{1}{q''_2} = \varepsilon_2$,

„ „ „ de M_1 et $M_2 = \frac{1}{q''_1} - \frac{1}{q''_2} = \varepsilon_2^1$,

d'où l'on tire

$$\varepsilon_2^1 = \varepsilon_2 - \varepsilon_1 \quad \ldots\ldots\ldots\ldots \quad (45)$$

Cette équation n'a pas été justifiée par les expériences de M. Jamin et de M. Quincke.

En vertu de l'équation (30) on a

$$q'' = \mu q_{,,}$$

et, par suite,

$$\varepsilon = \frac{1}{q_{,,}}\left(1 - \frac{1}{\mu}\right) \quad \ldots\ldots\ldots\ldots \quad (46)$$

La relation (10) $\frac{g}{g'} = \frac{h}{h'}$ et, par conséquent, la formule (46) s'obtient, comme il a déjà été démontré, non-seulement par l'application du principe de l'égalité entre les pressions, dont nous avons montré la légitimité, mais encore en laissant de côté ce principe, pourvu que l'on admette que la force exercée entre deux molécules d'éther varie en raison inverse d'une certaine puissance de la distance, cette puissance étant la même dans tous les milieux. Mais il est facile de voir que la formule (46) n'est pas en accord avec les faits, constatés par l'expérience. Ainsi, par exemple, $q_{,,}$ étant toujours positif, suivant cette formule le coefficient d'ellipticité ne peut pas avoir de valeurs négatives pour des valeurs de μ plus grandes que l'unité.

L'espèce particulière d'extinction des vibrations longitudinales qu'entraîne la supposition de Cauchy par rapport à $\omega_{,,}^2$ et ω''^2, est une absorption véritable, par laquelle une partie du mouvement ondulatoire est transformée en des oscillations des molécules d'éther, de sorte que l'éther renfermé dans les corps transparents, quoique un milieu parfaitement élastique par rapport aux vibrations transversales, est imparfaitement élastique par rapport aux vibrations longitudinales. Contre cette supposition, M. Lorenz a fait l'objection qu'en introduisant d'une telle manière des coefficients d'absorption, on peut déduire à peu près tous les résultats que l'on voudra. M. Strutt a signalé, en outre, l'impossibilité d'obtenir dans une théorie de l'élasticité imparfaite des équations différentielles donnant des vibrations longitudinales de la forme actuelle. De toutes ces raisons il faut donc inférer qu'on ne peut pas admettre que $\omega_{,,}^2$ et ω''^2 soient des quantités négatives.

Passons maintenant à la supposition

2:o), $\omega_{,,}^2$ et $\omega''^2 > 0$ et des quantités très-grandes.

Si, pour abréger, on pose

$$\frac{\omega^2}{\omega_{,,}^2 \sin^2\alpha} = \nu_{,,}^2 \text{ et } \frac{\omega^2}{\omega''^2 \sin^2\alpha} = \nu''^2,$$

on obtient, en ayant égard à l'équation (30)

$$\nu''^2 = \mu^2 \nu_{,,}^2$$

et par suite on a, dans le cas actuel,

$$\varkappa = \frac{\mu^2 - 1}{\mu^2 \sqrt{1 - \nu_{,,}^2} + \sqrt{1 - \mu^2 \nu_{,,}^2}}. \quad \ldots \quad (47)$$

Si l'on admet, comme le fait Green, que $\omega_{,,}^{2}$ et ω''^{2} sont des quantités à peu près infiniment grandes par rapport à ω^2 et ω'^2, et, par suite, que l'on a

$$\nu_{,,}^{2} = \nu''^{2} = 0 ,$$

on obtient

$$\varkappa = \frac{\mu^2 - 1}{\mu^2 + 1} . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

La supposition examinée ici ne peut pas rencontrer les mêmes objections que celle de Cauchy. En effet, la supposition que $\omega_{,,}^{2}$ et ω''^{2} sont des quantités positives n'entraîne point d'absorption, et, par conséquent, l'éther est regardé comme un milieu parfaitement élastique par rapport aux vibrations tant longitudinales que transversales. Suivant Green, on a, en outre, même pour les vibrations de la dernière espèce, la dilatation cubique $\upsilon = 0$[1]) En effet, la formule (2) donne, par exemple,

$$\upsilon = \frac{d\xi_{,,}}{dx} + \frac{d\eta_{,,}}{dy} ,$$

d'où l'on déduit, en vertu des relations (23^{b}), (34) et (36),

$$\upsilon = \frac{iB_{,,}}{\mathrm{V}} \cdot \mathrm{k}_{,,}^{2} = \frac{iB_{,,}}{\mathrm{V}} \cdot \frac{\mathrm{s}^2}{\omega_{,,}^{2}} ,$$

expression, qu'on peut évidemment réduire à zéro en supposant que $\omega_{,,}^{2}$ soit une quantité infiniment grande. De cette manière, ces vibrations deviennent à un certain degré comparables à celle qui, à la réflexion totale, se propage dans le second milieu, et dont plusieurs expérimentateurs ont prouvé l'existence. Cependant, comme nous l'avons déjà fait observer, la formule (48) n'est pas justifiée par l'expérience, les valeurs calculées de $\varkappa$ étant toujours plus grandes que celles déduites des observations. Cette remarque a plus de force encore quant à la formule (47), car, comme M. Eisenlohr l'a signalé[2]) la valeur de $\varkappa$ fournie par elle est toujours plus grande que celle que donne la formule (48).

Si l'on suppose que la force agissant entre deux molécules d'éther, varie en raison inverse de la $n^{\text{ième}}$ puissance de la distance, on obtient des formules (31)

$$\omega^2 = \frac{4 - n}{5} g , \quad \omega_{,,}^{2} = \frac{2 - 3n}{5} g .$$

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[1]) Strutt, *Phil. Mag.* (4) XLII, p. 89.

[2]) *Pogg. Ann.* CIV, p. 359.

Par conséquent, la quantité ω^2 étant toujours finie, il faut pour que $\omega_{,,}^{\ 2}$ soit d'une grandeur infinie, que l'on ait:

$$g = \infty \ , \ n = 4 \ .$$

De l'exposition donnée ci-dessus des différentes théories on voit donc clairement qu'il est impossible, quelle que soit au reste l'hypothèse admise par rapport aux quantités $\omega_{,,}^{\ 2}$ et ω''^2, d'établir des formules théoriques qui soient vérifiées d'une manière absolue par l'expérience, si l'on suppose qu'on puisse regarder comme discontinu le passage de l'éther compris dans un milieu, à celui compris dans un autre. Il faut donc rejeter cette supposition, et, par conséquent, admettre que la couche de transition a une épaisseur appréciable.

M. Lorenz[1]) a le premier donné une théorie montrant comment l'influence de cette épaisseur doit être envisagée, bien que ses formules ne soient pas complétement développées pour ce cas, son but principal étant de déterminer la direction des vibrations de la lumière polarisée.

M. Strutt[2]) a proposé une modification très-simple aux formules de Green, pour les accommoder aux résultats des expériences. En effet, il suppose que la couche intermédiaire, où se produit la variation totale de la densité de l'éther, soit assez mince, pour permettre d'en negliger l'influence sur les vibrations transversales, qui la traversent très-rapidement. Mais il n'en est pas de même des vibrations longitudinales, qui se propagent principalement dans cette même couche à densité variable. Ainsi, si l'on désigne les densités qui se rapportent à ces dernières vibrations ϱ_0 par, dans le premier milieu et par ϱ'_0 dans le second, ces quantités doivent en général différer des densités ϱ et ϱ'. On aura, par conséquent, au lieu des équations (29),

$$\omega^2 = \frac{M}{\varrho} \ , \ \omega'^2 = \frac{M}{\varrho'} \ , \ \omega_{,,}^{\ 2} = \frac{L+2M}{\varrho_0} \ , \ \omega''^2 = \frac{L+2M}{\varrho'_0}$$

d'où l'on tire

$$\frac{\omega^2}{\omega'^2} = \frac{\varrho'}{\varrho} = \mu^2 \text{ et } \frac{\omega_{,,}^{\ 2}}{\omega''^2} = \frac{\varrho'_0}{\varrho_0} = \mu_0^2 \ .$$

En vertu de cette dernière équation, on a, comme plus haut (pag. 15),

$$\nu''^2 = \mu_0^2 \nu_{,,}^{\ 2} \ ,$$

[1]) *Pogg. Ann.* CXIV, p. 238, (1861).

[2]) *Phil. Mag.* (4) XLII, p. 96.

et, par suite, pour déterminer $\varkappa$ la relation

$$\varkappa = \frac{\mu_0^2 - 1}{\mu_0^2\sqrt{1-\nu_{''}^2} + \sqrt{1-\mu_0^2\nu_{''}^2}},$$

qui se réduit à

$$\varkappa = \frac{\mu_0^2 - 1}{\mu_0^2 + 1}, \quad \ldots\ldots\ldots\ldots \quad (49)$$

si l'on admet que $\omega_{''}{}^2$ et ω''^2 sont des grandeurs presque infinies. μ_0 et, par suite $\varkappa$, sont des quantités inconnues que l'on peut déterminer par des observations sur m et δ [1].

Il faut convenir que cette manière de voir offre beaucoup de probabilité. La méthode de Green pour le traitement de la question des vibrations longitudinales ne donne prise à aucune objection. Aussi l'on n'est pas conduit par cette voie à une relation analogue à la formule fautive (45), parce que la grandeur ϱ_0 d'un milieu doit dépendre de la densité de l'autre. On observera, en outre, que, d'aprés ces vues, si l'on a $\varrho < \varrho$, il faut poser

$$\varrho < \varrho_0 \text{ et } \varrho' > \varrho_0',$$

d'où l'on tire

$$\frac{\varrho'}{\varrho} > \frac{\varrho_0'}{\varrho_0},$$

c'est-à-dire

$$\mu > \mu_0, \text{ quand on a } \mu > 1;$$

et de même on trouve

$$\mu < \mu_0, \text{ quand on a } \mu < 1.$$

Ces inégalités sont évidemment d'accord avec des faits constatés. Pareillement, on voit sans peine, en partant de ce point de vue, que μ_0 doit en général croître avec μ, comme l'indique l'expérience. Enfin,

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[1]) Au *Phil. Mag.* (3) XXVI, (1845) p. 287 (la note) et 293, O'Brien a donné des formules identiques au fond avec celles de M. Strutt, quoique d'une forme différente. Ainsi, d'après ses vues, l'indice de réfraction des vibrations longitudinales n'est pas égal à celui des vibrations transversales. Pour la démonstration de ses formules, on est renvoyé à un mémoire inséré dans *Cambridge Phil. Trans.*, Vol. VIII. Malheureusement, je n'ai pas pu me procurer ce mémoire, de sorte que je ne connais pas complètement les idées qui y sont émises. Il est probable, cependant, que O'Brien, en opérant comme le fait Green par rapport aux vibrations longitudinales, a établi ses conditions à la surface par la méthode de Cauchy, c'est-à-dire en remplaçan le principe d'égalité entre les pressions, par celui de continuité du mouvement.

M. Haughton [1]) et M. Kurz [2]) ont montré que les valeurs de m et δ, que l'on calcule des formules (39) et (42) en considérant la quantité $\varkappa$ comme une constante, peuvent reproduire les valeurs observées aussi bien, sinon mieux, que celles que l'on obtient en remplaçant $\varkappa$ par $\varepsilon \sin\alpha$; et M. Jamin paraît aussi être de la même opinion [3]).

Il y a cependant un fait, constaté par M Jamin, dont l'explication ne se trouve pas immédiatement. En effet, si l'on a $\mu > 1$, c'est-à-dire $\varrho' > \varrho$, il serait à attendre que l'on eût aussi $\varrho'_0 > \varrho_0$ ou $\mu_0 > 1$, et, par suite, $\varkappa$ toujours positif. Il existe néanmoins, comme on le sait, des corps, liquides pour la plupart, pour lesquels cette quantité est négative.

M. Strutt incline à l'opinion que ce phénomène doit être attribué à l'action de la couche d'air que condensent les corps à leur surface. Il est évident, en effet, que si cette couche pouvait devenir assez dense pour que son indice de réfraction fût plus grand que celui du corps réfléchissant, ce phénomène pourrait être expliqué par la formule (49). Si l'on se demande cependant quelle serait en ce cas la grandeur de la densité de cette couche d'air, et si, pour la calculer, l'on applique la formule connue

$$\frac{\mu - 1}{\varrho} = \frac{1{,}000294 - 1}{0{,}00129},$$

on trouvera, en prenant son indice égal à 1,5, la densité 2,2, fois plus grand que celle de l'eau. Bien que la légitimité de cette formule ne soit pas démontrée pour des condensations comparables à celle dont il s'agit maintenant, cela suffira néanmoins, ce me semble, à faire voir que l'action de l'air ne peut être tout au plus qu'une cause accessoire.

On pourrait avec plus de raison présumer que ce phénomène est lié de quelque manière avec la tension superficielle des corps, dont la réalité est suffisamment constatée pour les liquides, et dont l'existence même chez les corps solides a été rendue très-probable par les expériences de M. Quincke [4]). Il est évident, en effet, que le coefficient d'ellipticité pourrait devenir négatif, si cette tension était suivie d'une condensation de la couche superficielle du corps réfléchissant. Malheureusement, il est assez généralement admis aujourd'hui, que la densité de cette couche est moindre que celle de l'intérieur; mais il faut remarquer que cette supposition, due à Poisson, n'est

[1]) *Phil. Mag.* (4) VI, p. 81, (1853).

[2]) *Pogg. Ann.* CVIII, p. 588, (1859).

[3]) *Ann. de Chim. et de Phys.* (3) LIX, p. 424, (1860).

[4]) *Pogg. Ann.* CXXXIV, p. 356, (1868).

pas expérimentalement prouvée, et qu'au moins un physicien, M. Hagen [1]), est de l'opinion opposée. Dans tous les cas, il sera utile de noter les faits suivants.

Les substances qui ont été étudiées par rapport aux deux phénomènes de la polarisation par réflexion et de la capillarité, sont l'eau, l'éther iodhydrique, l'éther benzoïque, l'essence de térébenthine, l'amylène, et quelques autres liquides. Le coefficient de capillarité, appelé α par Wilhelmy [2]) et par M. Quincke [3]), et qui, selon M. Hagen, représente la tension superficielle, a la valeur 7,5 pour la première de ces substances, pour l'eau, dont le coefficient d'ellipticité est négatif; tandis que pour les autres, qui sont toutes des substances positives par rapport à la réflexion, les valeurs du coefficient α sont de beaucoup inférieures, variant entre 1,8 et 3,7.

De ces remarques, il résulte, ce me semble, que la polarisation elliptique produite par la réflexion sur des corps transparents est un phénomène assez compliqué, dont les causes ne sont pas toutes bien connues; mais que, cependent, les formules de Green, modifiées par M. Strutt, rendent compte, d'une manière très-satisfaisante des résultats des expériences, bien que cette modification ne soit pas effectuée par une méthode parfaitement rigoureuse.

II. Les théories de la réflexion métallique.

La polarisation elliptique produite par la réflexion de la lumière sur les métaux fut découverte par Brewster [4]) en 1815, et étudiée par lui avec soin. Les particularités nombreuses qu'il faisait connaître, furent ramenées à certains principes simples par M. Neumann [5]), qui donna aussi des formules empiriques, propres à reproduire les observations avec une exactitude suffisante. Les tentatives faites en vue d'établir la théorie mathématique de la réflexion métallique sont dues surtout à Mac Cullagh, à Cauchy et à O Brien.

Mac Cullagh [6]) a donné ses formules comme fournies principalement par l'induction et par l'analogie, que présente la réflexion métallique avec

[1]) *Pogg. Ann.* LXVII, p. 1, (1846).

[2]) *Pogg. Ann.* CXXI, p. 57, (1864).

[3]) *Pogg. Ann.* CXXXV, p. 621, (1868); CXXXIX, p. 10, 15, (1870).

[4]) Biot, *Traité de physique* IV, Paris 1816, p 579; — *Phil. Trans.*, 1830, p. 287.

[5]) *Pogg. Ann.* XXVI, p. 89, (1832); XL, p. 513, (1837).

[6]) *Proceed. of the Ir. Acad.* 1836—37, I, p. 2; — *Irish Trans.* XVIII, part. I, p. 70, (1837); — *Compt. Rend.* VIII, p. 961, (1839).

la réflexion totale. Dans les formules de Fresnel, il remplace l'indice de réfraction, c'est-à-dire le rapport des sinus des angles d'incidence et de réfraction, par une quantité complexe $ce^{i\gamma}$, c et γ étant deux constantes; et, de même, le rapport de leurs cosinus par une autre quantité complexe, mais variable. Il obtient de cette manière des formules s'accordant avec celles de Cauchy, que nous donnerons plus loin. Suivant sa conjecture, l'indice de réfraction des métaux est représenté par $\frac{c}{\cos\gamma}$. Dans ses calculs Mac-Cullagh a été guidé par certaines idées théoriques, qu'il n'a cependant fait connaître qu'en partie.

Cauchy a, de son côté, cherché à rattacher les phénomènes de la réflexion métallique à une théorie résultant d'une généralisation de celle donnée par lui pour les corps transparents. A la vérité, il n'a pas publié la démonstration de ses formules, mais il en a fait connaître les principes, et cela suffit pour retrouver ses résultats. Beer [1]) en a donné le premier une démonstration, mais sa méthode a le défaut de faire paraître le nombre des hypothèses introduites plus grand qu'il n'est en réalité. A la démonstration de M. Eisenlohr [2]), très-satisfaisante en général, on pourrait, au contraire, présenter l'objection qu'il n'y a pas signalé la nécessité d'une hypothèse nouvelle pour obtenir les résultats de Cauchy.

Nous allons maintenant exposer ses formules par une autre méthode. Les notations sont les mêmes que précédemment.

Si le second milieu n'est pas parfaitement transparent, l'expérience a montré que les projections de la vibration réfractée sont de la forme

$$P'e^{-U'x}\cos(u'x+vy-st+\varpi'),$$

et qu'elles sont, par suite, représentées par les parties réelles des expressions

$$\xi'=A'e^{-U'x+i(u'x+vy-st)},\ \eta'=B'e^{-U'x+i(u'x+vy-st)},\ \zeta'=C'e^{-U'x+i(u'x+vy-st)}. \quad . \quad (50)$$

On ne connait pas les relations que doivent vérifier les constantes qui entrent dans ces expressions, mais il est du moins très-probable que, encore dans les milieux non transparents, les vibrations lumineuses se propagent sans produire de compressions et de dilatations, c'est-à-dire que l'équation

$$\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}=0$$

[1]) *Pogg. Ann.* XCII, p. 402.

[2]) *Pogg. Ann.* CIV, p. 368.

doit être vérifiée par les projections de la vibration réfractée. Mais alors il faut pouvoir encore satisfaire à cette équation par les expressions (50), d'où l'on a

$$A'(u' + iU') + B'v = 0 \quad \ldots\ldots\ldots \quad (51)$$

Remarquons maintenant que les expressions (50) et la relation (51) s'obtiennent des expressions (18) et de la relation (24ª) en remplaçant u dans celles-ci par $u' + iU'$, et, par conséquent, que les nouvelles conditions relatives à la surface se déduisent par le même procédé de celles que nous avons établies pour les corps transparents, pourvu que l'on admette que les expressions (35) de la vibration longitudinale restent sensiblement les mêmes. Alors il est facile de voir qu'on peut obtenir très-simplement les valeurs des inconnues dans le cas actuel, en remplaçant la quantité réelle u' dans les formules (32), (38) etc. par la quantité complexe $u' + iU'$. Ainsi l'on a:

$$C_1 = C \cdot \frac{u - u' - iU'}{u + u' + iU'}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (52)$$

$$B_1 = B \cdot \frac{u - u' - iU'}{u + u' + iU'} \cdot \frac{uu' - v^2 - \varkappa v U' + i[uU' + \varkappa v(u + u')]}{uu' + v^2 + \varkappa v U' + i[uU' + \varkappa v(u - u')]} \quad \ldots \quad (53)$$

Décomposons maintenant la vibration incidente en deux: l'une perpendiculaire au plan d'incidence, l'autre située dans ce plan, et supposons que les amplitudes de ces composantes soient égales entre elles et prises pour unité. Soient encore les amplitudes des composantes correspondantes de la vibration réfléchie représentées par I_1 et I_2 et leurs différences de phase avec les composantes de la vibration incidente par Δ_1 et Δ_2 respectivement. On a donc

$$C = 1, \; B = \cos\alpha, \; C_1 = I_1 e^{i\Delta_1}, \; B_1 = \cos\alpha I_2 e^{i\Delta_2},$$

d'où l'on tire

$$C_1 = CI_1 e^{i\Delta_1}, \; B_1 = BI_2 e^{i\Delta_2} \quad \ldots\ldots\ldots \quad (54)$$

En portant ces valeurs dans les équations (52) et (53) on obtient

$$I_1^2 = \frac{(u - u')^2 + U'^2}{(u + u')^2 + U'^2}; \quad \ldots\ldots\ldots\ldots \quad (55)$$

si l'on pose $U' = k.p$, on déduit de là, en substituant $u = v\cot\alpha$ et $u' = v\cot\alpha'$

$$I_1^2 = \frac{\sin^2(\alpha - \alpha') + p^2 \sin^2\alpha'}{\sin^2(\alpha + \alpha') + p^2 \sin^2\alpha'}.$$

On peut déduire de la même manière des formules pour I_2^2, Δ_1 et Δ_2 [1]).

Les formules ainsi obtenues sont parfaitement générales et doivent renfermer les lois de la réflexion sur tous les milieux isotropes, mais il faut remarquer que leur application exige la connaissance de la quantité p et de l'indice de réfraction, et cela entraîne des difficultés, quand il s'agit des corps opaques et spécialement des métaux, où ces quantités varient probablement avec l'angle d'incidence. Pour en faciliter l'application dans ce cas, Cauchy a établi entre ces quantités une relation, fournie par un genre d'induction, relation à l'aide de laquelle on peut donner aux formules une forme très-commode.

Remarquons d'abord que l'équation (24^c) n'est plus applicable, quand il s'agit d'un corps opaque. En effet, si l'on remplace u' dans cette équation par $u' + iU'$, on obtient

$$k'^2 = (u' + iU')^2 + v^2,$$

d'où l'on tire, en observant que k'^2 est nécessairement positif en vertu de l'équation (24^b),

$$k'^2 = u'^2 - U'^2 + v^2 \text{ et } u'U' = 0.$$

Ainsi l'on doit avoir

$$U' = 0 \text{ ou bien } u' = 0.$$

Le premier cas correspond à la réflexion ordinaire sur les corps transparents, le second cas a lieu à la réflexion totale; mais ni l'un ni l'autre ne correspondent à la réflexion sur les corps opaques.

Cependant, la relation nécessaire peut s'obtenir par les considérations suivantes [2]). Pour les corps transparents on a trouvé la relation (12)

$$k^2 = u^2 + v^2 + w^2$$

ou

$$-k^2 = (iu)^2 + (iv)^2 + (iu)^2,$$

où la constante réelle et positive k est appelée par Cauchy la *caractéristique.* Lorsque, au contraire, le corps considéré cesse d'être transparent, les quantités iu, iv, iw, doivent être remplacées par $U+iu$, $V+iv$, $W+iw$, et la caractéristique nouvelle $k+iK$ est par analogie donnée par l'équation

$$-(k+iK)^2 = (U+iu)^2 + (V+iv)^2 + (W+iw)^2. \quad . \quad . \quad . \quad (56)$$

Or, la caractéristique étant constante dans le cas précédent, il est vraisemblable qu'elle le sera aussi dans ce cas-ci. Supposons donc qu'il en soit

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[1]) Voyez Beer, *Pogg. Ann.* XCII, p. 408. Cette déduction s'opère très-simplement par la méthode indiquée par Eisenlohr, *Pogg. Ann.* CIV, p. 351.

[2]) *Compt. Rend.* VIII, p. 43, (1839).

ainsi, c'est-à-dire que les quantités k et K soient des constantes, et que la première soit positive.

Il s'agit maintenant de transformer les formules (52) et (53) à l'aide de l'équation (56). Dans le cas actuel, cette relation prend la forme

$$-(\mathrm{k}'+i\mathrm{K}')^2=(-\mathrm{U}'+i\mathrm{u}')^2+(i\mathrm{v})^2. \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Posons

$$\mathrm{k}'+i\mathrm{K}'=\mathrm{k}\,c\,e^{i\gamma}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

c et γ étant deux constantes nouvelles. Alors on aura pour l'incidence normale

$$\Big/^{\alpha=0} \quad \mathrm{u}'+i\mathrm{U}'=\mathrm{u}'_0+i\mathrm{U}'_0=\mathrm{k}\,c\,e^{i\gamma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

Pour une incidence quelconque, on peut poser

$$\mathrm{u}'+i\mathrm{U}'=\mathrm{k}\,r\,e^{i\varrho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

r et ϱ étant des quantités variables. On aura donc, en vertu de l'équation (57)

$$c^2e^{2i\gamma}=r^2e^{2i\varrho}+\sin^2\alpha; \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

et, par suite,

$$c^2\sin 2\gamma=r^2\sin 2\varrho,\ c^2\cos 2\gamma=r^2\cos 2\varrho+\sin^2\alpha, \quad . \quad . \quad . \quad (62)$$

d'où l'on peut déduire les formules [1])

$$\left.\begin{aligned} r&=\left(\frac{\sin 2\gamma}{\sin 2\varrho}\right)^{\frac{1}{2}}c \\ \cot(2\varrho-\gamma)&=\cot\gamma\cos\left[2\,\mathrm{Arctang}\,\frac{\sin\alpha}{c}\right], \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (63)$$

qui serviront à déterminer r et ϱ, lorsque c et γ sont connus.

Ayant égard aux formules (54) et (60), on tire de l'équation (52)

$$I_1e^{i\Delta_1}=\frac{\mathrm{u}-\mathrm{k}\,r\,e^{i\varrho}}{\mathrm{u}+\mathrm{k}\,r\,e^{i\varrho}}=\frac{\dfrac{\cos\alpha}{r}-e^{i\varrho}}{\dfrac{\cos\alpha}{r}+e^{i\varrho}}.$$

[1]) Les transformations de ce genre s'opèrent très-facilement en observant que, si l'on a

$$\mathrm{tang}\,\alpha=\frac{n\sin 2\beta}{1-n\cos 2\beta},\ \text{on a aussi}\ \mathrm{tang}(\alpha+\beta)=\frac{1+n}{1-n}\,\mathrm{tang}\beta.$$

En posant, pour abréger,

$$\cot \Psi_1 = \cos \varrho \sin \left[2 \operatorname{Arctang} \frac{\cos \alpha}{r} \right], \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

on déduit de là

$$I_1^2 = \operatorname{tang} \left(\Psi_1 - \frac{\pi}{4} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

$$\operatorname{tang} \Delta_1 = \sin \varrho \operatorname{tang} \left[2 \operatorname{Arctang} \frac{\cos \alpha}{r} \right].$$

A l'aide de ces formules, on calculera l'intensité I_1^2 et la différence de phase Δ_1 du rayon réfléchi, dont les vibrations sont perpendiculaires au plan d'incidence.

Pour obtenir les formules correspondantes pour le rayon, dont les vibrations sont situées dans ce plan, on peut, dans une première approximation, négliger la quantité $\varkappa$ qui, au moins pour les corps transparents, est toujours très-petite. Alors on obtient de l'équation (53), en ayant égard aux formules (54) et (60),

$$I_2 \, e^{i \Delta_2} = \frac{\mathrm{u} - \mathrm{k} r e^{i \varrho}}{\mathrm{u} + \mathrm{k} r e^{i \varrho}} \cdot \frac{\mathrm{u} \mathrm{k} r e^{i \varrho} - \mathrm{v}^2}{\mathrm{u} \mathrm{k} r e^{i \varrho} + \mathrm{v}^2}.$$

En vertu de la relation (61), cette formule se réduit à

$$I_2 \, e^{i \Delta_2} = \frac{\frac{r}{c^2 \cos \alpha} - e^{i (2\gamma - \varrho)}}{\frac{r}{c^2 \cos \alpha} + e^{i (2\gamma - \varrho)}}.$$

En posant, pour abréger,

$$\cot \Psi_2 = \cos (2\gamma - \varrho) \sin \left[2 \operatorname{Arctang} \frac{r}{c^2 \cos \alpha} \right], \quad . \quad . \quad . \quad . \quad (66)$$

on tire de là

$$I_2^2 = \operatorname{tang} \left(\Psi_2 - \frac{\pi}{4} \right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

$$\operatorname{tang} \Delta_2 = \sin (2\gamma - \varrho) \operatorname{tang} \left[2 \operatorname{Arctang} \frac{r}{c^2 \cos \alpha} \right];$$

à l'aide de ces formules on calculera l'intensité I_2^2 et la différence de phase Δ_2.

Si l'on pose

$$\frac{I_2}{I_1} = \operatorname{tang}\theta\,, \quad \Delta_2 - \Delta_1 = \Delta\,,$$

on tire des équations (52), (53) et (54)

$$\operatorname{tang}\theta . e^{i\Delta} = \frac{\mathrm{u}\,\mathrm{k}\,r e^{i\varrho} - \mathrm{v}^2}{\mathrm{u}\,\mathrm{k}\,r e^{i\varrho} + \mathrm{v}^2} = \frac{e^{i\varrho} - \dfrac{\sin^2\alpha}{r\cos\alpha}}{e^{i\varrho} + \dfrac{\sin^2\alpha}{r\cos\alpha}}\,,$$

d'où l'on déduit les formules

$$\cos 2\theta = \cos\varrho \sin\left[2\,\operatorname{Arctang}\frac{\sin^2\alpha}{r\cos\alpha}\right], \quad \ldots\ldots \quad (68)$$

$$\operatorname{tang}\Delta = \sin\varrho \operatorname{tang}\left[2\,\operatorname{Arctang}\frac{\sin^2\alpha}{r\cos\alpha}\right], \quad \ldots\ldots \quad (69)$$

qui serviront à calculer l'azimut de la polarisation rétablie du rayon réfléchi θ et la différence de phase Δ[1]).

Soient A et Θ les valeurs acquises par α et θ, lorsque Δ sera égale à $\frac{\pi}{2}$, c'est-à-dire, leur valeurs principales. Donc on aura, d'après les formules (68) et (69)

$$\Big|^{\Delta=\frac{\pi}{2}}_{\varrho = 2\Theta}\,, \quad r = \sin\mathrm{A}\operatorname{tang}\mathrm{A} \ldots\ldots \quad (70)$$

Portons ces valeurs dans les équations (62), on tire de là les formules

$$\operatorname{tang}\frac{2(\gamma - \Theta)}{\operatorname{tang} 2\Theta} = -\cos 2\mathrm{A}\,, \quad \ldots\ldots \quad (71)$$

$$c = \sin\mathrm{A}\sqrt{\frac{\sin 4\Theta}{\sin 2(2\Theta - \gamma)}} = \operatorname{tang}\mathrm{A}\sqrt{\frac{\cos 2\Theta}{\cos 2(\gamma - \Theta)}}\,; \quad \ldots \quad (72)$$

à l'aide desquelles on peut déterminer les valeurs de c et γ, lorsque celles de A et Θ sont données par l'expérience.

La valeur de l'indice de réfraction s'obtient de la manière suivante. On a

$$\mathrm{u}' = \mathrm{v}\cot\alpha' = \mathrm{k}\,r\cos\varrho\,,$$

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[1]) *Compt. Rend.* VIII, p. 553, (1839); XXVI, p. 86, (1848).

d'où l'on tire

$$v^2(1+\cot^2\alpha') = \frac{v^2}{\sin^2\alpha'} = k^2\mu^2 = k^2 r^2\cos^2\varrho + v^2,$$

et enfin

$$\mu^2 = r^2\cos^2\varrho + \sin^2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Pour l'incidence normale cette expression se réduit à

$$\mu_0 = c\cos\gamma .$$

Lorsque α a sa valeur principale, on déduit à l'aide des formules (70),

$$\mu_A^2 = \operatorname{tang}^2 A\,(1 - \sin^2 A \sin^2 2\Theta) .$$

On voit par là que les formules de Cauchy donnent la valeur de μ_A toujours plus petite que ne l'indique la loi de Brewster.

Il est très-probable que Cauchy a cherché à établir rationnellement des équations différentielles, propres à réprésenter le mouvement vibratoire de l'éther dans les métaux, mais il n'a point publié de résultats se rapportant à ce but, La seule tentative connue de ce genre est due à O'Brien [1]).

Suivant les idées généralement admises, l'extinction de la lumière dans un corps opaque tient à ce qu'une partie de l'énergie du mouvement vibratoire passe de l'éther aux molécules pondérables, mais, si l'on tente à faire entrer cette condition dans le calcul, il s'oppose des difficultés. Pour éviter celles-ci O'Brien a pris pour point de départ la supposition que "les amplitudes des vibrations de l'éther sont grandes par rapport aux distances entre les molécules pondérables, et, par conséquent, que chaque élément d'éther éprouve durant son mouvement l'action de forces résistantes, des réactions des molécules pondérables, et qui sont caractérisées par la propriété de dépendre seulement de l'état de mouvement de l'élément, mais non de son déplacement de la position d'équilibre."

S'appuyant sur l'analogie qui, suivant cette supposition, existe entre les oscillations d'un pendule dans un milieu résistant et le mouvement vibratoire de l'éther, il est parvenu à montrer que les expressions des forces résistantes, mises en jeu par ce mouvement, doivent être de la forme

$$a_1\frac{d\varkappa}{dt} + a_2\frac{d^2\varkappa}{dt^2} + a_3\frac{d^3\varkappa}{dt^3} + \text{etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

[1]) *Phil. Mag.* (3) XXV, p. 326 et 521, (1844); (3) XXVI, p. 114 et 287.

$\varkappa$ étant le déplacement de l'élément, a_1 a_2 a_3 .... des constantes dépendantes de la constitution de l'éther et de la matière pondérable. Si l'on suppose, en outre, que les vibrations s'effectuent suivant la loi, qu'indique la partie réelle de l'expression

$$\Phi e^{-ist},$$

Φ étant une fonction indépendante de t, l'expression (74) peut se réduire à

$$P\frac{d\varkappa}{dt}+Q\frac{d^2\varkappa}{dt^2},$$

si l'on pose, pour abréger,

$$P=a_1-a_3 s^2+a_5 s^4-\ldots\ldots, \quad Q=a_2-a_4 s^2+a_6 s^4-\ldots\ldots .$$

Soit le second milieu de l'éther compris dans un métal. En appliquant au cas actuel les considérations précédentes, les projections sur les axes des résistances, exercées par la matière pondérable sur un élément d'éther, seront exprimées par

$$-\left(P'\frac{d\xi'}{dt}+Q'\frac{d^2\xi'}{dt^2}\right), \quad -\left(P'\frac{d\eta'}{dt}+Q'\frac{d^2\eta'}{dt^2}\right), \quad -\left(P'\frac{d\zeta'}{dt}+Q'\frac{d^2\zeta'}{dt^2}\right).$$

Pour avoir les équations du mouvement il faut, en vertu du principe de d'Alembert, remplacer dans les équations (4) (pag. 5) les X, Y, Z par les expressions

$$-\frac{d^2\xi'}{dt^2}-\left(P'\frac{d\xi'}{dt}+Q'\frac{d^2\xi'}{dt^2}\right), \quad -\frac{d^2\eta'}{dt^2}-\left(P'\frac{d\eta'}{dt}+Q'\frac{d^2\eta'}{dt^2}\right), \quad -\frac{d^2\zeta'}{dt^2}-\left(P'\frac{d\zeta'}{dt}+Q'\frac{d^2\zeta'}{dt^2}\right).$$

En opérant ainsi, ces équations peuvent s'écrire:

$$\left.\begin{aligned}
\frac{d^2\xi}{dt^2}+R'\frac{d\xi}{dt} &= (G'-H')\frac{d\upsilon}{dx}+H'\left(\frac{d^2\xi}{dx^2}+\frac{d^2\xi}{dy^2}+\frac{d^2\xi}{dz^2}\right),\\
\frac{d^2\eta}{dt^2}+R'\frac{d\eta}{dt} &= (G'-H')\frac{d\upsilon}{dy}+H'\left(\frac{d^2\eta}{dx^2}+\frac{d^2\eta}{dy^2}+\frac{d^2\eta}{dz^2}\right),\\
\frac{d^2\zeta}{dt^2}+R'\frac{d\zeta}{dt} &= (G'-H')\frac{d\upsilon}{dz}+H'\left(\frac{d^2\zeta}{dx^2}+\frac{d^2\zeta}{dy^2}+\frac{d^2\zeta}{dz^2}\right),
\end{aligned}\right\} \ldots . \quad (75)$$

en posant, pour abréger,

$$R'=\frac{P'}{1+Q'}, \quad G'=\frac{L+2M}{\varrho'(1+Q')}, \quad H'=\frac{M}{\varrho'(1+Q')},$$

P' et Q' étant donnés par les formules

$$P' = a_1 - a_3 s^2 + a_5 s^4 - \ldots\ , \quad Q' = a_2 - a_4 s^2 + a_6 s^4 - \ldots$$

Aux équations (75) on peut satisfaire par des intégrales de la forme (50),

$$\xi = A e^{-Ux+i(ux+vy-st)}\ , \quad \eta = B e^{-Ux+i(ux+vy-st)}\ , \quad \zeta = C e^{-Ux+i(ux+vy-st)}\ ,$$

les constantes, qui y entrent, devant vérifier l'un de deux systèmes d'équations: on doit avoir

$$A'(u'+iU')+B'v=0; \quad s^2-iR's=H'(k'+iK')^2; \quad (k'+iK')^2=(u'+iU')^2+v^2; \quad (76)$$

ou bien

$$\frac{A''}{u''+iU''}=\frac{B''}{v}, \quad C''=0; \quad s^2-iR's=G'(k''+iK'')^2; \quad (k''+iK'')^2=(u''+iU'')^2+v^2. \quad (77)$$

Le premier mode donne les vibrations transversales, le second les vibrations longitudinales.

Les relations (76^a) et (76^c) sont identiques aux relations (51) et (57) de Cauchy. Pareillement, les conditions relatives à la surface sont évidemment les mêmes dans les deux cas. Il en faut donc conclure, que les formules qui en ont été déduites par O'Brien, bien qu'elles se présentent sous une forme différente, sont au fond identiques avec celles de Cauchy, au moins si l'on néglige l'ellipticité, qui tient aux vibrations longitudinales. Quant à ces dernières vibrations, en effet, il faut remarquer que, suivant les formules (77), elles ne sont pas tout à fait analogues à celles que produit la réflexion sur des substances transparentes. Car ici la quantité u'' ne peut pas être égale à zéro, vu que cela entraîne la relation impossible

$$R' = 0.$$

L'analyse de O'Brien donne, en outre, la relation (76^b)

$$s^2 - iR's = H'[(u'+iU')^2+v^2]\ ,$$

ou, ce que revient au même,

$$s^2 = H'(u'^2+v^2-U'^2), \quad R's = -2H'u'U'. \quad \ldots \quad (78)$$

Supposons que le premier milieu soit de l'éther libre, et soient ω et ω' les vitesses respectives de propagation dans le premier et dans le second milieu. On aura donc

$$s^2 = \omega^2 k^2 = \omega'^2(u'^2+v^2)\ ,$$

d'où l'on tire

$$u'^2 = k^2(\mu^2 - \sin^2\alpha).$$

En portant ces valeurs dans les équations (78), on obtient, après l'élimination de U',

$$\left(\mu^2 - \frac{\omega^2}{H'}\right)(\mu^2 - \sin^2\alpha) = \frac{R'^2\omega^2}{4H'^2}\cdot\frac{1}{k^2}. \quad \ldots \quad (79)$$

Pour avoir une valeur approchée de l'indice de réfraction, on peut s'arrêter après le second terme des expressions des forces résistantes, ce qui réduira P', Q' et, par conséquent, R', H' à des constantes. En rappelant la formule $k = \frac{2\pi}{\lambda}$, on verra que les valeurs de μ, calculées à l'aide de la relation (79) pour une incidence donnée, vont en augmentant avec λ, c'est-à-dire que la dispersion des métaux doit en général être anomale et opposée à celle des corps transparents. Nous démontrerons cependant plus loin que l'équation (79) n'est pas compatible avec la relation (73).

Les premières expériences faites en vue de vérifier les formules théoriques sont dues à Mac Cullagh [1]), et il a trouvé que ses résultats sont représentés avec une grande exactitude, même par des formules simplifiées, qui s'accordent avec celles que nous donnerons à l'instant (80).

Des vérifications plus nombreuses ont été fournies par les recherches de M. Jamin [2]) sur la réflexion métallique, ce savant ayant comparé tous les faits trouvés expérimentalement avec les résultats calculés à l'aide des formules de Cauchy. Pour déduire les valeurs théoriques de l'intensité, il s'est servi des formules simplifiées. En effet, pour les métaux la quantité $\frac{1}{c}$ est assez petite pour qu'il soit permis d'en négliger les puissances supérieures. Alors les formules (63) se réduisent à

$$\varrho = \gamma, \quad r = c.$$

Or nous avons précédemment obtenu, pour l'incidence principale, les relations (70), d'où il vient

$$\gamma = 2\Theta \quad c = \sin A \operatorname{tang} A.$$

En portant ces valeurs dans les équations (64)... (67), on aura, pour calculer les intensités, les formules

$$\left.\begin{aligned} I_1^2 &= \operatorname{tang}\left(\Psi_1 - \frac{\pi}{4}\right), \quad \cot\Psi_1 = \cos 2\Theta \sin\left[2 \operatorname{Arctang} \frac{\cos A \cos\alpha}{\sin^2 A}\right], \\ I_2^2 &= \operatorname{tang}\left(\Psi_2 - \frac{\pi}{4}\right), \quad \cot\Psi_2 = \cos 2\Theta \sin\left[2 \operatorname{Arctang} \frac{\cos A}{\sin^2 A \cos\alpha}\right]. \end{aligned}\right\} \quad \ldots \quad (80)$$

De cette comparaison il résulte une concordance aussi grande que possible, entre les valeurs observées et les valeurs calculées, c'est-à-dire que les différences sont toujours des quantités de l'ordre des erreurs d'observation. Il faut remarquer, cependant, que le procédé employé pour mesurer les intensités, n'est pas susceptible d'une grande perfection.

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[1]) *Proceed. of the Ir. Acad.* 1842—43, VII, p. 375.

[2]) *Compt. Rend.* XXI, p. 430, (1845), et les Tom. suiv.; — ***Ann. de Chim. et de Phys.*** (3) XIX, p. 296, (1847); (3) XXII, p. 311, (1848).

Parmi les résultats de M. Jamin que la théorie de Cauchy n'avait pas prévus, il faut mentionner en premier lieu qu'il a constaté que, pour les métaux observés, les incidences principales A vont en décroissant du rouge au violet, mais qu'il ne subsiste pas de loi analogue générale pour la quantité Θ. On peut conclure de ces faits que la dispersion des métaux doit en général être anomale; mais, suivant les calculs effectués par Beer [1]) et par M. Eisenlohr [2]), elle n'est pas toujours opposée à celle des corps transparents.

Les expériences de MM. de la Provostaye et Desains [3]) sur la réflexion de la chaleur rayonnante ont aussi donné des résultats qui s'accordent avec ceux que l'on déduit des formules de Cauchy.

M. Haughton [4]) s'est occupé de l'étude expérimentale de la réflexion de la lumière sur les métaux, mais ses observations n'ont pas été comparées avec les formules théoriques.

Enfin, dans ses vastes recherches optiques, M. Quincke [5]) a étudié par plusieurs méthodes la réflexion et la réfraction métalliques. Il a reconnu que les formules de Cauchy reproduisent les résultats de l'expérience d'une manière satisfaisante non seulement quand il s'agit de la réflexion ordinaire, mais encore lorsque la réflexion s'opère sur des feuilles métalliques très-minces et diaphanes. Pour calculer le rapport des intensités et la différence de phase, il s'est servi des formules simplifiées suivantes

$$\left.\begin{aligned} \cos 2\theta &= \cos 2\Theta \sin\left[2 \operatorname{Arctang} \frac{\sin\alpha \operatorname{tang}\alpha}{\sin A \operatorname{tang} A}\right], \\ \operatorname{tang}\Delta &= \sin 2\Theta \operatorname{tang}\left[2 \operatorname{Arctang} \frac{\sin\alpha \operatorname{tang}\alpha}{\sin A \operatorname{tang} A}\right], \end{aligned}\right\} \quad \ldots \quad (81)$$

dont le degré d'approximation est égal à celui des formules (80).

Il a tenté aussi d'effectuer des mesures directes de l'indice de réfraction des métaux, mais ce problème difficile n'a pas encore été résolu d'une manière définitive.

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[1]) *Pogg. Ann.* XCII, p. 417.

[2]) *Pogg. Ann.* CIV, p. 375.

[3]) *Compt. Rend.* XXIV, p. 684, (1847); — *Ann. de Chim. et de Phys.* (3) XXVII, p. 121, (1849).

[4]) *Phil. Trans.* 1863, p. 81.

[5]) *Pogg. Ann.* CXIX, p. 368; CXX, p. 599, (1863); CXXVIII, p. 541; CXXIX, p. 177, (1866); CXXXII, p. 321, 561, (1867); CXLII, p. 177, 380, (1871).

En résumé, dans tous les cas où il a été possible de comparer avec les observations les formules (64)...(69), on les a trouvées toujours en concordance avec l'expérience quand il s'est agi de la réflexion métallique.

Quant à l'équation (79), au contraire, il faut remarquer qu'elle n'est pas compatible avec la relation (73). L'expérience a montré, en effet, que et r et ϱ sont des quantités à peu près constantes pour les métaux. En ayant égard à ce fait, on voit facilement qu'il est impossible de satisfaire à l'équation (79) par des valeurs de μ^2 tirées de la formule (73), attendu que H' et R' sont des constantes, au moins pour une valeur déterminée de λ. C'est le principal défaut de la théorie de O'Brien, qui ayant pour but d'établir rationnellement l'équation (57), mériterait sans cela de l'attention, quoique son hypothèse fondamentale ne s'accorde pas bien avec les idées de Cauchy sur la constitution des corps.

III. Réflexion sur les corps imparfaitement transparents et non métalliques.

Après la découverte de M. Airy que la réflexion sur le diamant produit la polarisation elliptique, deux autres savants anglais, Powell [1]) et Dale [2]), s'appliquèrent à l'étude expérimentale des phénomènes de ce genre. Leurs recherches firent constater la présence de la polarisation elliptique dans le rayon réfléchi, pour un grand nombre de substances plus ou moins opaques. A la vérité, la plupart de ces corps renfermaient une quantité plus ou moins grande de métal, à laquelle on pouvait attribuer leur propriété de polariser la lumière elliptiquement; mais ce phénomène se présentait aussi chez des substances ne renfermant point de métal, comme l'encre de chine et l'indigo.

M. Jamin a depuis reconnu que tous les corps tant imparfaitement que parfaitement transparents, changent par réflexion la polarisation rectiligne en polarisation elliptique, et il a encore fait voir que les formules établies par Cauchy pour les corps parfaitement transparents, s'appliquent également bien à la réflexion sur des corps peu transparents. Il faut remarquer, cependant, qu'aucune des substances qu'a examinées M. Jamin, n'est du nombre de celles qui présentent le phénomène de *la coloration superficielle.*

Cette propriété remarquable, que possèdent un grand nombre de corps, a été étudiée principalement par Haidinger [3]) et par M. Stokes [4]). Il

[1]) *Phil. Trans.* 1843, p. 35; 1845, p. 269; — 13th *Rep. of Brit. Assoc.* 1843.
[2]) 16th *Rep. of Brit. Assoc.* 1846.
[3]) *Wien. Sitz. Ber.* VIII, p. 97, (1852) où sont exposés la plupart de ses résultats.
[4]) *Phil. Mag.* (4) VI, p. 393, (1853).

résulte de leurs recherches que ces substances participent à la fois des propriétés des corps transparents et de celles des métaux, en ce qu'ils se comportent vis-à-vis des rayons de certaines couleurs, comme s'ils étaient métalliques, en leur communiquant une polarisation très-elliptique, et vis-à-vis des rayons d'autres couleurs comme les corps transparents.

Pour deux corps appartenants à ce groupe, l'indigo et le fer oligiste, M. van der Willigen [1]) a déterminé les constantes qui caractérisent la réflexion, par rapport aux différentes parties du spectre; et il a trouvé que la valeur principale de l'angle d'incidence va en décroissant avec la longueur d'onde pour les couleurs vis-à-vis desquelles les substances présentent un fort pouvoir réflecteur, c'est-à-dire se comportent comme les métaux, mais que cette valeur reste constante ou va en croissant avec la longueur d'onde pour les autres couleurs. Ces lois expérimentales ont été vérifiées d'une manière encore plus prononcée par les recherches de M. E. Wiedemann [2]) sur la polarisation elliptique produite par la réflexion sur le rouge d'aniline (fuchsine) et sur le violet d'aniline. Il a effectué, sous un assez grand nombre d'incidences, des mesures de la direction des axes de l'ellipse dans le rayon réfléchi et du rapport de ces grandeurs, mais il n'a pas comparé les formules théoriques avec les résultats de l'expérience. Une telle comparaison doit cependant offrir beaucoup d'intérêt. A la vérité, il est à présumer, par avance, que les formules établies pour la réflexion métallique reproduiront d'une manière satisfaisante les observations exécutées sur les substances à coloration superficielle, quand il s'agit des couleurs vis-à-vis desquelles ces substances se comportent comme les métaux; mais on ne peut pas s'attendre à trouver aussi une telle concordance pour les autres couleurs. Il est du moins très-facile de prouver qu'à la réflexion sur certains corps opaques, il se produit des phénomènes dont ces formules ne rendent pas compte.

En effet, si un rayon, pénétrant par l'une des faces latérales d'un prisme de verre, va tomber sur la face de l'hypothénuse sous une incidence suffisamment grande, on sait que la totalité de la lumière est réfléchie. Ce phénomène peut encore avoir lieu, si l'on applique sur la face réfléchissante des couches de certaines substances opaques, et Wollaston a cherché, en partant de cette propriété, à déterminer l'indice de réfraction de ces substances. Supposons maintenant, pour fixer les idées, que les vibrations du rayon incident soient perpendiculaires au plan d'incidence. On aura donc, pour calculer l'intensité du rayon réfléchi, l'expression (55)

[1]) *Pogg. Ann.* CXVII, p. 464, (1862).

[2]) *Ber. der Sächs. wiss. Gesellschaft*, Math.-Phys. Cl. 1872.

$$I_1^2 = \frac{(u - u')^2 + U'^2}{(u + u')^2 + U'^2},$$

qu'on ne peut réduire à l'unité, comme l'exige la réflexion totale, qu'en posant $u' = 0$. Or pour déduire les formules de Cauchy, il faut supposer, comme nous l'avons fait voir, que la caractéristique $k' + iK'$ du milieu opaque, donné par la formule (57)

$$(k' + iK')^2 = (u' + iU')^2 + v^2,$$

est une quantité complexe constante. Mais, toutes les fois que la réflexion totale a lieu, on tire de cette rélation

$$\text{ou } k' = 0, \text{ ou bien } K' = 0.$$

On voit donc que, du moins pour les substances qui présentent ce phénomène, la caractéristique n'est pas une quantité constante, et que, par conséquent, les formules de Cauchy n'y sont pas applicables.

Pour acquérir une connaissance plus détaillée des causes qui déterminent la polarisation elliptique du rayon réfléchi, et, en particulier, pour trouver jusqu'à quel point les formules théoriques sont vérifiées par l'expérience, j'ai commencé, il y a deux ans, quelques recherches sur la réflexion à la surface de diverses substances opaques, et spécialement de la fuchsine, la découverte remarquable de M. Christiansen de la dispersion anomale ayant dirigé l'attention vers les propriétés optiques de cette substance. Je l'ai étudiée à l'état solide, déposée sur la face de l'hypothénuse d'un prisme de verre, la réflexion s'opérant à cette surface commune aux deux corps. Les premiers prismes que j'employai, quoique d'un très-bon usage pour d'autres buts, laissent cependant beaucoup à désirer quand il s'agit d'expériences de ce genre, et c'est seulement depuis l'été dernier, après m'être procuré deux nouveaux prismes, construits pour ce but par MM. Steinheil, que j'ai obtenu des résultats suffisamment exacts, que je vais rapporter ici.

a) *Mesure du rapport des amplitudes et de la différence de phase.*

L'instrument employé dans ces expériences, est de la même construction que celui qui a servi à M. Jamin dans ses recherches sur les corps transparents, et, par conséquent, il ne sera pas besoin d'en faire ici la description complète. Voici cependant quelques détails.

Comme les deux prismes de Nicol appartenant à l'instrument, produisaient une déviation trop grande du rayon transmis, ils furent remplacés par deux autres d'une construction plus parfaite. Pour le nouveau polarisateur, en effet, dont la longueur est de 62 millimètres, la déviation ne monte qu'à 3 minutes tout au plus. et pour l'analyseur, qui mesure une longueur

de 49 millimètres, elle est presque insensible. Le compensateur employé a le défaut que la frange obscure centrale se déplace un peu, quand varie l'azimut du rayon polarisé incident sur cet appareil. Cependant, pour des valeurs égales et de signes contraires de cet azimut, le déplacement offre aussi des valeurs égales et de signes contraires, de sorte qu'on peut faire disparaître des moyennes l'erreur provenant de ce déplacement.

Il était évidemment nécessaire, dans ces recherches, d'étudier séparément la réflexion des différentes couleurs. Dans ce but, j'ai essayé au commencement, un procédé que m'avait proposé M. Ångström. On fait tomber sur la surface réfléchissante de la lumière blanche et plane polarisée, et le rayon réfléchi, après avoir traversé le compensateur et le nicol oculaire, est reçu dans un spectroscope à vision directe. La polarisation rectiligne des rayons des diverses couleurs est rétablie au moyen des parties latérales du prisme mobile du compensateur. Quoique les expériences préliminaires eussent en général assez bien réussi, je fus pourtant obligé d'abandonner cette méthode, parce que mon instrument était trop flexible pour pouvoir supporter sans inconvénient le poids d'un spectroscope, même plus léger que celui qui était à ma disposition.

Cette méthode ressemble à celle dont s'est servi M. E. Wiedemann dans ses recherches sur le même sujet. Elle diffèrent surtout en ce que, dans son appareil, le faisceau réfléchi est ramené à l'état de polarisation rectiligne au moyen d'une lame de mica d'un quart d'onde.

Les expériences dont je vais exposer les résultats, sont exécutées, à l'aide du procédé employé par M. Jamin dans ses recherches sur les couleurs des métaux, et par M. van der Willigen. Un faisceau très-mince de lumière solaire est renvoyé par un héliostat dans une direction horizontale et fixe, et, en le recevant d'abord sur une lentille achromatique, puis sur un prisme de flint-glass, tourné au minimum de la déviation, on obtient un spectre pur, où l'on voit un nombre considérable de raies. Ce prisme glisse sur une règle dans la direction du rayon incident, de sorte qu'il est possible de faire tomber sur le polarisateur successivement les divers rayons simples. Après avoir traversé ce nicol, le faisceau incident est reçu, avant de se réfléchir, dans un spectroscope, où la longueur d'onde en est déterminée avec l'exactitude suffisante. L'analyse des rayons réfléchis s'opère de la manière connue au moyen du compensateur et du nicol oculaire.

Pour avoir les valeurs absolues des azimuts du rayon incident et du rayon réfléchi, il faut déterminer les points zéro des cercles azimutaux. J'ai effectué cette détermination par un procédé indiqué par M. Kurz [1]). On

[1]) *Pogg. Ann.* CVIII, p. 586.

fait tomber sur la surface polie d'une plaque de crown-glass, sous une incidence constante, de la lumière polarisée par le premier nicol, et l'on observe les azimuts du rayon réfléchi pour les divers azimuts du rayon incident.

Les deux tubes de cuivre dans lesquels sont enchâssés les nicols, portent, pour l'appréciation des incidences, à leurs deux extrémités des fils croisés. Une lame de cuivre, percée en son milieu d'une fente verticale de largeur convenable, est placée devant le compensateur pour protéger l'œil contre la lumière superflue. Il faut remarquer, enfin, que, ayant la vue assez courte, je n'avais besoin d'aucune lentille pour voir distinctement les franges produites par le compensateur.

Au centre du grand cercle, qui reste toujours horizontal dans ces recherches, est disposée une petite règle pouvant tourner autour de l'axe de l'instrument. Une plaque glissant sur cette règle, porte une plate-forme à vis calantes, sur laquelle se place la substance à observer. Cette disposition permet de placer verticalement la surface réfléchissante et en outre de la rapprocher ou de l'éloigner du centre du cercle.

La substance étudiée par ces expériences était de la fuchsine, comme nous l'avons déjà dit. Elle était appliquée sur la face de l'hypothénuse d'un prisme de crown-glass, haut de $27{,}5^{mm.}$ et qui a pour section principale un triangle rectangle isocèle, dont la base a $38{,}5^{mm.}$ de longueur. Le tableau suivant en donne l'indice de réfraction pour les raies principales de Frauenhofer.

Raies	*Indice*
B	1,5117
C	1,5127
D	1,5154
E	1,5188
F	1,5218
G	1,5273
H_1	1,5322

Dans ce prisme aussi, il se manifeste les phénomènes produits par les verres trempés, quoique ce défaut se rencontre ici à un degré comparativement petit. Pour que l'influence n'en soit point sensible, les mesures de la différence de phase ne furent pas étendues à des incidences plus grandes que de 62 degrés.

La fuchsine se composait principalement d'acétate de rosaniline mélangé avec une certaine quantité d'arsénite de la même base. La solution de cette substance dans de l'esprit-de-vin est filtrée et évaporée jusqu'à ce qu'elle prenne un reflet vert métallique; alors on la laisse se refroidir. En-

suite on en verse une petite quantité sur la face de l'hypothénuse du prisme bien nettoyé, sur laquelle sont tirées à la paraffine des bandes déliées pour limiter la couche mince ainsi obtenue, qu'on fait sécher complètement. Elle n'est diaphane que pour la lumière rouge d'une grande intensité. Si le même procédé est répété une ou deux fois, on aura une couche parfaitement opaque. Il ne faut pas l'employer immédiatement, parce que les constantes de réflexion sont au premier abord sujettes à des variations assez considérables. Pour prévenir, autant que possible, des changements durant les observations, j'ai couvert, avant de les commencer, la fuchsine d'une couche de vernis d'asphalte. J'ai fait aussi des essais avec de l'acétate de rosaniline plus pur, mais les surfaces en prennent bientôt un aspect cristallin, et alors on ne peut plus les employer.

Afin d'avoir rapidement, pour une incidence donnée α, la position de l'alidade qui porte l'analyseur, j'ai calculé, à l'aide de la formule

$$\beta = 45^\circ + \operatorname{Arcsin}[\mu \sin(\alpha - 45^\circ)],$$

une table, qui donne, pour des incidences successives, les valeurs de l'angle 2β, que fait le rayon émergeant du prisme sur lequel est appliquée la fuchsine, avec le rayon incident sur ce prisme. Soient φ_1 la lecture sur le cercle horizontal faite au vernier de l'analyseur, quand il est traversé par le rayon réfléchi sous une incidence quelconque, et φ_0, cette lecture quand la lumière incidente est reçue directement dans l'analyseur sans se réfléchir; alors φ_1 se calculera pour une incidence donnée par la formule

$$\varphi_1 = \varphi_0 \pm (2\beta - 180^\circ),$$

où l'on doit prendre le signe + ou le signe — suivant que le rayon incident dévie à droite ou à gauche par l'effet de la réflexion.

Les observations sont exécutées pour les raies de Frauenhofer B, C, D, E, F, et en outre pour deux autres, l'une située dans l'indigo, l'autre dans le violet, désignés respectivement ci-dessous par I et par V.

Les résultats des expériences sont renfermés dans les tableaux suivants, où l'on désignera

par α l'angle d'incidence;
„ A la valeur principale de α;
„ ϑ l'azimut du rayon incident;
„ ϑ_1 l'azimut du rayon réfléchi, la polarisation en ayant été rétablie;
„ θ la valeur que prendrait ϑ_1, si ϑ était égal à 45 degrés;
„ Θ la valeur principale de θ;
„ n le nombre indiquant la position du micromètre du compensateur, en prenant pour point de départ la position où le compensateur ne produit point de différence de phase;

par N la valeur de n pour une différence de phase égale à un quart d'onde;
„ d la différence de phase, introduite par la réflexion, entre les deux composantes de la vibration réfléchie, la longueur d'un quart d'onde étant prise comme l'unité.

Les valeurs observées de ϑ, et de n sont consignées dans les deuxièmes et les quatrièmes colonnes ou dans les troisièmes et les cinquièmes, suivant que le rayon incident a dévié à droite ou à gauche par l'effet de la réflexion. Les nombres qui y sont donnés, sont au surplus, sous la plupart des incidences, les moyennes des déterminations correspondantes à deux valeurs de ϑ, égales et de signes contraires. Remarquons, en outre, que les expériences ont été disposées de manière que les moyennes finales de ϑ, et n, qui ne sont pas données dans les tableaux, peuvent être regardées comme fournies par des observations simultanées pour les diverses couleurs. Il est bien entendu qu'on s'est servi de ces dernières moyennes pour déduire les valeurs de θ et de α. Ces valeurs étant représentées graphiquement, les grandeurs A et Θ sont déterminées par une interpolation très-facile.

Avec les résultats expérimentaux on trouve dans ces tableaux des nombre calculés en regard. Pour les raies D, E, F, I, ils ont été déduits des formules (63), (68) et (69), établies pour la réflexion métallique, les valeurs de c et γ ayant été calculées au moyen des relations (71) et (72).

Pour la raie C le calcul s'est effectué par les formules (26) et (29) établies pour les milieux transparents. Afin d'augmenter la précision, la plupart des observations ont été employées à déduire les valeurs de μ et de $\varkappa$.

Les valeurs données pour la raie B ont été calculées au moyen de l'un et l'autre groupe de formules.

Réflexion, dans le crown-glass, sur la fuchsine.

Tableau I. Raie B.

$\vartheta = 75°$ $\lambda = 686{,}7$ $N = +8{,}92$

α	ϑ_1		n		θ					d				
	à droite	à gauche	à droite	à gauche	observé	calculé(1)	différ. (1)	calculé(2)	différ. (2)	observé	calculé(1)	différ. (1)	calculé(2)	différ. (2)
35°	58° 54′	59° 24′	0,16	0,18	24° 10′	24° 44′	− 34′	24° 53′	− 43′	− 0,019	− 0,013	− 0,006	− 0,009	− 0,010
39	51 52	52 8	0,19	0,12	18 56	19 14	− 18	19 18	− 22	− 0,017	− 0,018	+ 0,001	− 0,014	− 0,003
44	36 14	37 26	0,20	0,11	11 21	11 31	− 10	11 47	− 26	− 0,017	− 0,032	+ 0,015	− 0,027	+ 0,010
47	21 44	24 22	0,52	0.13	6 30	6 34	− 4	6 40	− 10	− 0,036	− 0,056	+ 0,020	− 0,053	+ 0,017
49	10 12	12 4	1,59	0,93	3 1	3 14	− 13	3 20	− 19	− 0,141	− 0,115	− 0,026	− 0,108	− 0,033
50	4 42	4 6	4,30	2,85	1 11	1 36	− 25	1 42	− 31	− 0,401	− 0.236	− 0,165	− 0,211	− 0,190
51	2 14	1 21	9,28	—	0 29	0 37	− 8	0 52	− 23	− 1,040	− 1,218	+ 0,178	− 1,148	+ 0,108
52	7 48	7 42	16,73	15,99	2 5	2 0	+ 5	1 46	+ 19	− 1,834	− 1,814	− 0,020	− 1,788	− 0,046
53	13 28	13 25	17,32	17,07	3 40	3 39	+ 1	3 19	+ 21	− 1,928	− 1,900	− 0,028	− 1,890	− 0,038
55	23 40	23 41	17,63	17,30	6 42	6 56	− 14	6 38	+ 4	− 1,958	− 1,949	− 0,009	− 1,944	− 0,014
58	37 47	36 50	17,81	17,62	11 32	11 45	− 13	11 31	+ 1	− 1,986	− 1,971	− 0,015	− 1,967	− 0,019
61	46 53	46 14	17,87	17,62	15 48	16 18	− 30	16 5	− 17	− 1,989	− 1,980	− 0,009	− 1,975	− 0,014

$A = 50°52'$ $\Theta = 0° 36'$

(1) $\mu = 1{,}230$ $\varkappa = 0{,}010$

(2) $c = 1{,}233$ $\gamma = 0° 43'$

Tableau II. Raie C.

$\lambda = 656{,}2$

$\vartheta = 75°$ $N = +8{,}51$

α	ϑ_1		n		θ			d		
	à droite	à gauche	à droite	à gauche	observé	calculé	différ.	observé	calculé	différ.
35° 0′	60° 6′	60° 10′	0,18	0,18	25° 1′	25° 23′	−22′	−0,021	−0,008	−0,013
39	53 0	52 58	0,16	0,19	19 34	20 5	−31	−0,021	−0,011	−0,010
44	38 24	38 50	0,23	0,24	12 5	12 35	−30	−0,027	−0,019	−0,008
47	26 28	25 56	0,35	0,20	7 31	7 46	−15	−0,032	−0,032	0,000
49	15 12	15 27	0,42	0,31	4 12	4 28	−16	−0,043	−0,055	+0,012
50	10 6	8 49	1,02	0,32	2 34	2 48	−14	−0,079	−0,087	+0,008
51	3 7	3 0	2,70	1,68	0 49	1 10	−21	−0,257	−0,214	−0,043
51 30	1 56	1 5	—	—	0 24	0 28	− 4	—	−0,604	—
52	3 10	2 39	13,30	13,04	0 47	0 41	+ 6	−1,547	−1,623	+0,076
53	8 22	9 38	16,12	16,45	2 26	2 16	+10	−1,913	−1,893	−0,020
55	19 31	20 17	16,87	16,77	5 32	5 33	− 1	−1,977	−1,957	−0,020
58	33 42	34 12	16,98	16,87	10 14	10 21	− 7	−1,989	−1,978	−0,011
61	44 6	44 41	17,07	16,87	14 42	14 57	−15	−1,994	−1,986	−0,008

$A = 51°30'$ $\Theta = 0°24'$ $\mu = 1{,}264$ $\varkappa = 0{,}006$

Tableau III. Raie D.

$\lambda = 589{,}2$

$\vartheta = 65°$ $N = -7{,}57$

α	ϑ_1		n		θ			d		
	à droite	à gauche	à droite	à gauche	observé	calculé	différ.	observé	calculé	différ.
35° 0′	48° 38′	48° 36′	− 0,47	− 0,50	27° 53′	28° 16′	−23′	0,064	0,079	−0,015
40	41 22	41 13	− 0,85	− 0,98	22 17	22 40	−23	0,121	0,125	−0,004
44	33 48	33 44	− 1,32	− 1,54	17 19	17 42	−23	0,189	0,188	+0,001
48	25 8	25 12	− 2,21	− 2,48	12 21	12 32	−11	0,310	0,305	+0,005
51	18 35	18 53	− 3,56	− 3,95	8 59	9 2	− 3	0,496	0,480	+0,016
53	14 46	15 30	− 5,10	− 5,37	7 11	7 17	− 6	0,692	0,684	+0,008
54	13 56	14 58	− 6,12	− 6,42	6 51	6 46	+ 5	0,829	0,821	+0,008
55	13 40	14 18	− 7,23	− 7,49	6 37	6 35	+ 2	0,973	0,970	+0,003
55 12	13 22	13 37	− 7,86	− 7,53	6 23	6 36	−13	0,990	0,996	−0,006
56	14 14	15 19	− 8,33	− 8,49	7 0	6 47	+13	1,111	1,120	−0,009
57	15 3	15 45	− 9,26	− 9,41	7 19	7 18	+ 1	1,233	1,255	−0,022
59	18 32	19 2	−10,81	−10,98	9 1	9 3	− 2	1,439	1,459	−0,020
61	22 49	23 52	−11,89	−12,00	11 22	11 20	+ 2	1,578	1,590	−0,012

$A = 55°12'$ $\Theta = 6°36'$ $c = 1{,}4221$ $\gamma = 8°56'$

Tableau IV. Raie E.

$\lambda = 526{,}9$

$\vartheta = 65°$ $N = -6{,}70$

α	ϑ_1 à droite	ϑ_1 à gauche	n à droite	n à gauche	θ observé	θ calculé	θ différ.	d observé	d calculé	d différ.
35° 0′	46° 58′	47° 1′	−1,33	−1,43	26° 34′	26° 47′	−13′	0,206	0,225	−0,019
39	41 19	41 55	−2,12	−2,16	22 30	22 44	−14	0,319	0,325	−0,006
44	34 5	35 14	−3,47	−3,50	17 52	18 12	−20	0,520	0,522	−0,002
47	30 50	32 44	−4,56	−4,62	16 7	16 22	−15	0,685	0,688	−0,003
49	30 20	31 31	−5,51	−5,52	15 37	15 44	− 7	0,823	0,815	+0,008
50	29 49	31 16	−5,94	−5,96	15 23	15 37	−14	0,888	0,881	+0,007
51	30 20	31 54	−6,32	−6,40	15 43	15 39	+ 4	0,947	0,948	−0,001
51 34	30 25	32 7	−6,60	−6,64	15 49	15 44	+ 5	0,988	0,986	+0,002
52	30 35	32 3	−6,75	−6,79	15 50	15 50	0	1,011	1,014	−0,003
53	30 58	32 39	−7,25	−7,22	16 8	16 9	− 1	1,080	1,077	+0,003
55	33 7	34 2	−8,01	−7,98	17 12	17 7	+ 5	1,193	1,195	−0,002
58	36 36	36 46	−9,00	−8,89	19 9	19 13	− 4	1,335	1,341	−0,006
61	40 13	40 49	−9,80	−9,71	21 44	21 46	− 2	1,456	1,457	−0,001

$A = 51° 48′$ $\Theta = 15° 48′$ $c = 1{,}1784$ $\gamma = 19° 53′$

Tableau V. Raie F.

$\lambda = 486{,}0$

$\vartheta = 65°$ $N = -6{,}13$

α	ϑ_1 à droite	ϑ_1 à gauche	n à droite	n à gauche	θ observé	θ calculé	θ différ.	d observé	d calculé	d différ.
35° 0′	40° 42′	40° 46′	−2,18	−2,21	21° 53′	22° 10′	−17′	0,358	0,361	−0,003
39	35 40	35 56	−3,41	−3,52	18 35	18 39	− 4	0,566	0,558	+0,008
42	33 56	34 13	−4,49	−4,85	17 31	17 22	+ 9	0,762	0,750	+0,012
44	33 12	34 20	−5,41	−4,93	17 19	17 21	− 2	0,891	0,887	+0,004
45	33 39	34 42	−5,74	−5,88	17 34	17 36	− 2	0,948	0,953	−0,005
45 30	33 51	34 46	−5,98	−6,09	17 39	17 47	− 8	0,985	0,984	+0,001
46	34 42	35 24	−6,18	−6,25	18 7	18 0	+ 7	1,014	1,015	−0,001
47	35 27	36 20	−6,51	−6,67	18 39	18 33	+ 6	1,075	1,075	0,000
48	36 32	37 7	−6,87	−6,98	19 15	19 11	+ 4	1,130	1,129	+0,001
50	38 10	39 6	−7,42	−7,63	20 26	20 43	−17	1,228	1,225	+0,003
53	41 50	42 44	−8,21	−8,39	22 59	23 21	−22	1,354	1,340	+0,014
56	45 46	46 58	−8,74	−8,89	26 4	26 4	0	1,438	1,429	+0,009
60	48 24	50 25	−9,30	−9,46	28 33	29 29	−56	1,531	1,521	+0,010

$A = 45° 45′$ $\Theta = 17° 54′$ $c = 0{,}9249$ $\gamma = 18° 26′$

Tableau VI. Raie I.

$\lambda = 447{,}5$

$\vartheta = 65°$ $N = -5{,}61$

α	ϑ_1 à droite	ϑ_1 à gauche	n à droite	n à gauche	θ observé	θ calculé	θ différ.	d observé	d calculé	d différ.
35° 0′	30° 32′	31° 26′	−1,65	−1,54	15° 39′	15° 34′	+0° 5′	0,284	0,252	+0,032
38	24 20	25 30	−2,69	−2,88	12 13	10 45	+1 28	0,496	0,459	+0,037
40	20 24	21 6	−4,25	−3,65	10 1	8 45	+1 16	0,704	0,712	−0,008
41	18 54	19 13	−5,03	−5,11	9 9	8 28	+0 41	0,912	0,866	+0,046
41 13	17 32	19 5	−5,25	−5,34	8 46	8 28	+0 18	0,944	0,903	+0,041
42	18 48	19 42	−5,63	−5,94	9 15	8 41	+0 34	1,031	1,030	+0,001
43	20 18	19 30	−7,01	−6,98	9 35	9 26	+0 9	1,247	1,176	+0,071
45	23 12	23 24	−7,89	−8,06	11 21	12 17	−0 56	1,422	1,383	+0,039
48	32 42	33 6	−8,84	−8,99	16 47	17 3	−0 16	1,589	1,540	+0,049
52	42 18	42 26	−9,37	−9,50	23 2	23 46	−0 44	1,682	1,648	+0,034
56	49 4	50 28	−9,62	−9,73	28 52	29 36	−0 44	1,725	1,692	+0,033
60	54 53	54 47	−9,73	−9,96	33 30	33 56	−0 26	1,755	1,716	+0,039

	A	Θ		
Déduits des observations:	41°36′	9° 0′		
Employés dans le calcul:	41°48′	8°36′	$c = 0{,}874$	$\gamma = 7°37'$

Tableau VII. Raie V.

$\lambda = 422{,}6$

$\vartheta = 65°$ $N = -5{,}25$

α	ϑ_1 à droite	ϑ_1 à gauche	n à droite	n à gauche	θ observé	d observé
35°	39° 30′	34° 54′	− 1,36	− 1,57	19° 30′	0,28
39	27 30	26 18	− 2,10	− 2,52	13 18	0,44
41	10 18	10 0	—	—	4 48	—
42	8 6	11 6	—	− 5,67	4 30	1,08
43	8 48	9 6	− 7,56	− 7,45	4 12	1,43
44	9 18	11 12	− 9,77	− 9,45	4 48	1,83
45	12 24	11 6	−10,29	− 9,76	5 30	1,91
47	17 30	16 54	−10,34	− 9,24	8 12	1,86
50	27 30	26 0	−10,39	− 9,87	13 12	1,93
53	36 6	38 24	− 9,97	−10,34	19 30	1,94
57	45 42	47 6	−10,03	− 9,92	26 6	1,90
61	52 42	52 36	− 9,92	−10,08	31 24	1,90

$A = 41°36'$ $\Theta = 4°36'$

Les valeurs de A et de Θ pour les raies observées se trouvent réunies dans le tableau suivant.

Tableau VIII.

	Raies.						
	B	C	D	E	F	I	V
A	50°52′	51°30′	55°12′	51°48′	45°45′	41°36′	41°36′
Θ	0 36	0 24	6 36	15 48	17 54	9 0	4 36
μ_0	1,230	1,264	1,405	1,108	0,877	0,864	—

On voit, par ce tableau, que l'azimut principal croît du violet au bleu, pour décroître ensuite jusqu'au commencement du rouge, où il devient égal à zéro, et que, vers le rouge extrême, il se produit en dernier lieu une légère croissance. L'incidence principale, à peu près constante dans le violet, où l'ellipticité de la lumière réfléchie est comparativement petite, va en croissant de l'indigo à l'orange, c'est-à-dire dans la partie du spectre qui renferme tous les rayons auxquels la fuchsine imprime une polarisation elliptique prononcée, atteint sa plus grande valeur à la raie D, pour décroître ensuite dans le rouge. On retrouve donc ici la loi qu'a énoncée M. van der Willigen en se fondant sur les résultats obtenus par ses recherches sur l'indigo et sur le fer oligiste. Les nombres suivants montrent que le point du spectre pour lequel l'incidence principale atteint au maximum, coïncide avec la raie D.

α	λ	θ	d
55°12′	586.5	7°16′	1,002
	589,2 D	6 23	0,990
	591,8	5 46	1,003

Le tableau VIII contient aussi des valeurs calculées de l'indice de réfraction au passage du crown-glass de dans la fuchsine pour le cas de l'incidence normale. Il est déduit pour les raies D, E, F, I de la formule (73)

$$\mu_0 = c \cos \gamma.$$

La dispersion en résultant est anomale et très-considérable.

Les tableaux I et II montrent que la fuchsine se comporte dans le cas actuel comme une substance à *réflexion négative* vis-à-vis des rayons rouges. Ce phénomène peut donc se présenter à la surface de deux substances à l'état solide même pour un indice plus grand que l'unité. Lorsque, au contraire, la réflexion sur la fuchsine a lieu dans l'air, elle est positive pour toutes les couleurs, comme l'a reconnu M. E. Wiedemann. Des valeurs de

A et de Θ trouvées par lui pour ce cas, sont renfermées dans les quatrième et septième colonnes du tableau suivant. Pour faciliter la comparaison, j'ai exprimé, dans les deuxième et cinquième colonnes, les grandeurs correspondantes tirées de mes observations.

Tableau IX.

Raies	A			Θ		
	VERRE-FUCHSINE		AIR-FUCHSINE	VERRE-FUCHSINE		AIR-FUCHSINE
	observé	calculé		observé	calculé	
C	51°30′	56° 2′	66° 3′	0°24′	—	4°48′
D	55 12	56 34	65 45	6 36	15° 1′	12 41
E$\frac{1}{2}$b	51 18	51 2	59 48	16 12	23 4	18 47
F	45 45	44 15	53 0	17 54	24 12	17 45
F$\frac{1}{2}$G	41 48	38 26	50 23	10 48	1 46	1 9

Les nombres calculés qui se trouvent dans ce tableau, ont été déduits au moyen des formules que nous donnerons plus loin.

En parcourant les colonnes des différences des tableaux III, IV et V, on voit que l'accord entre les valeurs observées et les valeurs calculées est aussi complet que possible pour les raies D, E, F. Il faut remarquer, en outre, que les formules simplifiées (80) et (81) ne reproduisent pas ces observations avec une exactitude suffisante. Ainsi, par exemple, il résulte des formules en question que le minimum de θ doit coïncider avec la valeur principale de cet angle, résultat que l'expérience ne justifie pas pour le cas actuel.

Pour la raie I, les différences entre les valeurs observées et les calculées, sont plus grandes, et pour la raie V j'ai supprimé les valeurs calculées, parce qu'elles ne sont point compatibles avec les observations. Ce défaut de concordance tient évidemment en grande partie aux erreurs d'observation, qui étaient assez considérables pour ces deux couleurs, à cause de la faible intensité des rayons réfléchis; mais il doit aussi être attribué à ce que les formules théoriques cessent d'être applicables pour ces couleurs. C'est ce que l'on peut faire voir de la manière suivante. Si, pour abréger, on pose dans les formules (68) et (69) (pag. 26)

$$\frac{\sin^2\alpha}{r\cos\alpha} = \operatorname{tang}\varpi,$$

on peut les remplacer par celles-ci:

$$\operatorname{tang}\varrho = \sin\Delta \operatorname{tang} 2\theta\ ,\quad \cos 2\varpi = \cos\Delta \sin 2\theta. \quad . \quad . \quad . \quad . \quad (82)$$

En désignant par χ un angle donné par l'expression

$$\operatorname{tang}\chi = \frac{\operatorname{tang}\varpi}{\operatorname{tang}\alpha},$$

on tire des équations (63) les formules

$$\operatorname{tang}(2\gamma - \varrho) = \operatorname{tang}\varrho\cos 2\chi\,,\quad c = \sin\alpha\sqrt{\frac{\sin 2\varrho}{\sin 2(\varrho-\gamma)}} \quad . \quad . \quad . \quad . \quad (83)$$

Les observations ayant fait connaître les grandeurs θ et Δ, la quantité ϱ se déduit de la première des relations (82), ensuite les formules (83) servent à calculer les valeurs de c et γ pour les diverses incidences. Ce calcul, exécuté pour les raies E, I, V, a donné les résultats suivants:

Tableau X.

Raie E			Raie I			Raie V		
α	log c	γ	α	log c	γ	α	log c	γ
35° 0′	0,0767	18°12′	35° 0′	9,938	8°29′	35° 0′	9,970	12°18′
39	0,0715	19 14	38	9,942	9 22	39	9,974	10 14
44	0,0703	19 20	40	9,943	8 50	41	—	—
47	0,0717	19 31	41	9,936	8 5	42	9,948	3 55
49	0,0694	19 42	41 13	9,937	7 47	43	9,951	2 46
50	0,0703	19 31	42	9,940	8 6	44	9,954	1 2
51	0,0714	19 59	43	9,936	7 8	45	9,962	0 38
51 34	0,0704	19 58	45	9,943	6 54	47	9,973	1 33
52	0,0721	19 56	48	9,942	6 52	50	9,978	1 6
53	0,0708	19 50	52	9,946	6 46	53	9,975	1 15
55	0,0710	20 1	56	9,948	6 39	57	9,976	2 33
58	0,0730	20 2	60	9,949	6 27	61	9,978	2 50
61	0,0716	19 53	—	—	—	—	—	—

Ce tableau montre que les quantités c et γ sont sensiblement constantes pour la raie E, et, en ayant égard aux erreurs d'observation, on en peut dire autant de la quantité c pour les deux autres raies, mais il n'en est plus de même pour γ, qui diminue à mesure que l'incidence augmente.

Pour les raies B et C, les nombres calculés au moyen des formules établies pour les corps transparents, reproduisent les observations avec une fidélité satisfaisante. Le tableau I montre de plus que les résultats trouvés expérimentalement pour la raie B sont aussi représentés avec une approximation suffisante par les formules établies pour la réflexion métallique, bien que, la réflexion étant négative, l'extinction du rayon réfracté ne puisse pas déterminer l'ellipticité de ce rayon.

De cette comparaison il résulte, ce me semble, que les formules établies pour la réflexion métallique reproduisent toujours les observations avec exactitude, quand l'incidence principale est plus grande que de 50 degrés, mais qu'elles ne sont applicables pour des valeurs de cette incidence inférieures à 45 degrés qu'à condition que l'azimut principal soit assez considérable.

En ayant égard à ces faits, on est conduit à admettre que l'ellipticité observée par M. Jamin chez plusieurs substances opaques, comme le sélénium, la houille, l'anthracite, le bitume de Judée, le goudron de gaz etc.[1]), doit être en partie attribuée à l'extinction du rayon réfracté, bien qu'il ait prouvé que les phénomènes de la réflexion sur ces substances sont représentés par les formules établies pour les corps transparents.

Lorsqu'un rayon propagé dans un milieu transparent, va tomber sur la surface réfléchissante d'une substance opaque, l'intensité et la polarisation du rayon réfléchi sont, selon la théorie de Cauchy, déterminées à l'aide de la constante complexe désignée sous le nom de caractéristique. Cette constante ne change pas de valeur, si le premier milieu transparent est remplacé par un autre, la substance opaque restant la même.

Soit λ la longueur de l'onde dans le premier milieu transparent, λ_1 dans le second et ainsi de suite, on a (pag. 24)

$$\frac{2\pi}{\lambda} c e^{i\gamma} = \frac{2\pi}{\lambda_1} c_1 e^{i\gamma_1} = \frac{2\pi}{\lambda_2} c_2 e^{i\gamma_2} \text{ etc.}$$

d'où l'on tire

$$\gamma = \gamma_1 = \gamma_2 = \text{etc.}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

et encore

$$\frac{c}{\lambda} = \frac{c_1}{\lambda_1} = \frac{c_2}{\lambda_2} = \text{etc.}$$

Ces dernières équations peuvent s'écrire

$$c = \mu_1 c_1 = \mu_2 c_2 = \text{etc.},$$

en désignant par μ_1, μ_2, etc. les indices de réfraction respectifs du second, du troisième etc. des milieux transparents par rapport au premier. Des expériences ayant donné les constantes A et Θ pour l'un de ces cas, les équations (84) et (85), jointes aux formules (71) et (72), peuvent servir à calculer ces grandeurs pour d'autres cas.

M. Quincke a fait réfléchir sur de l'argent de la lumière orangée se propageant dans diverses substances transparentes. Les résultats de ses mesures sont consignés dans le tableau suivant[2]). En regard de ceux-ci,

[1]) *Ann. de Chim. et de Phys.* (3) XXIX, p. 303; (3) XXXI, p. 179.

[2]) *Pogg. Ann.* CXXVIII, p. 561.

on trouvera des nombres calculés par le procédé que nous venons de décrire. Les observations faites sur la réflexion dans l'air y sont prises pour points de départ.

Tableau XI. *Réflexion sur l'argent*

dans	μ	A		Θ	
		observé	calculé	observé	calculé
Air	1,000	74°19′	—	43°48′	—
Eau	1,336	71 28	70°17′	44 3	43°52′
Essence de térébenthine	1,474	69 16	68 59	43 21	43 54
Air	1,000	74 50	—	43 20	—
Flint-glass	1,626	69 48	68 3	41 22	43 29
Air	1,000	75 57	—	44 1	—
Crown-glass	1,514	69 5	70 23	42 28	44 5

L'accord entre les valeurs observées et les valeurs calculées n'est pas parfaitement satisfaisant, mais en considérant que les déterminations de ce genre sont nécessairement entachées d'erreurs causées par le défaut d'homogénéité du métal, il faut admettre que les différences ne sont point assez grandes pour qu'il soit permis d'en tirer une objection contre la théorie de Cauchy.

En partant des valeurs des constantes A et Θ obtenues à la réflexion dans l'air sur la fuchsine et renfermées dans le tableau IX, j'ai calculé, à l'aide des mêmes formules, ces grandeurs pour la réflexion dans le crown-glass. Les résultats de ce calcul se trouvent dans le même tableau. La concordance entre les valeurs calculées de l'incidence principale et celles qui se déduisent de mes expériences, est plus complète pour les raies D, E$\frac{1}{2}$b, F, que l'on n'aurait pu s'y attendre, vu que la fuchsine employée n'a probablement pas été de la même espèce dans les deux séries d'expériences. Sur les rayons rouges, au contraire, l'influence de cette circonstance est très-prononcée.

b) *Mesure de l'intensité.*

Les observations précédentes ont prouvé que les formules établies pour la réflexion métallique s'appliquent également bien à la réflexion sur la fuchsine et, suivant toute probabilité, à la réflexion sur les substances à coloration superficielle en général, toutes les fois qu'il s'agit des couleurs vis-à-vis desquelles ces substances se comportent comme des métaux. Il

n'était pas, cependant, sans intérêt de chercher une autre vérification de ces formules dans des mesures directes de l'intensité des rayons réfléchis.

Des expériences de ce genre ont été faites pour les métaux par M. Jamin. Un faisceau polarisé dans les azimuts de 0 ou 90 degrés est reçu sur une lame, moitié en verre, moitié en métal, de telle sorte qu'il tombe sur la ligne de séparation des deux parties, et le faisceau réfléchi est analysé au moyen d'un prisme biréfringent: on voit alors dans cet analyseur deux images, dont chacune est formée de deux moitiés réfléchies l'une par le verre, l'autre par le métal. En cherchant les azimuts pour lesquels deux moitiés, une de chaque image, ont des intensités égales, on détermine l'intensité du rayon réfléchi sur le métal par rapport à celle du rayon réfléchi par le verre, laquelle peut être calculée au moyen des formules de Fresnel.

J'essayai cette méthode pour mesurer l'intensité de la lumière réfléchie par la fuchsine. Quelques essais suffirent cependant à me convaincre de l'impossibilité d'obtenir par ce procédé des résultats satisfaisants, et il faut certainement posséder l'habileté de l'illustre expérimentateur qui l'a employée, pour que les erreurs d'observation ne deviennent pas énormes. Il est, en effet, extrêmement difficile de rendre égales les intensités de deux moitiés des images non adjacentes, parce que les deux autres moitiés présentent des intensités toutes différentes. Pour cette raison j'ai préféré la méthode suivante.

Je me suis servi du même instrument dans ces expériences que dans celles dont j'ai déjà rendu compte. Le compensateur et le nicol polarisant ont été éloignés, et ce dernier est remplacé par un collimateur, dont la fente, qui est placée horizontalement, a $1{,}5^{mm.}$ de long et $0{,}6^{mm.}$ de large, et dans le tube duquel est fixé un prisme de Rochon. En traversant ce prisme, le faisceau incident se sépare en deux faisceaux polarisés, dont les plans de polarisation sont perpendiculaires entre eux. Le prisme est disposé de façon que ces faisceaux soient compris dans le même plan vertical, et qu'ils se trouvent parallèles en sortant de la lentille du collimateur. Si ces faisceaux, dont la distance est d'environ $2{,}5^{mm.}$, sont reçus sur l'analyseur, ils donnent dans le champ de vision d'une petite lunette portée sur la même alidade, deux images rectangulaires qui se touchent; et il n'y a pas de difficulté à trouver l'azimut de l'analyseur pour lequel les intensités de ces images sont égales. Cela doit avoir lieu dans quatre positions symétriques par rapport à deux lignes, l'une horizontale et l'autre verticale. Les résultats des observations faites dans les mêmes circonstances, diffèrent rarement de plus d'un degré, et à l'ordinaire de quantités beaucoup plus petites.

Pour employer cet appareil à des mesures de l'intensité, on prépare, par la méthode connue, un spectre pur, et les rayons de la couleur qu'on

veut étudier sont reçus d'abord sur un nicol dont l'axe optique se confond avec le prolongement de celui du collimateur. La lumière, après s'y être polarisée, pénètre dans le collimateur, et l'on reçoit les deux faisceaux qui en émergent, d'abord directement sur l'analyseur pour en rendre les intensités égales. Dans ce but on oriente la section principale de ce nicol dans l'azimut de 45 degrés; ensuite on fait tourner le polarisateur jusqu'à ce que les deux images prennent des intensités égales, et on laisse le polarisateur dans cette position durant les expériences. Cette détermination faite, on fixe le prisme de crown-glass, dont la face d'hypothénuse est à moitié couverte de la couche de fuchsine, sur le support à vis calantes qui se trouve au centre du cercle horizontal; en plaçant verticalement la surface réfléchissante, on fait affleurer en même temps la ligne horizontale qui limite la fuchsine, au plan horizontal passant par l'axe optique de l'analyseur, de sorte que l'un des faisceaux incidents soit réfléchi sur la fuchsine et l'autre sur l'air. Les faisceaux réfléchis sont reçus dans l'analyseur, et l'on détermine l'azimut σ pour lequel les intensités des images sont égales.

Dans les recherches dont je vais énoncer les résultats, la lumière réfléchie sur la fuchsine était toujours polarisée dans le plan d'incidence. Soit R^2 l'intensité du rayon réfléchi sur l'air, on a, l'intensité du rayon incident étant prise pour l'unité,

$$R^2 = \frac{\operatorname{tang}^2(\alpha - \alpha')}{\operatorname{tang}^2(\alpha + \alpha')}, \quad \text{si } \alpha < \operatorname{Arcsin}\frac{1}{\mu},$$

et

$$R^2 = 1, \qquad \text{si } \alpha > \operatorname{Arcsin}\frac{1}{\mu}.$$

Pour calculer l'intensité, du rayon réfléchi par la fuchsine, on obtiendra donc la formule

$$I_1^2 = R^2 \frac{\operatorname{tang}^2\sigma}{\cos^4(\alpha - \beta)},$$

où l'on a, comme précédemment (pag. 37),

$$\beta = 45^0 + \operatorname{Arcsin}[\mu \sin(\alpha - 45^0)].$$

Les résultats de ces expériences, qui se rapportent aux raies D, E, F, sont renfermés dans les tableaux suivants. On les trouvera dans la deuxième ou la troisième colonne, suivant que le rayon incident a dévié à droite ou à gauche par l'effet de la réflexion. Chaque détermination résulte des observations faites dans les quatre positions de l'analyseur.

Les nombres calculés, donnés dans ces tableaux, sont fournis par les formules (64) et (65), établies pour la réflexion métallique.

Réflexion, dans le crown-glass, sur la fuchsine.

Tableau XII. Raie D.

α	σ			I_1^2		
	à droite	à gauche	moyenne	observé	calculé	différ.
37°	64°50′	65°41′	65°15′	0,067	0,065	+0,002
45	18 5	17 54	18 0	0,106	0,087	+0,019
51	19 42	20 8	19 55	0,132	0,111	+0,021
55	21 14	22 15	21 45	0,162	0,133	+0,029
59	23 14	23 7	23 10	0,190	0,162	+0,028
64	25 22	26 32	25 57	0,254	0,210	+0,044
71	29 12	29 28	29 20	0,367	0,311	+0,056
80	31 48	32 4	31 56	0,583	0,534	+0,049

	A	Θ	
Employé dans le calcul:	55°12′	6°36′	(Tab. III).

Tableau XIII. Raie E.

α	σ			I_1^2		
	à droite	à gauche	moyenne	observé	calculé	différ.
37°	65°16′	65°20′	65°18′	0,080	0,073	+0,007
44	18 49	19 40	19 15	0,122	0,095	+0,027
49	20 10	21 30	20 50	0,145	0,118	+0,027
51	21 19	22 52	22 6	0,166	0,130	+0,036
53	22 3	23 5	22 34	0,175	0,142	+0,033
58	25 51	26 18	26 5	0,247	0,182	+0,065
68	30 20	30 0	30 10	0,377	0,305	+0,072
80	33 48	33 32	33 40	0,670	0,581	+0,089

	A	Θ	
Employé dans le calcul:	51°48′	15°48′	(Tab. IV).

Tableau XIV. Raie F.

α	σ			I_1^2				
	à droite	à gauche	moyenne	observé	calculé (1)	différ. (1)	calculé (2)	différ. (2)
37°	60°44′	60°36′	60°40′	0,057	0,067	−0,010	0,073	−0,016
42	17 10	15 40	16 25	0,087	0,086	+0,001	0,095	−0,008
45	18 42	17 37	18 9	0,107	0,101	+0,006	0,114	−0,007
47	20 2	19 44	19 53	0,131	0,112	+0,019	0,129	+0,002
50	21 26	21 2	21 14	0,152	0,133	+0,019	0,155	−0,003
55	24 14	24 5	24 9	0,205	0,178	+0,027	0,210	−0,005
66	31 26	29 58	30 42	0,385	0,325	+0,060	0,377	+0,008
80	34 12	33 48	34 0	0,692	0,641	+0,051	0,685	+0,007

		A	⊖	
Employé dans le calcul:	(1)	45°45′	17°54′	(Tab. V).
	(2)	43°42′	17°54′	

On voit, par ces tableaux, que la précision des mesures est assez satisfaisante, car l'intensité observée croît régulièrement avec l'incidence. De l'autre côté, les colonnes des différences montrent que les nombres calculés sont plus petits que les nombres fournis par les observations, si les valeurs principales de l'incidence et de l'azimut prises pour point de départ dans le calcul, sont celles qui résultent des expériences décrites plus haut (Tab. VIII, pag. 43). Il faut se rappeler, cependant, que dans ces dernières expériences, le faisceau incident s'est réfléchi sur la partie centrale de la surface de la fuchsine, tandis que dans les observations dont je viens de rendre compte, la réflexion a eu lieu tout près du bord de la couche de fuchsine, où l'état moléculaire ne peut pas être le même qu'au centre de la couche. Par conséquent, il est très-probable que, dans ces deux cas, les valeurs de A et de ⊖ ne sont pas identiques non plus, et que c'est de cette circonstance que dépend la différence indiquée entre les valeurs observées et les valeurs calculées.

Je n'ai pas encore eu occasion de déterminer directement les valeurs des constantes de réflexion qui correspondent à mes observations; mais afin de montrer que les formules théoriques peuvent représenter ces résultats avec exactitude, j'ai déduit pour la raie F, au moyen de ces formules, les valeurs calculées (2). On voit, par la colonne des différences correspondante que ces nombres sont complètement d'accord avec les résultats expérimentaux. La valeur de A employé dans ce calcul, ne diffère que de 2 degrés de celle qu'on a trouvée plus haut, et ⊖ a la même grandeur dans l'un et l'autre calculs.

Des observations renfermées dans les tableaux précédents, il résulte que, si la lumière incidente est blanche, la couleur du rayon réfléchi doit varier avec l'incidence. Pour l'incidence normale on trouve, en effet, que la fuchsine réfléchit plus de lumière verte et jaune que de lumière bleue, tandis que pour une incidence de 80 degrés, le cas est inverse. En observant que le rouge et le violet n'entrent que pour une faible part dans la lumière réfléchie, on peut donc en inférer que la couleur de la fuchsine appliquée sur le verre et regardée à travers cette substance, se changera de vert en bleu lorsque l'incidence augmente, pour devenir blanche à l'incidence rasante. Cette conclusion s'accord parfaitement avec l'expérience.

c) *Relation entre les pouvoirs réflecteur et absorbant.*

Lorsque un faisceau lumineux est reçu sur la surface polie d'une substance peu transparente, la quantité de lumière pénétrant dans le corps est d'autant plus petite que le pouvoir réflecteur en est plus grand. Quand la lumière incidente est blanche, et que le pouvoir réflecteur de la substance est différent vis-à-vis les diverses couleurs, il faut donc que le spectre de la lumière transmise présente des maxima et des minima, quand même les coefficients d'extinction des divers rayons seraient égales. Ainsi, par exemple, si la substance était une couche de fuchsine très-mince, on verrait dans le vert et le bleu du spectre une large bande noire, parce que ce corps présente un fort pouvoir réflecteur vis-à-vis ces couleurs. Tout cela est bien évident et se confirme du reste par l'expérience. Mais M. Stokes [1]) a reconnu, en outre, que le spectre de la lumière transmise par la dissolution d'une substance opaque offre aussi des bandes obscures, correspondantes aux rayons des couleurs que la substance, en l'état solide, réfléchit le plus fortement; et ce phénomène se manifeste lors même que la solution est si étendue que l'influence que la matière dissoute exerce sur la réflexion, doit être presque insensible.

Il n'est pas par là, cependant, parfaitement constaté que les bandes observées sont dues en totalité à l'absorption, car un faisceau passant à travers un milieu s'affaiblit aussi par la diffusion intérieure, et si le milieu possède cette propriété à un degré éminent, l'effet en peut devenir très-sensible. Afin d'examiner, de ce point de vue, le phénomène en question j'ai fait l'expérience suivante. Un vase cubique de verre, dont le côté a 2 déci-

[1]) *Phil. Mag.* (4) VI, p. 393.

mètres de longueur, est rempli d'une dissolution de fuchsine aqueuse et étendue. Sur une des faces latérales on projette un spectre pur tout près de la surface du liquide, de sorte qu'on puisse voir d'en haut y pénétrer la lumière. Les rayons rouges la traversent sans difficulté, mais à mesure qu'on advance dans le spectre, l'absorption devient de plus en plus énergique, et dès l'autre côté de la raie D on ne distingue plus de trace de la lumière pénétrante. Dans les parties bleues et violettes, la couche superficielle émet une lueur rougeâtre, qui n'est pourtant pas de la lumière diffuse, mais produite par la fluorescence.

Il paraît donc que, dans la plupart des cas, l'absorption est la seule cause des bandes obscures observées dans le spectre de la lumière transmise par les solutions de substances opaques. Comme il n'y a pas de raison de supposer que le cas soit différent, lorsque les substances se trouvent en l'état solide, on est ainsi conduit à admettre qu'à mesure que le pouvoir réflecteur est plus grand le coefficient d'absorption le sera aussi. Si la couche traversée est assez épaisse pour que la totalité de la lumière y entrant soit absorbée, on obtient le résultat assez curieux que la quantité absorbée d'une certaine sorte de lumière est d'autant plus petite que le coefficient d'absorption correspondant est plus grand.

Il est probable, cependant, que cette loi ne doit être interprétée dans un sens absolu. La réflexion est caractérisée par deux constantes distinctes, et pour que cette loi soit vraie dans toutes les circonstances, il faut qu'il y ait entre ces constantes une certaine relation dont on n'a pourtant pas encore prouvé l'existence.

A défaut de mesures exactes des coefficients d'absorption des corps solides, on peut en déduire les valeurs au moyen des formules de Cauchy, dans les cas auxquels elles sont applicables. On trouvera ainsi que la loi en question est en général verifiée pour les métaux, du moins si l'on considère chaque métal à part. Pour la fuchsine, au contraire, cela ne paraît pas être le cas, comme le montrent les nombres suivants. La première ligne indique l'intensité du rayon réfléchi, à l'incidence normale, dans le crown-glass, la seconde, le coefficient d'absorption correspondant U_0 (pag. 24).

	Raie D	Raie E	Raie F
Intensité:	0,036	0,037	0,028
Coefficient d'absorption:	3569	7258	5754

Quoique la lumière jaune soit réfléchie dans une plus grande proportion que la lumière bleue, on voit que le coefficient d'absorption de celle-

là est plus petit que celui de celle-ci. Cependant il ne faut pas attacher à ce résultat une trop grande importance, car, bien que la légimité des formules de Cauchy soit suffisamment constatée pour la réflexion, on n'en peut pas dire autant quand il s'agit des phénomènes de la réfraction.

Les expériences dont je viens d'exposer les résultats, et que j'ai l'intention de continuer, ont été faites dans le laboratoire physique de l'université d'Upsal, avec des appareils appartenant à cette institution.

PRODROMUS

MONOGRAPHIÆ OEDOGONIEARUM

AUCTORE

VEIT BRECHER WITTROCK.

CUM TABULA.

(Reg. Societati Scientiarum Upsaliensi traditum die 6 Dec. 1873.)

UPSALIÆ
EXCUDIT ED. BERLING REG. ACAD. TYPOGRAPHUS
MDCCCLXXIV.

Quum septem fere anni elapsi sint, quibus in *Oedogoniearum* cognoscenda familia operam ac studium collocavimus, ut harum plantarum, si fieri posset, monographia ad certum quendam finem adduceremus, tamen illud nunc propositum nostrum recessisse potius quam accessisse nobis videtur. Opinione enim nostra multo plures in illa familia occurrerunt formæ, omnibusque ex rebus concludere licet minime omnes, quas natura bene distinxit formas, cognitas nobis esse. Nec potest dubitari, quin pervestigatio terrarum, quas ad hanc cognoscendam familiam nemo adhuc satis perlustravit, maximeque earum extra Europam quæ sunt, novarum copiam formarum haud exiguam præbitura nobis sit. Atque etiam in iis, quæ ad morphologiam harum plantarum et physiologiam pertinent, licet præstantia in hac re effecerit celeberrimus PRINGSHEIM, tamen multa adhuc habemus inexplorata; velut in oosporarum germinatione tres tantum species sunt observatæ [1]). Itaque quum haud exiguo temporis spatio opus esse videatur, ut in cognitione familiæ *Oedogoniearum* ad aliquam saltem perveniamus perfectionem, huic nos cognitioni nonnihil prodesse arbitramur, si quæ inchoata in his atque manca consecuti sumus, ea jam nunc proferemus.

Est igitur *Oedogoniearum* familia satis definita, cui proximæ, quamquam non paullum dispares, hinc *Coleochetæ*, hinc *Sphæropleeæ* sunt. Tres illæ familiæ nobis una classe videntur contineri, quæ ex media maximaque earum *Oedogoniaceæ* [2]) nominetur. Consistit familia *Oedogoniearum* e duobus

[1]) Quarum cel. PRINGSHEIM unam disquisivit, alteram cel. CLEVE, tertiam cel. JURANYI; et observationes horum virorum doctissimorum inter se etiam discrepantes sunt. Quæ in hac re nos conati sumus, parum adhuc processerunt.

[2]) Character hujus classis ita concipi possit: *Oedogoniaceæ*, plantæ cellulares (ex aqua dulci) cellulis chlorophylliferis fila confervoidea, simplicia vel ramosa, raro discos parenchymaticos, monostromaticos formantibus; organis femineis oogoniis; org.

bene distinctis generibus, *Oedogonio* Link et *Bulbochæte* Ag[1]). Est in utroque genere et monoica sectio et dioica. Species dioicæ generis *Oedogonii* duas subsectiones formant; altera tales continet species, quæ plantas masculas nanas (nannandres) et in plantis femineis epiphyticas[2]) habent (species dioicæ nannandræ); in altera masculæ plantæ eadem magnitudine formaque ac femineæ et non epiphyticæ sunt (species dioicæ macrandræ).

Nannandres non eodem gradu in diversis speciebus sunt evolutæ. Quæ monoicis maxime affines sunt, plantas masculas tam paullum evolutas habent, ut unam tantum cellulam (scil. ante formationem spermatozoidarum binarum) contineant (nannandres unicellulares; vide fig. 6 et 11 n). Hæc cellula, ex qua una tota constat planta, spermogonii etiam munere fungitur, quippe quæ partita duas mobiles proferat cellulas masculas, spermatozoidas. Aliæ autem species nonnullæ ejusmodi sunt, ut nannandres ex cellulis minime duabus consistant, una inferiore, quæ permanet vegetativa (stipes), una vel paucis cellulis superioribus, quarum cum singulæ duas gignant spermatozoidas, cellulæ sunt spermogonicæ. Hæ vero aut solita cellularum partitione exoriuntur; tum spermogonium, cel. Pringsheim secuti, interius vocamus; aut per partitionem oedogoniaceam (i. e. ita, ut in transversum membrana cellulæ matris rumpatur) exteriusque tum spermogonium nominatur (vide fig. 13 et 15 n.) Nannandres interiore spermogonio unicellulare id habent; qui exteriore sunt spermogonio, hoc habent vel univel pluri-cellulare. Species nonnullæ, quæ nannandres possident spermogonio exteriore, stipitem eorum interdum habent pluricellularem[3]), iisque apparet speciebus concatenari nannandras et macrandras formas. Nannandres proprio sunt genere zoosporarum (androsporæ Pringsh.) orti, quæ modo — id quod plerumque fieri solet — in quibusdam cellulis (androsporangia) femineorum individuorum (vide fig. 10, 11, 15 a) gignuntur (species gynandrosporæ Pringsh.), modo in quibusdam cellulis, quæ in propriis individuis neutris (vide fig. 12 a) occurrunt (spec. idioandrosporæ).

masculis spermogoniis, spermatozoidas ciliatas proferentibus; oosporis, foecundatione genitis, zoosporas germinatione procreantibus; agamosporis mobilibus (zoosporis) singulis in singulis cellulis vegetativis genitis. (Zoosporæ tales adhuc non observatæ apud *Sphæropleeas.*)

[1]) Cel. PRINGSHEIM in præstantissimo illo opere "Morphologie der Oedogonieen" et familia quod habet et genera proprii optime tractavit.

[2]) Nannandres in individuis femineis parasiticos habent etiam animalia quædam Crustacea: *Chondracanthidæ*, *Lernæopodidæ* nonnullaque *Cirripedia.*

[3]) Apud *Oed. Braunii* Kütz. nannandres vidimus stipitem habentes usque ad 5-cellularem, apud *Oed. macrandrum* Wittr. et *Oed. sexangulare* Clev. 2—3-cellularem, apud *Oed. Aster* Wittr., alia 2-cellularem.

In genere *Bulbochæte* species dioicæ macrandræ desunt; eo plures contra species sunt dioicæ nannandræ, habentque et hæ modo nannandres unicellulares, modo bi- vel pluri-cellulares; pluricellulares vero nannandres qua interioribus, (vide fig. 18 et 20 n) qua exterioribus (vide fig. 22 n) prædíti spermogoniis sunt. Longe plurimæ species nannandræ generis *Bulbochætes* gynandrosporæ sunt; una tantum species idioandrospora[1]) observata adhuc est.

Oogonia utriusque generis ante foecundationem aperturam foecundationis in membrana efficiunt. In aliquot generis *Oedogonii* speciebus hæc apertura formam habet pori tenuis, qui in primaria fit oogonii membrana, nulla hujus antecedente ruptione annuliformi[2]) (oogonia poro aperta; vide fig. 1 et 13 oo.) In plerisque, si non in omnibus, harum specierum conspicitur sub illo poro membrana secundaria, quæ suo ordine apertura poriformi perforatur. In aliis ejusdem generis speciebus annuliformis effingitur ruptio membranæ oogonii, idque in medio fit plerumque; quo facto secundadaria pars membranacea inseritur, in qua porus foecundationis (oogonia circumscissa poro in circumscissione; vide fig. 6 et 11 oo). In aliis autem membrana oogonii prope apicem rumpitur, non tamen plene, ita ut superior membranæ pars cum inferiore uno loco cohæreat. Superior deinde membranæ pars uno latere velut operculum extollitur; dum hæc rimiformis formatur apertura, exsistit intra eam membrana secundaria poro perforata (oogonia operculo aperta; vide fig. 3 et 17 oo).

Oogonia generis *Bulbochætes* Ag. vidimus duplici formari modo. Interdum oogonia horizontali cellularum matrum partitione oriuntur. Cellulæ suffultoriæ tum ambæ (in sectione optica verticali visæ) tetragonæ fiunt et oogonium solum sustinent (vide 22 ss'). Talia nos oogonia erecta nominavimus. Plerumque autem oogonia post obliquam cellulæ aut matris aut aviæ partitionem exsistunt. Cellula suffultoria tum una fit pentagona, altera tetragona[3]), feruntque communiter juxta oogonium cellulam vegetativam trivialem vel setam terminalem. Oogonia hæc patentia nominentur. — Spermogonia etiam hujus generis duplici sunt indole: tum erecta, partitione horizontali orta, tum patentia, partitione obliqua.

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[1]) *B. polyandra* Cleve (vide fig. 19 et 20).

[2]) Observatur tamen sæpenumero ejusmodi ruptionis initium ut linea vel rima subtilissima circa oogonium eadem altitudine atque est porus foecundationis (vide fig. 4 oo.)

[3]) Apud species oogoniis globosis prædítas cellula mater partitione obliqua dividitur, superiorque tum cellula suffultoria pentagona fit, tetragona inferior (vide fig. 18 et 20 ss); apud species contra oogoniis ellipsoideis prædítas cellula avia partitione obliqua dividitur, habetque tum cellula suffultoria inferior formam pentagonam, superior tetragonam (vide fig. 21 et 22 ss).

Habet hæc familia, ut supra dictum jam est, magnam formarum copiam, quarum characteres plerumque sunt constantissimi [1]). Hanc vero constantiam non solum ii obtinent characteres, qui aperte majoris sunt momenti, velut dispositio organorum fructificationis atque universa eorum natura, sed tales etiam, qui specie saltem minoris momenti sunt, velut dimensiones cellularum, sedes nannandrium in plantis femineis [2]) cet.

In iis, quæ sequentur, id elaborandum nobis putabamus, ut, quantum dispositio in simplicem seriem pateretur, in disponendis illius generis speciebus naturam sequeremur. Scimus quidem veram specierum affinitatem maxime ita fieri conspicuam, si in tabulam genealogicam redigantur, quippe cum ex eadem omnes radice trahant originem; cum vero dubitari vix possit, quin minor tantum pars familiæ formarum cognita adhuc sit, ejusmodi nunc dispositio præmatura nobis videtur.

Quod ad geographiam familiæ pertinet, plane est cosmopolitica. Ex utroque ejus genere species cum in arcticis [3]) tum in tropicis regionibus sunt observatæ. Ubi maxima formarum ubertate familia gaudeat, nondum potest dijudicari. In enumeratione vero sequente longe plurimas Sueciam præbere species ex eo pendere videtur, quod Suecia in *Oedogoniis* multo diligentius quam ceteræ terrarum partes est perlustrata. Quantum specierum numerum singulæ nobis præbuerint terræ, ex his intelligitur: e Suecia habemus species 102, e Norvegia 29, Dania 15, Germania 46, Hollandia 1, Austria 5, Pannonia 1, Helvetia 1, Italia 2, Gallia 22, Lusitania 2, Britannia 18, Hibernia 12, Insulis Spetsbergensibus 1, Groenlandia 9, India occi-

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[1]) Ex multis exemplis, quibus species miram illam characterum constantiam ostendunt, hæc afferantur: exemplaria *Oe. crispi* (Hass.) Wittr., ex Andibus Americæ Meridionalis sumta, in minimis rebus, atque etiam in cellularum dimensionibus, plane cum sueticis congruunt exemplaribus: tum exemplaria specierum et in Groenlandia et in Europa occurentia, velut *Oe. upsaliense* Wittr., *Bulb. intermedia* De Bary c. pl., nihil omnino inter se sunt disparia; denique exemplaria *Bulb. elatioris.* Pringsh. indica et *Bulb. rhadinosporæ* Wittr. tasmanica cum exemplaribus sueticis omnibus ex partibus congruunt. In familia tam magna etiam inesse species, quæ magnam variandi facultatem possideant, perspicuum est.

[2]) In plerisque speciebus nannandris, *Oedogonii* saltem generis, sedes in plantis femineis nannandrium semper fere fixa est atque stabilis, in nonnullis enim ipsi oogonio assident (vide fig. 6 et 11), in aliis cellulæ proxime infra oogonium (cellulæ suffultoriæ; vide fig. 13 et 15). In paucis tantum Oedogonii speciebus, in multis vero *Bulbochætes* nannandres in sedibus eligendis non tam sunt delicati.

[3]) Genus *Bulbochæte* Ag. in Insulis Spetsbergensibus deesse videtur; in Groenlandia eo frequentius est.

dentali 1, Brasilia 6, Bolivia 1, India cis Gangem 3, India trans Gangem 4, Tasmania 4. — Species omnes, quas ex universa cognovimus terra, numero sunt 141 [1]).

Omnes familiæ species dulcis sunt aquæ [2]). Pleræque sunt epiphyticæ, partim in Phanerogamis (velut *Batrachiis* et *Gramineis*), partim, et quidem plerumque, in foliis Muscorum aquaticorum aliarumque Cryptogamarum aquæ dulcis [3])

His viris doctissimis maxime mihi gratiæ sunt agendæ, quod per eos formas *Oedogoniearum* e diversis orbis terrarum partibus [4]) videndi mihi facultas est parata: celeberrimis Professoribus J. E. Areschoug, Al. Braun, W. Carruthers, Doctoribus P. T. Cleve, G. Eisen, J. D. Hooker, Th. M. Fries, Prof. C. Jessen, Drr. F. R. Kjellman, T. O. B. N. Krok, N. G. W. Lagerstedt, Cand. C. I. Lalin, Prof. S. Lovén, Cand. P. M. Lundell, A. N. Lundström, Drr. P. Magnus, E. von Martens, S. J. Nilsson, Cand. C. F. O. Nordstedt, G. E. Ringius, Dr. J. Spångberg, Cand. C. Örström.

[1]) Omnium, quæ infra describentur, specierum exemplaria fructificantia ipsi examinavimus; excipiuntur sola *Oedogonium urbicum*, *Oe. depressum*, *Bulbochæte elachistandra*, *B. crassa*, *B. anomala*, in quibus describendis cel. Pringsheim secuti sumus.

[2]) Tres quidem species (*Oedog. crispum* (Hass.) Wittr., *Oe. undulatum* (Bréb.) Al. Br., *Bulb. rectangularis* Wittr.) in parte maxime septentrionali Maris Baltici inventæ sunt; est autem aqua illic omnino fere dulcis.

[3]) Collectiones *Oedogoniearum* optime in vasis vitreis, glycerinum dilutum vel spiritum vini continentibus, conservantur.

[4]) Norvegiam, Britanniam, Hiberniam, Daniam, Germaniam, Austriam regionesque diversas Sueciæ, *Oedogonieis* inprimis studendi causa, ipse visitavi.

OEDOGONIEÆ De Bar.

Ueb. Oedog. u. Bolb. p. 94; Pringsh. Beitr. z. Morph. d. Alg. I, p. 68.

GENUS I. OEDOGONIUM Link.

Epist. de Alg. p. 5; Pringsh. l. c.

SECTIO I. Species monoicæ.

A. Species oogoniis processubus medianis semper detitutis.

a. Species oosporis globosis vel subglobosis.

α. Species oogoniis globosis vel subglobosis.

1. Oe. Petri Wittr. mscr.

Oe. [1]) oogoniis singulis (rarissime binis), pyriformi-globosis, poro foecundationis paullum supra medium sito apertis; oosporis subdepresso-globosis, oogonia prope complentibus; spermogoniis 1—2cellularibus, hypogynis vel epigynis vel subepigynis; spermatozoidis singulis (?); cellula terminali obtusa;

crassitudine cellularum vegetativarum 6—7 μ [2]), altitud. 5—7plo major;
„ oogoniorum 21—24 „ „ 22—29 μ;
„ oosporarum 20—23 „ „ 17—19 „;
„ cell. spermogoniorum $5\frac{1}{2}$—6 „ „ 10—11 „.

Hab. in Suecia, Germania, Hibernia.

[1]) In descriptionibus specierum hujus generis hæc sunt animadvertendæ: in forma oogoniorum describenda partis eorum mitralis rationem a nobis habitam non esse; oosporam, quam diximus oogonium complere, partem tamen mitralem non replere (excipe *Oe. acrosporum* De Bar.); deinde, si nihil aliud dicitur, membranam oosporarum lævem esse, cellulas vegetativas triviales cylindricas lævesque esse, cellulas suffultorias cellulis vegetativis trivialibus esse similes, spermatozoidas, binas ortas, divisione horizontali cellularum spermogoniorum formari. — De vi et significatione terminorum techn., quos in speciebus describendis adhibuimus, neque interpretati in exordio sumus, explicatio videnda est figurarum.

[2]) μ = 0,001 millemetri.

2. **Oe. cryptoporum** Wittr.

Dispos. Oed. Suec. p. 19.

Oe. oogoniis singulis, elliptico- ad subdepresso-globosis, poro mediano apertis; oosporis subdepresso-globosis, oogonia fere complentibus; spermogoniis 2—7cellularibus, sparsis (plerumque subhypogynis); spermatozoidis singulis (?);

crassit. cell. veget. 7—9 μ, altit. 4—6plo major;
„ oogon. 24—25 „ „ 26—27 μ;
„ oospor. 22—23 „ „ 19—21 „;
„ cell. spermog. 6—8 „ „ 7—11 „.

Hab. in Suecia, Norvegia, Dania.

β vulgare Wittr. mscr.

Var. oogoniis 2—5 continuis vel singulis; spermogoniis 1—4cellularibus, subepigynis vel hypogynis vel sparsis;

crassit. cell. veget. 5—8 μ, altit. 3—5plo major;
„ oogon. 18—25 „ „ 18—26 μ;
„ oospor. 16—22 „ „ 13—18 „;
„ cell. spermog. 5—7 „ „ 9—12 „.

Hab. in Suecia, Norvegia, Germania, Britannia.

3. **Oe. capitellatum** Wittr. mscr.

Oe. piliferum Wittr. Dispos. Oed. Suec. p. 122; *non* Auersw.

Oe. oogoniis singulis, subdepresso-globosis, in medio anguste sed manifeste circumscissis, poro in circumscissione sito apertis; oosporis depresso-globosis, oogonia complentibus; spermogoniis 1—2cellularibus, hypogynis vel epigynis; spermatozoidis singulis (?); cellulis vegetativis trivialibus capitellatis; cellula terminali piliformi;

crassit. cell. veget. 6—9 μ, altit. 3—7plo major;
„ oogon. 24—25 „ „ 22—23 μ;
„ oospor. 22—23 „ „ 18—19 „;
„ cell. spermog. 6—7 „ „ 8—9 „.

Hab. in Suecia.

4. **Oe. sphærandrum** Wittr. et Lund. mscr.

Oe. oogoniis 2—4 continuis vel singulis, subpyriformi- ad subdepresso-globosis, operculo apertis, rima angustissima; oosporis subdepresso-globosis, oogonia fere complentibus; spermogoniis sparsis, 2—6cellularibus, cellulis subglobosis; spermatozoidis singulis; cellulis vegetativis trivialibus plerumque capitellatis, interdum leviter trinodulosis;

crassit. cell. vegetativ. 5—10 μ, altit. 1 $^1/_2$—3 $^1/_2$plo major;
„ oogon. 18—23 „ „ 18—21 μ;
„ oospor. 16 $^1/_2$—19 „ „ 14—18 „;
„ cell. spermog. 6—7 $^1/_2$ „ „ 5—6 „.
Hab. in Suecia.

5. **Oe. curvum** Pringsh.

Beitr. z. Morph. d. Alg. I, p. 69, tab. 5, fig. 3.

Oe. oogoniis 2—7 continuis vel singulis, depresso-globosis, poro mediano apertis; oosporis depresso-globosis, oogonia complentibus; spermogoniis 3— pluricellularibus, in suprema parte fili sitis; spermatozoidis singulis; parte fili superiore arcuata vel spiraliter contorta;

crassit. cell. veget. 5—10 μ, altit. 1 $^1/_2$—4plo major;
„ oogon. 23—25 „ „ 20—24 μ;
„ oospor. 20—23 „ „ 16 $^1/_2$—19 „;
„ cell. spermog. 7—9 „ „ 7—9 „.
Hab. in Suecia, Germania (sec. cel. Pringsheim), Hibernia.

6. **Oe. cymatosporum** Wittr. et Nordst.

in Wittr. Dispos. Oed. Suec. p. 121.

Oe. oogoniis singulis, raro binis, subdepresso-globosis, poro mediano vel paullum supra medium sito apertis; oosporis depresso-globosis, oogonia prope complentibus; membrana oosporarum scrobiculata (in sectione optica transversali undulata), scrobiculis profundis densisque; spermogoniis 1—4cellularibus, subepigynis, hypogynis, subhypogynis vel sparsis; spermatozoidis singulis;

crassit. cell. veget. 8—10 μ, altit. 4—7plo major;
„ oogon. 24—34 „ „ 27—36 μ;
„ oospor. 22—31 „ „ 19—27 „;
„ cell. spermog. 8—9 „ „ 9—15 „.
Hab. in Suecia, Gallia.

7. **Oe. læve** Wittr. mscr.

Oe. oogoniis singulis, depresso-globosis, poro mediano apertis; oosporis depresso-globosis, oogonia complentibus; spermogoniis 1—2cellularibus, subepigynis; spermatozoidis singulis (?);

crassit. cell. veget. 10—12 μ, altit. 2—6plo major,
„ oogon. 32—38 „ „ 28—30 μ;
„ oospor. 30—35 „ „ 23—26 „;
„ cell. spermog. 9—10 „ „ 9—13 „.
Hab. in Gallia.

8. **Oe. minus** Wittr.

Oe. punctato-striatum De Bar. var. *minor* Wittr. Dispos. Oed. Suec. p. 123; Rabenh. Alg. Eur. n:o 2277; Aresch. Alg. Scand. exsic. Ser. nov., fasc. 7 & 8 n:o 352.

Oe. oogoniis singulis, depresso-globosis, in medio manifeste circumscissis, poro in circumscissione sito apertis, oosporis depresso-globosis, oogonia fere complentibus; spermogoniis 1—10cellularibus, subepigynis vel subhypogynis vel rarius sparsis; spermatozoidis singulis; cellulis vegetativis trivialibus capitellatis; membrana cellularum vegetativarum et oogoniorum punctulis densis, spiraliter dispositis ornata;

crassit. cell. veget. 9—13 μ, altit 3—6plo major;
„ oogon. 34—46 „ „ 28—42 μ;
„ oospor. 30—42 „ „ 26—36 „;
„ cell. spermog. 9—12 „ „ 3—5 „.

Hab. in Suecia, Norvegia, Hibernia.

9. **Oe. obsoletum** Wittr. mscr.

Oe. vernale Wittr. [1]) Oedog. nov. p. 1.

Oe. oogoniis singulis, globosis ad subdepresso-globosis, poro paullum supra medium sito apertis; oosporis eadem forma ac oogoniis, hæc fere complentibus; spermogoniis 1—3cellularibus, subepigynis; spermatozoidis singulis;

crassit. cell. veget. 9—14 μ, altit. 3—5plo major;
„ oogon. 34—38 „ „ 34—40 μ;
„ oospor. 31—34 „ „ 28—31 „;
„ cell. spermog. 8—9 „ „ 12—16 „.

Hab. in Suecia.

10. **Oe. fragile** Wittr.

Dispos. Oed. Suec. p. 120.

Oe. oogoniis singulis, globosis vel suboboviformi-globosis, poro superiore apertis; oosporis oogonia complentibus; spermogoniis 1—3cellularibus hypogynis vel subepigynis; spermatozoidis binis;

crassit. cell. veget. 12—17 μ, altit. 4—5plo major;
„ oogon. 42—47 „ „ 44—50 μ;
„ oospor. 40—44 „ „ 39—44 „;
„ cell. spermog. 12—15 „ „ 10—12 „. Tab. I, fig. 1.

Hab. in Suecia.

[1]) Cel. Hassall antea, in Ann. and Magaz. of Nat. Hist. vol. 11, 1843, speciem alteram hujus generis sub nomine *vernali* descripsit.

11. **Oe. Zig-zag** Clev.

in Wittr. Dispos. Oed. Suec. p. 120.

Oe. oogoniis singulis, globosis, poro superiore apertis; oosporis oogonia complentibus; spermogoniis unicellularibus; oogoniis spermogoniisque atque cellulis vegetativis alternis;

crassit. cell. veget. 17—18 μ, altit. 2—4plo major;
„ oogon. 50—51 „ „ 53—54 μ;
„ oospor. 47—48 „ „ 49—50 „;
„ cell. spermog. 17 „ „ 10—11 „.

Hab. in Suecia.

12. **Oe. Pyrulum** Wittr.

Oedog. nov. p. 2.

Oe. oogoniis singulis, globosø-pyriformibus, operculo apertis, rima angusta; oosporis globosis, oogonia non plane complentibus; spermogoniis unicellularibus, epigynis vel hypogynis vel rarius subepigynis; cellula terminali apice brevi-acuta, ceterum eadem forma ac cellulis vegetativis trivialibus;

crassit. cell. veget. 8—11 μ, altit. $3^1/_1$—7plo major;
„ oogon. 30—33 „ „ 31—34 μ;
„ oospor. 20—29 „ „ 27—28 „;
„ cell. spermog. 8—9 „ „ 10—11 „.

Hab. in Suecia, Norvegia.

13. **Oe. vernale** (Hass.) Wittr.

(?)*Vesiculifera vernalis* Hass. Descr. on. Freshw. Conf. p. 434. (?)*Vesiculifera Candollii* Hass. Hist. of Brit. Freshw. Alg. p. 208, t. 52, f. 9. — *Non Oe. vernale* Wittr. Oedog. nov. p. 1.

Oe. oogoniis singulis, oboviformi-globosis, operculo apertis, rima angusta; oosporis globosis, oogonia non plane complentibus; spermogoniis 2cellularibus, subepigynis;

crassit. cell. veget. 10—16 μ, altit. $4\,^1/_2$—6plo major;
„ oogon. 39—45 „ „ 45—51 μ;
„ oospor. 34—38 „ „ 34—39 „;
„ cell. spermog. 10—12 „ „ 8—9 „.

Hab. in Suecia.

14. **Oe. crispum** (Hass.) Wittr.

Vesiculifera crispa Hass. Hist. of Brit. Freshw. Alg. p. 203, tab. 52, f. 8 [1]).

[1]) Vidimus specimina originalia Hassalliana e herbario cel. Professoris J. E Areschoug.

Oedogonium rostellatum Pringsh. Beitr. z. Morph. d. Alg. I, p. 69, tab. 5, fig. 1; Rabenh. Alg. Eur. n:o 2275. *Oe. pulchellum* Al. Br. in Rabenh. Alg. Eur. n:o 2095.

Oe. oogoniis singulis, oboviformi-globosis, operculo apertis, rima angusta; oosporis subglobosis, oogonia non plane complentibus; spermogoniis 2—5cellularibus, hypogynis vel subepigynis; spermatozoidis binis; cellula terminali obtusa;

crassit. cell. veget. 12—18 μ, altit. 2—4 ½plo major;
„ oogon. 37—49 „ „ 42—54 μ;
„ oospor. 33—46 „ „ 34—46 „;
„ cell. spermog. 9—14 „ „ 7—12 „.

Hab. in Suecia, Norvegia, Dania, Germania, Gallia, Britannia, Austria et in Andibus Boliviensibus.

15. **Oe. autumnale** Wittr. mscr.

Oe. oogoniis singulis, oboviformi-globosis, operculo apertis, rima angustissima; oosporis subglobosis, oogonia complentibus; spermogoniis 1—2cellularibus, subepigynis vel hypogynis vel sparsis; spermatozoidis binis; cellula terminali apice brevi-acuta;

crassit. cell. veget. 16—19 μ, altit. 1½—2½plo major;
„ oogon. 39—43 „ „ 45—51 μ;
„ oospor. 37—40 „ „ 39—43 „;
„ cell. spermog. 15—18 „ „ 9—10 „.

Hab. in Suecia.

16. **Oe. varians** Wittr. et Lund. mscr.

Oe. oogoniis singulis, depresse oboviformi-globosis, poro superiore apertis; oosporis globosis, oogonia non plane complentibus; spermogoniis 1—9cellularibus, sparsis; spermatozoidis binis;

crassit. cell. veget. 12—16 μ, altit. 3—8plo major;
„ oogon. 35—46 „ „ 34—55 μ;
„ oospor. 31—39 „ „ 30—36 „;
„ cell. spermog. 11—15 „ „ 5—6 „.

Hab. in Suecia.

17. **Oe. plusiosporum** Wittr. mscr.

Oe. oogoniis singulis, subelliptico-globosis, poro paullum supra medium sito vel submediano apertis; oosporis globosis, oogonia non plane complentibus; spermogoniis 2—6cellularibus, subepigynis (rarissimis) [1]; cellula terminali obtusa;

[1]) Species hæc forsan trioica est.

crassit. cell. veget. 14—19 μ, altit. 2 $^1/_2$—3 $^1/_2$plo major;
„ oogon. 38—45 „ „ 42—50 μ;
„ oospor. 33—39 „ „ 31—37 „;
„ cell. spermog. 12—14 „ „ 8—12 „.
Hab. in Suecia.

18. **Oe. tyrolicum** Wittr. mscr.

Oe. oogoniis singulis, elliptico-globosis, poro superiore apertis; oosporis globosis, oogonia non complentibus; spermogoniis 1—4cellularibus, subepigynis vel subhypogynis; spermatozoidis binis;

crassit. cell. veget. 15—24 μ, altit. 4—5plo major;
„ oogon. 48—53 „ „ 62—70 μ;
„ oospor. 42—48 „ „ 42—48 „;
„ cell. spermog. 12—21 „ „ 10—11 „.
Hab. in Austria.

19. **Oe. polymorphum** Wittr. et Lund. mscr.

Oe. oogoniis singulis, (plerumque uno solo in unoquoque exemplare), oboviformi-globosis, poro superiore apertis; oosporis globosis, oogonia non plane complentibus; spermogoniis 1—5cellularibu, sæpe terminalibus, subepigynis vel sparsis[1]); spermatozoidis binis; cellula terminali obtusa vel apiculata vel setiformi;

crassit. cell. veget. 8—14 μ, altit. 4—10plo major;
„ oogon. 30—33 „ „ 30—35 μ;
„ oospor. 25—30 „ „ 25—30 „;
„ cell. spermog. 8 „ „ 6 $^1/_2$—7 „;
„ cellulæ basalis 13—18 „ „ 4—5plo major.
Hab. in Suecia.

20. **Oe. curtum** Wittr. et Lund.
in Wittr. Dispos. Oed. Suec. p. 121.

Oe. oogoniis 2—4 continuis vel singulis, oboviformi-globosis, poro superiore apertis; oosporis oogonia complentibus; spermogoniis 2—4cellularibus, subepigynis, sæpe terminalibus: spermatozoidis binis; cellula terminali obtusa;

crassit. cell. veget. 12—16 μ, altit. 2—3plo major;
„ oogon. 42—45 „ „ 42—48 μ:
„ oospor. 39—41 „ „ 38—45 „;
„ cell. spermog. 11—13 „ „ 9—11 „.
Hab. in Suecia.

[1]) Interdum oogonia et spermogonia in diversis exemplaribus occurrunt, efficitque igitur species hæc trioica nexum quendam inter species monoicas et dioicas.

21. Oe. nodulosum Wittr.

Om Gotl. och Öl. Sötv. Alg. p. 22, tab. 1, fig. 8–10.

Oe. oogoniis singulis vel binis, oboviformi-globosis, operculo apertis, rima mediocri; oosporis globosis vel subglobosis, oogonia prope complentibus; spermogoniis 1—3cellularibus, hypogynis vel subepigynis; spermatozoidis binis; cellulis vegetativis trivialibus bis undulato-constrictis (itaque trinodulosis); cellula terminali apiculata;

crassit. cell. veget. 23—29 μ, altit. 1 $^1/_2$—4 $^1/_2$plo major;
„ oogon. 48—57 „ „ 56—72 μ;
„ oospor. 46—53 „ „ 50—56 „;
„ cell spermog. 18—25 „ „ 7—9 „. Tab. I, fig. 2, 3.

Hab. in Suecia.

22 Oe. dictyosporum Wittr. mscr.

Oe. oogoniis singulis, ellipsoideo-globosis, poro apertis; oosporis eadam forma ac oogoniis hæc non complentibus, episporio reticulato; spermogoniis subepigynis 3cellularibus; spermatozoidis binis;

crassit. cell. veget; 12—16 μ, altit. 1 $^3/_4$—3plo major;
„ oogon. 39—40 „ „ 45—46 μ;
„ oospor. 31—32 „ „ 37—38 „;
„ cell. spermog. 12—13 „ „ 4 $^1/_2$—5 „.

Hab. in Brasilia.

23. Oe. Vaucherii (Le Cl.) Al. Br.

Ueb. Chytrid. p. 40, t. 2, f. 13 (?). — (?) *Prolifera Vaucherii* Le Cl. Sur. gen. Prolif. p. 474, t. 24, f. 4.

Oe. oogoniis singulis, oboviformi-globosis vel subglobosis, poro superiore apertis; oosporis globosis vel subglobosis, oogonia non plane complentibus; spermogoniis 2—4cellularibus, subepigynis vel hypogynis; spermatozoidis binis;

crassit. cell. veget. 20—30 μ, altit. 1 $^1/_2$—4plo major;
„ oogon. 40—55 „ „ 45—65 μ;
„ oospor. 35—50 „ „ 35—52 „;
„ cell. spermog. 17—24 „ „ 6—11 „.

Hab. iu Suecia, Dania, Germania.

β. Species oogoniis ellipsoideis vel oviformibus.

24. Oe. urbicum Wittr. mscr.

Oe. tumidulum Pringsh. Beitr z. Morph. d. Alg. I, p. 69. t. 5, f. 2; *non* Kütz. [1]).

[1]) Cel. *Kützing* in Decadibus Algarum speciem alteram hujus generis sub nomine *tumidulo* distribuit et descripsit.

Oe. oogoniis singulis, ellipsoideis, poro superiore apertis; oosporis globosis, oogonia non complentibus; spermogoniis plerumque 2cellularibus; spermatozoidis binis; cellulis suffultoriis chlorophyllo prope destitutis;

crassit. cell veget. 16 1/2 μ, altit. 2 1/2—6plo major;
„ oospor. 33—45 „ „ 33—45 μ.

Hab. in Germania (sec. Pringsh.).

25. Oe. nobile Wittr. mscr.

Oe. oogoniis singulis (rarissime binis), oviformibus, operculo apertis; rima angusta; oosporis globosis, oogonia non complentibus, in parte inferiore eorum sitis; spermogoniis 2—3cellularibus, hypogynis; spermatozoidis binis.

crassit. cell. veget. 17—20 μ, altit. 5—7plo major;
„ oogon. 60—62 „ „ 84—90 μ;
„ oospor. 51—54 „ „ 51—54 „;
„ cell. spermog. 18—19 „ „ 12—13 „.

Hab. in Norvegia.

b. Species oosporis ellipsoideis vel oviformibus.

26. Oe. paludosum (Hass.) Wittr.

Dispos. Oed. Suec. p. 124. (?) *Vesiculifera paludosa* Hass. Hist. of Brit. Freshw. Alg. p. 199, t. 52, f. 3.

Oe. oogoniis singulis, ellipsoideis, plerumque subobliquis, poro superiore apertis; oosporis ellipsoideis, oogonia plane complentibus (membrana oosporarum maturarum in latere interiore longitudinaliter costata?); spermogoniis 1—8cellularibus, sparsis, plerisque in parte fili superiore sitis; spermatozoidis binis, divisione obliqua sæpe ortis.

crassit. cell. veget. 15—20 μ, altit. 3—7plo major;
„ oogon. 39—48 „ „ 66—84 μ;
„ oospor. 36—45 „ „ 54—63 „;
„ cell. spermog. 14—16 „ „ 6—13 „.

Hab. in Suecia, Norvegia.

27. Oe. upsaliense Wittr.

Dispos. Oed. Suec. p. 125. *Oe. tumidulum* Aresch. Alg. Scand. exsic., fasc. 5, n:o 236.

Oe. oogonis singulis oboviformibus vel subellipsoideis, poro superiore apertis[1]); oosporis oogonia complentibus; spermogoniis 1—2(rarissime3)-cellularibus, subhypogynis;

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[1]) Rima transversa angustissima interdum discerni potest; vide tab. 1, fig. 4.

crassit. cell. veget. 13—20 μ, altit. 4—8plo major;
„ oogon. 45—50 „ „ 66—100 μ;
„ oospor. 42—47 „ „ 60—75 „;
„ cell. spermog. 15—18 „ „ 7—10 „. Tab. I, fig. 4.
Hab. in Suecia, Germania, Groenlandia.

28. **Oe. gracillimum** Wittr. et Lund. mscr.

Oe. oogoniis singulis, oblongis, operculo apertis, rima sublata; oosporis oblongo-ellipsoideis, oogonia non complentibus; spermogoniis unicellularibus subepigynis; spermatozoidis binis;
crassit. cell. veget. $3^1/_2$—5 μ, altit. 5—6plo major;
„ oogon. 14—16 „ „ 34—42 μ;
„ oospor. 13—15 „ „ 24—31 „;
„ cell. spermog. 3—$3^1/_2$ „ „ 4 „.
Hab. in Suecia.

29. **Oe. oblongum** Wittr.

Oedog. nov. p. 2.

Oe. oogoniis singulis, oblongis, operculo apertis, rima lata; oosporis ellipsoideis, partem inferiorem oogoniorum complentibus; spermogoniis 1—3-cellularibus, subhypogynis vel subepigynis; spermatozoidis binis;
crassit. cell. veget. 9—11 μ, altit. 3—6plo major;
„ oogon. 24—26 „ „ 41—48 μ;
„ oospor. 21—22 „ „ 30—33 „;
„ cell. spermog. 9 „ „ 7—9 „.
Hab. in Suecia, Dania.

30. **Oe. pachydermum** Wittr. et Lund.

in Wittr. Dispos. Oed. Suec. p. 125.

Oe. oogoniis singulis, ellipsoideis (membrana post foecundationem valde incrassata), operculo apertis, rima mediocri; oosporis ellipsoideis, oogonia non plane complentibus; spermogoniis 1—2cellularibus, hypogynis, vel subepigynis, sæpe terminalibus; spermatozoidis binis; cellula terminali apiculata;
crassit. cell. veget. 21—27 μ, altit. $1^1/_2$—2 $^1/_2$plo major;
„ oogon. 50—70 „ „ 75—100 μ;
„ oospor. 40—60 „ „ 50—80 „;
„ cell. spermog. 18—21 „ „ 10—12 „.
Hab. in Suecia.

B. Species oogoniis processubus verticillatis in medio instructis.

a. Species oosporis subglobosis.

31. Oe. Itzigsohnii De Bar.

Ueb. Oedog. und Bolb. p. 56. t. 3, p. 29—32.

Oe. oogonis singulis, ellipsoideis, processubus medianis 7—10, obtuse conicis; oog. infra medium circumscissis, poro in circumscissione sito apertis, a vertice visis stellatis, 7—10radiatis, incisuris inter radios profundis, rotundatis; oosporis globosis, oogonia non complentibus: spermogoniis 1—2cellularibus; cellula terminali obtusa vel apiculata;

crassit. cell. veget. 8—10 μ, altit. 3—6plo major;
„ oogon. 34—38 „ „ 32—40 μ;
„ oospor. 22—23 „ „ 22—23 „;
„ cell spermog. 8—9 „ „ 9—15 „.

Hab. in Suecia, Germania.

32. Oe. mammiferum Wittr. mscr.

Oe. oogoniis singulis, rarius binis, oblongis, processubus medianis 12, mammæformibus, 6 majoribus, 6 minoribus; oog. a vertice visis stellatis, radiis 12, 6 majoribus cum 6 minoribus alternis; incisuris inter radios rotundatis; oosporis depresso-globosis, oogonia non complentibus; spermogoniis 1cellularibus, subepigynis;

crassit. cell. veget. 5—8 μ, altit. 4—7plo major;
„ oogon. 27 „ „ 25—30 μ;
„ oospor. 17—20 „ „ 15—16 „;
„ cell. spermog. 6 „ „ 7 „.

Hab. in Suecia, Norvegia.

b. Species oosporis subellipsoideis.

33. Oe. excisum Wittr. et Lund.

in Wittr. Oedog. nov. p. 3, t. 1, f. 1—4.

Oe. oogoniis singulis, subbiconico-oblongis, processubus medianis 9, rotundatis, parvis; oog. profunde circumscissis, a vertice visis orbicularibus, margine leviter undulata; oosporis ellipsoideis, in medio quasi constrictis, oogonia non complentibus; spermogoniis 1—2cellularibus, subepigynis vel hypogynis; cellula terminali obtusa; parte suprema fili curvata;

crassit. cell. veget. $3^1/_2$—5 μ, altit. 5—6plo major;
„ oogon. 13—15 „ „ 18—25 μ;
„ oospor. 9—12 „ „ 15—18 „;
„ cell. spermog. 3—$3^1/_2$ „ „ 6—7 „.
Hab. in Suecia, Austria.

SECTIO II. Species dioicæ.

SUBSECTIO I. Species nannandræ.

A. Spec. nannandribus unicellularibus.

a. Spec. oogoniis processubus verticillatis in medio instructis.

34. **Oe. platygynum** Wittr.

Oedog. nov. p. 1.

Oe. gynandrosporum (vel idio-androsporum?); oogoniis singulis (rarissime binis), depresso-oboviformibus, processubus medianis 7—12, rotundatis; oog. infra medium circumscissis, poro in circumscissione sito apertis; a vertice visis orbicularibus, margine sinuata, sinubus 7—12 (plerumque 8); oosporis subdepresso-globosis, oogonia fere complentibus; androsporangiis 1—3cellularibus; cellula terminali obtusa; nannandribus oboviformibus, minimis, in oogoniis sedentibus;

crassit. cell. veget. 6—10 μ, altit. 2—5plo major;
„ oogon. 21—30 „ „ 16—24 μ;
„ oospor. 17—24 „ „ 15—20 „;
„ cell. androsporang. 6—8 „ „ 7—8 „;
„ nannandr. $4^1/_2$—5 „ „ $8^1/_2$—$9^1/_2$ „. Tab. I, fig. 5—9.
Hab. in Suecia, Norvegia, Germania, Hibernia.

35. **Oe. oelandicum** Wittr. mscr.

Oe. gynandrosporum; oogoniis singulis vel binis, depresso-globosis, poro superiore apertis, a vertice visis orbicularibus, margine levissime undulata; oosporis depresso-globosis, oogonia fere complentibus; androsporangiis 2—6cellularibus, subepigynis (interdum terminalibus); cellulis vegetativis trivialibus capitellatis; cellula terminali obtusa; nannandribus oboviformibus, in oogoniis sedentibus;

crassit. cell. veget. 10—15 μ, altit. $2^1/_2$—9plo major;
„ „ androsp. 7—10 „ „ 12—18 μ;
„ oogon. 36—44 „ „ 27—32 „;
„ oospor. 33—40 „ „ 26—30 „;
„ nannandr. 7 „ „ 12 „.
Hab. in Suecia.

b. Species oogoniis processubus medianis semper destitutis.

α. Spec. oosporis globosis vel subglobosis.

36. **Oe. Rothii** (Le Cl.) Pringsh.

Beitr. z. Morph. d. Oedog. I, p. 69, t. 5, f. 4. (?) *Prolifera Rothii* Le Cl. Surgen. Prolif. p. 476, t. 23, f. 8.

Oe. gynandrosporum, oogoniis singulis vel 2—6 continuis, globosis vel subdepresso-globosis, poro mediano apertis; oosporis subdepresso-globosis, oogonia fere complentibus; androsporangiis 2—4cellularibus, subhypogynis; nannandribus oboviformibus, in oogoniis sedentibus;
crassit. cell. veget. 6—8 μ, altit. 3—8plo major;
„ oogon. 20—21 „ „ 16—19 μ.
Hab. in Germania (sec. cel. Pringsheim), Norvegia.

37. **Oe. decipiens** Wittr.

Dispos. Oedog. Suec. p. 126. *Oe. vesicatum* De Bar. Ueb. geschl. Zeug. bei Alg. p. 224, t. 5.

Oe. gynandrosporum, oogoniis singulis vel 2—3 continuis. subdepresso-globosis, in medio angustissime circumscissis, poro in circumscissione sito apertis; oosporis subdepresso-globosis, oogonia non plane complentibus; androsporangiis 2—6cellularibus; nannandribus oboviformibus, in oogoniis sedentibus.
crassit. cell. veget. 10—12 μ, altit. 3—5plo major;
„ oogon. 32—38 „ „ 30—40 μ;
„ oospor. 29—34 „ „ 25—28 „;
„ cell. androsp. 9—10 „ „ 11—15 „;
„ nannandr. 6—7 „ „ 14—15 „.
Hab. in Germania (sec. cel. De Bary), Suecia.

38. **Oe. Areschougii** Wittr.

Dispos. Oed. Suec. p. 122.

Oe. gynandrosporum [1]), oogoniis 2—6 continuis vel singulis, subdepresso-globosis, in medio late circumscissis, poro in circumscissione sito apertis; oosporis exacte globosis, oogonia longe non complentibus; androsporangiis 1—6cellularibus, hypogynis vel subepigynis vel rarius sparsis; cellula terminali (quæ interdum est androsporangium) obtusa; nannandribus oboviformibus, in oogoniis sedentibus;

crassit. cell. veget. 8—12 μ, altit. 4—6plo major;
„ oogon. 38—39 „ „ 36—40 μ;
„ oospor. 22—24 „ „ 22—24 „;
„ cell. androsp. 10—11 „ „ 10—12 „;
„ nannandr. 6—7 „ „ 14—15 „. Tab. I, fig. 10, 11.

Hab. in Suecia, Groenlandia.

39. **Oe. megaporum** Wittr.

Oedog. nov. p. 3, t. 1, f. 5, 6.

Oe. idioandrosporum; oogoniis 2—6 continuis vel singulis, pyriformibus, operculo apertis, rima angusta et indistincta, poro foecundationis magno; oosporis depresso-globosis, oogonia non plane complentibus; cellula terminali (quæ interdum est oogonium) obtusa; androsporangiis 4cellularibus; nannandribus late oboviformibus, in oogoniis sedentibus;

crassit. cell. veget. 13—17 μ, altit. 3—6plo major;
„ oogon. 40—42 „ „ 40—45 μ;
„ oospor. 33—38 „ „ 27—30 „;
„ cell. androsp. 10—11 „ „ 19—22 „;
„ nannandr. 11—12 „ „ 13—16 „.

Hab. in Suecia.

40. **Oe. pluviale** Nordst.

in Rabenh. Alg. Eur. n:o 2257; Wittr. & Nordst. in Wittr. Oedog. nov. p. 7; Aresch. Alg. Scand. exsic. Ser. nov. fasc. 7 & 8, n:o 353. *Oe. diplandrum* Jur. Beitr. z. Morph. d. Oedog. p. 27, t. 1—3. ? *Vesiculifera dissiliens* Hass. Hist. of Brit. Freshw. Alg. p. 202, t. 50, f. 7.

Oe. idioandrosporum; oogoniis singulis, raro 2—3 continuis, oboviformi-globosis vel subglobosis, operculo terminali apertis, rima angusta; oosporis subglobosis, oogonia fere complentibus; cellula terminali obtusa; plantis an-

[1]) Hanc speciem l. c. monoicam descripsimus, androsporangia non recte pro spermogoniis habentes.

drosporangiferis paullo gracilioribus quam femineis; androsporangiis [1]) 6—10-cellularibus; nannandribus late oboviformibus, in oogoniis sedentibus;

crassit. cell. veget. 18—28 μ, altit. par ad 3plo major;
„ oogon. 34—39 „ „ 34—45 μ;
„ oospor. 32—37 „ „ 31—40 „;
„ cell. androsp. 17—19 „ „ 6—11 „;
„ nannandr. 10 „ „ 15 „.

Hab. in Suecia, Pannonia (sec. cel. Juranyi).

41. Oe. alternans Wittr. et Lund. mscr.

Oe. gynandrosporum; oogoniis singulis, cum cellulis vegetativis vel androsporangiis plerumque alternis, globosis, poro superiore apertis; oosporis oogonia complentibus; androsporangiis 1—2cellularibus, epigynis; cellula terminali obtusa; nannandribus oblongis, inferne constrictis, in cellulis suffultoriis sedentibus.

crassit. cell. veget. 21—40 μ, altit. 2—5plo major;
„ oogon. 65—80 „ „ 65—86 μ;
„ oospor. 59—74 „ „ 59—75 „;
„ cell. androsp. 18—25 „ „ 16—25 „;
„ nannandr. 12—17 „ „ 40—46 „.

Hab. in Suecia.

42. Oe. undulatum (Bréb.) Al. Br.

in De Bary, Ueb. Algengatt. Oed. u. Bolb. p. 94. *Conferva undulata* Bréb. *Cymatonema confervaceum* Kütz. Tab. Phyc. III, t. 47, f. 1; Rabenh. Alg. Sachs. n:o 632.

Oe. oogoniis singulis vel binis, ellipsoideo-globosis vel subglobosis, poro inferiore apertis; oosporis ellipsoideo-globosis vel subglobosis, oogonia fere explentibus; cellulis vegetativis quater undulato-constrictis; cellula terminali (quæ interdum est oogonium) obtusa; nannandribus elongato-obconicis, in cellulis suffultoriis sedentibus;

crassit. cell. veget. 15—17 μ, altit. 3—5plo major;
„ oogon. 51—56 „ „ 57—75 μ;
„ oospor. 46—50 „ „ 48—60 „;
„ nannandr. 9—10 „ „ 65—70 „;

Hab. in Suecia, Norvegia, Germania, Gallia.

[1]) Nannandribus non observatis, androsporangia antea spermogonia putavimus.

β. Species oosporis subellipsoideis.

43. **Oe. cyathigerum** Wittr.

Dispos. Oed. Suec. p. 131, t. 1, f. 6 et 7.

Oe. idioandrosporum; oogoniis binis vel singulis, oboviformibus vel quadrangulari-ellipsoideis, poro superiore apertis; oosporis oogonia complentibus; cellulis suffultoriis tumidis; cellula terminali (quæ non raro est oogonium) obtusa; androsporangiis pluricellularibus; nannandribus cyathiformibus, paullulum curvatis, in cellulis suffultoriis sedentibus;

crassit. cell. veget. trivial. 24—30 μ, altit. 2—5plo major;
„ „ suffultor. 44—48 „ „ 2plo „ ;
„ „ oogon. 57—66 „ „ 70—90 μ;
„ „ oospor. 51—60 „ „ 60—75 „;
„ „ cell. androsp. 23—30 „ „ 12—30 „;
„ „ nannandr. 12—15 „ „ 50—54 „.

Hab. in Suecia, Germania.

B. Species nannandribus bicellularibus, spermogonio interiore.

44. **Oe. depressum** Pringsh.

Beitr. z. Morph. d. Alg. I. p. 69, t. 5, f. 5.

Oe. gynandrosporum; oogoniis singulis, depresso-globosis, poro mediano apertis; oosporis depresso-globosis, oogonia non explentibus; androsporangiis 2cellularibus; nannandribus oblongo-oboviformibus, tertia parte brévioribus quam oogoniis, in his sedentibus;

crassit. cell. veget. 8—9 μ, altit. 3—6plo major;
„ oogon. 28 „ „ 26 μ;
„ oospor. 23 „ „ $17^1/_5$ μ.

Hab. in Germania (sec. cel. Pringsheim).

C. Species nannandribus bi-pluricellularibus, spermogonio exteriore.

a. Species oosporis lævibus.

α. Species oosporis globosis vel subglobosis.

45. **Oe. flavescens** (Hass.) Wittr.

Dispos. Oed. Suec. p. 127. (?) *Vesiculifera flavescens* Hass. Hist. of Brit. Freshw. Alg. p. 206, t. 53, f. 9.

Oe. idioandrosporum; oogoniis singulis, oboviformi-globosis (interdum subhexagono-globosis), poro paullum supra medium sito apertis; oosporis globosis, oogonia non plane complentibus (interdum subhexagono-globosis, oogonia complentibus); androsporangiis 1—9cellularibus; nannandribus paullulum curvatis, in cellulis suffultoriis sedentibus, spermogonio uni- (vel bi- ?) cellulari;

crassit.	cell. veget.	18—21 μ,	altit.	4½—6plo major;
„	oogon.	49—52 „	„	51—60 μ;
„	oospor.	45—49 „	„	45—49 „;
„	cell. androsp.	17—20 „	„	8—18 „;
„	stip. nannandr.	11—12 „	„	39—45 „;
„	cell. spermog.	9—10 „	„	15—20 „. Tab. I, fig. 12—14.

Hab. in Suecia.

46. Oe. irregulare Wittr.

Dispos. Oed. Suec. p. 128.

Oe. oogoniis singulis, globosis, poro superiore apertis; oosporis oogonia complentibus; nannandribus rectis, prope oogonia vel in his sedentibus, spermogonio 2—4cellulari;

crassit.	cell. veget;	16—20 μ,	altit.	2½—4plo major;
„	oogon.	37—45 „	„	36—43 μ;
„	oospor.	36—40 „	„	34—38 „;
„	stip. nannandr.	12—15 „	„	20—24 „;
„	cell. spermog.	10—12 „	„	6—8 „.

Hab. in Suecia.

47. Oe. Braunii Kütz.; Pringsh.

Beitr. z. Morph. d. Alg. I, p. 70, t. 5, f. 6; (?) Kütz. Spec. Alg. p. 366.

Oe. gynandrosporum; oogoniis singulis, ellipsoideo-globosis, poro mediano apertis; oosporis globosis, oogonia non plane complentibus; androsporangiis 1—2cellularibus; nannandribus paullulum curvatis, prope oogonia (sæpissime in cellulis suffultoriis) sedentibus, spermogonio unicellulari;

crassit.	cell. veget.	13—15 μ,	altit.	2—4plo major;
„	oogon.	30—33 „	„	33—36 μ;
„	oospor.	27—29 „	„	27—29 „;
„	cell. androsp.	14—15 „	„	11—12 „;
„	stip. nannandr.	7 „	„	15 „;
„	cell. spermog.	5 „	„	9 „.

Hab. in Suecia, Norvegia, Dania, Germania (sec. cel. Pringsheim), Britannia.

48. Oe. lundense Wittr.

Oedog. nov. p. 4.

Oe. oogoniis 2—4 continuis vel singulis, suboboviformi-globosis, operculo apertis, rima angustissima et indistincta, poro magno; oosporis globosis, oogonia fere complentibus; cellula terminali obtusa; nannandribus in oogoniis sedentibus, stipite curvato, spermogonio 2—?cellulari;

crassit.	cell. veget.	15—17 μ,	altit.	1 ½—2 ½plo major;
„	oogon.	32—34 „	„	34—35 μ;
„	oospor.	30—31 „	„	30—31 „;
„	stip. nannandr.	12 „	„	27—30 „;
„	cell. spermog.	10 „	„	6—8 „.

Hab. in Suecia.

49. Hohenackerii Wittr. mscr.

Oe. tumidulum Hohen. Alg. mar. sicc. n:o 404, ex parte.

Oe. gynandrosporum; oogoniis singulis, globosis, operculo apertis, rima angusta; oosporis globosis, oogonia complentibus; androsporangiis hypogynis vel subepigynis; nannandribus paullum curvatis, prope oogonia sedentibus;

crassit.	cell. veget.	12—15 μ,	altit.	2—3plo major;
„	oogon.	32—33 „	„	34—35 μ;
„	oospor.	30—31 „	„	29—30 „;
„	cell. androsp.	11—12 „	„	10—12 „;
„	stip. nannandr.	9 „	„	18 „;
„	cell. spermog.	6 „	„	5 „.

Hab. in India cis Gangem.

50. Oe. propinquum Wittr.

Dispos. Oed. Suec. p. 129.

Oe. oogoniis singulis vel binis, globosis vel oboviformi-globosis, operculo apertis, rima angustissima; oosporis globosis, oogonia fere complentibus; nannandribus subrectis, prope oogonia vel in his sedentibus, spermogonio unicellulari;

crassit.	cell. veget.	11—13 μ,	altit.	2—4plo major;
„	oogon.	33—36 „	„	39—45 μ;
„	oospor.	30—31 „	„	30—31 „;
„	stip. nannandr.	11 „	„	20 „;
„	cell. spermog.	7 „	„	6 „.

Hab. in Suecia, Gallia.

51. Oe. macrandum Wittr.

Dispos. Oedog. Suec. p. 130, t. 1, f. 3—5.

Oe. oogoniis singulis vel binis (raro ternis), oboviformibus vel globoso-oboviformibus, operculo apertis, rima angustissima; oosporis globosis vel oboviformi-globosis, oogonia non plane complentibus; cellula terminali brevissime apiculata; nannandribus valde curvatjs, in oogoniis sedentibus, (stipite sæpe 2—3cellulari), spermogonio pluri- (ad 7-) cellulari;

crassit. cell. veget. 15—16 μ, altit. 3—5plo major;
„ oogon. 36—40 „ „ 43—54 μ;
„ oospor. 31—34 „ „ 33—39 „;
„ stip. nannandr. 12 „ „ 24—33 „;
„ cell. spermog. 9 „ „ 9 „.

Hab. in Suecia, Norvegia, Gallia, Britannia.

52. Oe. crassiusculum Wittr.

Dispos. Oedog. Suec. p. 132.

Oe. gynandrosporum; oogoniis singulis vel binis, globoso-oboviformibus vel subglobosis, poro superiore apertis; oosporis ellipsoideo-globosis vel globosis, membrana incrassata, oogonia fere complentibus; androsporangiis 2—5cellularibus; nannandribus subrectis, in cellulis suffultoriis vel prope has sedentibus, spermogonio 1-(?)cellulari;

crassit. cell. veget. 27—30 μ, altit. $3\frac{1}{2}$—5plo major;
„ oogon. 54—60 „ „ 60—75 μ;
„ oospor. 51—57 „ „ 52—63 „;
„ cell. androsp. 26—28 „ „ 10—18 „;
„ stip. nannandr. 13 „ „ 60 „;
„ cell. spermog. 7—9 „ „ „.

Hab. in Suecia.

β. Species oosporis ellipsoideis vel oviformibus.

53. Oe. birmanicum Wittr. mscr.

Inter *Oe. Rothii* Zell. in Rabenh. Alg. Eur. n:o 2347.

Oe. oogoniis singulis vel 2—4 continuis, oboviformi-ellipsoideis, poro superiore apertis; oosporis ellipsoideo-globosis vel globoso-ellipsoideis, oogonia non plane complentibus; cellulis suffultoriis tumidis; cellula terminali (quæ interdum est oogonium) obtusa; nannandribus subrectis, in cellulis suffultoriis sedentibus;

crassit. cell. veget. 14—23 μ, altit. 4—5plo major;
„ „ suffult. 30—34 „ „ $1^3/_4$—$2^1/_2$ „ „ ;
„ oogon. 42—47 „ „ 45—75 μ;
„ oospor. 37—45 „ „ 40—46 „;
„ stip. nannandr. 11—12 „ „ 39—42 „;
„ cell. spermog. 6 „.
Hab. in India trans Gangem.

54. **Oe. Borisianum** (Le Cl.) Wittr.

Dispos. Oedog. Suec. p. 132. *Prolifera Borisiana* Le Clerc Sur gen. Prolif. p. 175, t. 23, f. 6. *Oe. apophysatum* Al. Br. [1]) in Kütz. Spec. Alg. p. 366; Kütz. Tab. Phyc. III, t. 35, f. 5; *non* Pringsh. — ?*Oe. setigerum* Vaup. Iagtt. öv. Oedog. p. 17, t. 1.

Oe. gynandrosporum (vel idioandrosporum?); oogoniis singulis vel binis, oboviformibus, poro superiore apertis; oosporis oboviformibus, oogonia fere complentibus; cellulis suffultoriis tumidis; androsporangiis 2—?cellularibus; cellula terminali (quæ sæpius est oogonium) obtusa; nannandribus paullum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 15—21 μ, altit. 3—5plo major;
„ „ suffult. 31—33 „ „ 2 „ „ ;
„ oogon. 45—50 „ „ 60—75 μ;
„ oospor. 40—44 „ „ 51—54 „;
„ cell. androsp. 17—18 „ „ 15—20 „;
„ stip. nannandr. 18 „ „ 45—57 „;
„ cell. spermog. 10 „ „ 21 „.
Hab. in Suecia, Germania, Gallia, Britannia.

55. **Oe. concatenatum** (Hass.) Wittr.

Vesiculifera concatenata Hass. [2]) Hist. of Brit. Freshw. Alg. p. 201, t. 50, f. 6, (fig. non bona). *Oe. apophysatum* Pringsh. Beitr. z. Morph. d. Alg. I, p. 71, t. 5, f. 9; *non* Al. Br.

Oe. gynandrosporum, oogoniis 2—6 continuis vel singulis, oviformibus vel quadrangulari-ellipsoideis, poro superiore apertis; oosporis oogonia complentibus exosporio subtilissime poroso; cellulis suffultoriis tumidis; androsporangiis 2—4cellularibus; cellula terminali obtusa; nannandribus curvatis, in cellulis suffultoriis sedentibus, spermogonio bi—quadricellulari;

[1]) Specimina originalia in herbario cel. Professoris Al. Braun examinavimus.

[2]) Vidimus specimina originalia Hassalliana ex herbario cel. Professoris J. E. Areschoug.

crassit. cell. veget. 25—40 μ, altit. 3—10plo major;
„ „ suffult. 58—62 „ „ 2½ „ „ ;
„ oogon. 70—83 „ „ 90—105 μ;
„ oospor. 65—76 „ „ 87—95 „;
„ cell. androsp. 27—28 „ „ 30—36 „;
„ stip. nannandr. 20—25 „ „ 55—75 „;
„ cell. spermog. 13—15 „ „ 22—25 „.
Hab. in Suecia, Germania, Britannia.

56. **Oe. sexangulare** Clev.

in Wittr. Dispos. Oed. Suec. p. 131.

Oe. gynandrosporum, oogoniis singulis (raro binis), sexangulari-ellipsoideis, poro paullum supra medium sito apertis; oosporis oogonia complentibus; androsporangiis 2—3cellularibus; nannandribus paullum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 9—16 μ, altit. 3½—7plo major;
„ oogon. 29—32 „ „ 33—38 μ;
„ oospor. 27—30 „ „ 31—36 „;
„ cell. androsp. 13—14 „ „ 10—14 „;
„ stip. nannandr. 7—9 „ „ 21—27 „;
„ cell. spermog 6—7 „ „ 9—12 „.
Hab. in Suecia, Norvegia.

57. **Oe. acrosporum** De Bar.

Ueb. Oedog. u. Bolb. p. 60—64 et 94, t. 3, f. 1—12.

Oe. idioandrosporum [1]); oogonio solitario, terminali, ellipsoideo, operculo apicali [2]), minimo, mox decidente (vel evanescente?) aperto; oospora oogonium plane complente, membrana in latere interiore longitudinaliter costata; cellula suffultoria plerumque subtumida; cellula terminali obtusa; nannandribus curvatis, in cellula suffultoria sedentibus, stipite (plerumque) bicellulari, cellula stipitis superiore longissima, spermogonio 1—2cellulari;

crassit. cell. veg. plant. fem. 10—14 μ, altit. 2—7plo major;
„ „ suffult. 15—18 „ „ 2—3 „ „ ;
„ oogon. 30—35 „ „ 45—51 μ;
„ cell. infer. stip. nann. 9—12 „ „ 24—32 „;
„ „ super. „ „ 6—8 „ „ 55—65 „;
„ „ spermog. 6—8 „ „ 14—15 „.
Hab. in Suecia, Norvegia, Germania, Britannia.

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[1]) Sec. cel. A. De Bary, l. c. p. 63.

[2]) Cel. C. F. O. Nordstedt hoc organum primus observavit.

58. **Oe. ciliatum** (Hass.) Pringsh.

Beitr. z. Morph. d. Alg. I, p. 70, t. 5, f. 8. *Vesiculifera ciliata* Hass. Hist. of Brit. Freshw. Alg. p. 202, t. 52, f. 2. *Oe. piliferum* Auersw. in Rabenh. Alg. Sachs. n:o 474.

Oe. gynandrosporum; oogoniis 2—7 continuis vel singulis, oviformibus, operculo apertis, rima lata; oosporis oviformibus, oogonia fere complentibus; androsporangiis 2—8cellularibus; cellula terminali setiformi; nannandribus curvatis, in oogoniis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 15—23 μ, altit. $2^1/_2$—4plo major;
„ oogon. 43—50 „ „ 55—72 μ;
„ oospor. 40—46 „ „ 47—57 „;
„ cell. androsp. 18—20 „ „ 16—20 „;
„ stip. nannandr. 15—20 „ „ 29—31 „;
„ cell. spermog. 8—10 „ „ 10—11 „.

Hab. in Suecia, Germania, Britannia.

59. **Oe. crispulum** Wittr. & Nordst.

in Wittr. Oedog. nov. p. 5.

Oe. oogoniis singulis, rarius binis, oviformibus, operculo apertis, rima lata; oosporis partem inferiorem oogoniorum complentibus, globoso-ellipsoideis, membrana crenulata; nannandribus in oogoniis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 4—7 μ, altit. $2^1/_2$—4plo major;
„ oogon. 17—18 „ „ 24—27 μ;
„ oospor. 15—16 „ „ 17—21 „;
„ stip. nannandr. 5—6 „ „ 12—13 „;
„ cell. spermog. 4—$4^1/_2$ „ „ 5—$5^1/_2$ „.

Hab. in Suecia.

b. Species oosporis echinatis.

α. Species oosporis globosis.

60. **Oe. stellatum** Wittr.

Dispos. Oed. Suec. p. 129.

Oe. gynandrosporum; oogoniis singulis vel 2—3 continuis, oboviformi-globosis, poro superiore apertis; oosporis oogonia fere complentibus, globosis, echinis conicis, spiraliter dispositis, spiris fere septem; androsporangiis 1—2cellularibus; cellula terminali gracili, subhyalina, apice obtuso; nannandribus rectis, in cellulis suffultoriis sedentibus, spermogonio bicellulari;

crassit. cell. veget. 15—35 μ, altit. 2½—5plo major;
„ oogon. 51—61 „ „ 58—70 μ;
„ oospor. (c. echin.) 50—58 „ „ 50—58 „; longitud.
echinor. 4—4½ „;
„ cell. androsp. 14—19 „ altit. 20—27 „;
„ stip. nannandr. 11—13 „ „ 45—52 „;
„ cell. spermog. 6—9 „ „ 8—10 „. Tab. I, fig. 15.
Hab. in Suecia.

61. Oe. Cleveanum Wittr.

Dispos. Oed. Suec. p. 129. *Oe. echinospermum* Pringsh. Beitr. z. Morph. d. Alg. I, p. 70, t. 5, f. 7; *non* Al. Br.

Oe. gynandrosporum, oogoniis singulis, subglobosis, poro inferiore apertis; oosporis oogonia fere complentibus, globosis, echinis conicis, spiraliter dispositis; androsporangiis 4—6cellularibus; nannandribus paullulum curvatis in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 18—26 μ, altit. 3—7plo major;
„ oogon. 52—60 „ „ 59—63 μ;
„ oospor. (c. echin.) 49—57 „ „ 51—59 „; longit.
echinor. 4 „;
„ cell. androsp. 18—22 „ altit. 9—18 „;
„ stip. nannandr. 10—11 „ „ 29—30 „;
„ cell. spermog. 8—8½ „ „ 14—16 „.
Hab. in Suecia, Germania, Gallia.

62. Oe. hispidum Nordst.

in Wittr. Dispos. Oed. Suec. p. 128.

Oe. gynandrosporum; oogoniis singulis, sæpe terminalibus, ellipsoideo-globosis, poro inferiore apertis; oosporis oogonia non plane complentibus, globosis, echinis subuliformibus; androsporangiis 2cellularibus; cellula terminali obtusa; nannandribus paullulum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 9—14 μ, altit. 3½—5plo major;
„ oogon. 36—42 „ „ 45—52 μ;
„ oospor. (c. echin.) 34—39 „ „ 36—40 „; longit.
echinor. 3 „;
„ cell. androsp. 10 „ altit. 6—8 „;
„ stip. nannandr. 7—8 „ „ 17—18 „;
„ cell. spermog. 5—6 „ „ 7—9 „.
Hab. in Suecia, Gallia.

63. **Oe. Aster** Wittr.

Oedog. nov. p. 4.

Oe. oogoniis singulis, globosis, poro mediano apertis; oosporis oogonia fere complentibus. globosis, echinis subuliformibus; cellula terminali (quæ interdum est oogonium) obtusa; nannandribus paullulum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget.	7—10 μ,	altit.	7—9plo	major;	
„ oogon.	33—34 „	„	34—39 μ;		
„ oospor. (c. echin.)	31—32 „	„	31—32 „;		longit.
		echinor.	2 „;		
„ stip. nannandr.	6—7 „	altit.	23—25 „;		
„ cell. spermog.	5—6 „	„	7—8 „.		

Hab. in Suecia.

64. **Oe. echinospermum** Al. Br. [1]).

in Kütz. Spec. Alg. p. 366; Kütz. Tab. Phyc. III, t. 36, f. 2; De Bar. Ueb. Oedog. u. Bolb. t. 3, f. 13—22 et 33; Rabenh. Alg. Eur. n:o 1817; *non* Pringsh.

Oe. gynandrosporum vel idioandrosporum; oogoniis singulis, ellipsoideo-globosis vel subglobosis, poro mediano apertis; oosporis oogonia fere complentibus, globosis, echinis subuliformibus; androsporangiis 2—5cellularibus; nannandribus paullulum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget	18—30 μ,	altit.	$2^1/_2$—$4^1/_2$plo	major;	
„ oogon.	40—50 „	„	42—57 μ;		
„ oospor. (c. echin.)	38—47 „	„	38—49 „;		longit.
		echinor.	3 „;		
„ cell. androsp.	21—25 „	altit.	9—15 „;		
„ stip. nannandr.	12—15 „	„	30—35 „;		
„ cell. spermog.	10—12 „	„	12—15 „.		

Hab. in Suecia, Germania. Gallia.

β. Species oosporis ellipsoideis.

65. **Oe. Hystrix** Wittr.

Dispos. Oedog. Suec. p. 133.

Oe. gynandrosporum (vel idioandrosporum?); oogoniis singulis, ellipsoideis ad globoso-ellipsoideis, poro mediano apertis; oosporis oogonia com-

[1]) Specimina originalia in herbario cel. Auctoris vidimus.

plentibus, echinis subuliformibus; androsporangiis 2—3cellularibus; cellula terminali obtusa; nannandribus paullum curvatis, in cellulis suffultoriis sedentibus, spermogonio unicellulari;

crassit. cell. veget. 17—22 μ, altit. $2^1/_2$—$4^1/_2$plo major;
„ oogon. 38—44 „ „ 45—65 μ;
„ oospor. (c. echin.) 37—42 „ „ 43—55 „; longit.
echinor. $2^1/_2$ „;
„ cell. androsp. 17—18 „ altit. 13—18 „;
„ stip. nannandr. 10—11 „ „ 22—23 „;
„ cell. spermog. $7^1/_2$—8 „ „ 12—14 „.

Hab. in Suecia.

SUBSECTIO II. Species dioicæ macrandræ.

a. Species oosporis echinatis.

66. **Oe. suecicum** Wittr.

Oedog. nov. p. 5.

Oe. oogoniis singulis, globosis ad ellipsoideo-globosis, poro mediano apertis; oosporis oogonia fere complentibus, globosis, echinis subuliformibus; plantis masculis eadem prope crassitudine ac femineis; spermogoniis 2—4-cellularibus, in parte superiore fili sitis; cellula terminali obtusa;

crassit. cell. veget. 9—14 μ, altit. 4—6plo major;
„ oogon. 32—38 „ „ 36—40 μ;
„ oospor. (c. echin.) 31—37 „ „ 31—37 „; longit.
echinor. 2 „;
„ cell. spermog. 11—12 „ altit. 13—16 „.

Hab. in Suecia, Norvegia, Dania, Austria, Gallia.

b. Species oosporis lævibus.

α. Species oogoniis non vel paullulum tumidis.

67. **Oe. capillare** (Lin.) Kütz.

Phyc. gener. p. 255, t. 12, f. II 1—10; Rabenh. Alg. Eur. n:ris 1180 et 1417. *Conferva capillaris* Lin. Spec. Plant. p. 1636. *Oe. regulare* Vaup. Bidr. t. Oedog. Morph. p. 213, t. I, f. 1—10.

Oe. oogoniis singulis, non tumidis, cylindricis, poro superiore apertis; oosporis globosis vel cylindrico-globosis (in sectione optica longitudinali tunc

subtetragonis), oogonia non complentibus; plantis masculis eadem prope crassitudine ac plantis femineis; spermogoniis 1—4cellularibus, iis cellulisque vegetativis alternis; spermatozoidis binis;

crassit. cell. veget. 35—55 μ, altit. par—2plo major;
„ oogon. „ „ „ „ „ —$1^1/_2$ „ ;
„ oospor. 30—52 „ „ 39—63 μ;
„ cell. spermog. 30—48 „ „ 5—6 „.

Hab. in Suecia, Dania (sec. cel. Vaupell), Germania.

68. **Oe. stagnale** Kütz.

Spec. Alg. p. 368; Tab. Phyc. III, t. 41, f. 2.

Oe. oogoniis singulis, vix tumidis, subcylindricis, poro superiore apertis; oosporis subcylindricis (in sectione optica longitudinali rectangularibus, angulis leviter rotundatis) vel globoso-ellipsoideis, in medio plerumque subconstrictis, oogonia fere complentibus; plantis masculis paullulo gracilioribus quam femineis; spermogoniis 1—3cellularibus, iis cellulisque vegetativis alternis;

crassit. cell. veg. plant. fem. 42—46 μ, altit. $1^1/_4$—2plo major;
„ „ „ „ masc. 38—40 „ „ „ „ „ „ ;
„ oogon. 49—51 „ „ 65—75 μ;
„ oospor. 47—49 „ „ 50—66 „;
„ cell. spermog. 36—38 „ „ 7—9 „.

Hab. in Germania.

69. **Oe. capilliforme** Kütz.

Spec. Alg. p. 367(?); Tab. Phyc. III, t. 37, f. 3(?); Wittr. Gotl. o. Öl. Sötv. Alg. p. 21.

Oe. oogoniis singulis, paullulum tumidis, oboviformibus, poro superiore apertis; oosporis subellipsoideo-globosis vel cylindrico-globosis (in sectione optica longitudinali tunc subtetragonis), oogonia non complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 2—10cellularibus, iis cellulisque vegetativis sæpe alternis; cellula terminali obtusa;

crassit. cell. veg. plant. fem. 32—38 μ, altit. $1^1/_4$—3plo major;
„ „ „ „ masc. 25—30 „ „ $1^1/_2$— „ „ ;
„ oogon. 42—48 „ „ 51—62 μ;
„ oospor. 37—45 „ „ 40—50 „;
„ cell. spermog. 20—25 „ „ 8—10 „.

Hab. in Suecia.

β. Species oogoniis manifeste tumidis.

αα. Species oosporis globosis vel subglobosis.

70. **Oe. calcareum** Clev.

in Wittr. Dispos. Oedog. Suec. p. 135. ?*Vesiculifera compressa* Hass. Hist. of Brit. Freshw. Alg. p. 204. t. 53, f. 4.

Oe. oogoniis singulis (rarissime binis) depresso-globosis, poro mediano apertis; oosporis oogonia complentibus; plantis masculis eadem prope crassitudine ac femineis; spermogoniis 2—5cellularibus, spermatozoidis singulis(?);

crassit. cell. veget. 11—14 μ, altit. 2—4plo major;
„ oogon. 27—30 „ „ 21—23 μ;
„ oospor. 26—28 „ „ 20—21 „;
„ cell. spermog. 10—11 „ „ 9—12 „.

Hab. in Suecia.

71. **Oe. rufescens** Wittr.

Dispos. Oedog. Suec. p. 134; Aresch. Alg. Scand. exsic. fasc. 7 & 8, n:o 350.

Oe. oogoniis singulis vel 2—3 continuis, globosis vel oboviformi-globosis, poro mediano apertis; oosporis globosis, oogonia complentibus; plantis masculis eadem fere crassitudine ac femineis; spermogoniis 6—12cellularibus, spermatozoidis singulis(?);

crassit. cell. veget. 8—10 μ, altit. 5—6plo major;
„ oogon. 22—24 „ „ 24—30 μ;
„ oospor. 21—23 „ „ 20—22 „;
„ cell. spermog. 6—8 „ „ 8—12 „.

Hab. in Suecia.

72. **Oe. Lundellii** Wittr. mscr.

Oe. oogoniis singulis (raro binis), subdepresso-globosis, poro mediano vel paullum supra medium sito apertis; oosporis depresso-globosis, oogonia non plane complentibus; plantis masculis paullulo gracilioribus quam femineis; spermogoniis 3—10cellularibus, spermatozoidis singulis(?);

crassit. cell. veg. plant. fem. 9—12 μ, altit. 3—4plo major;
„ „ „ „ masc. 8—9 „ „ „ „ „ „ „;
„ oogon. 22—25 „ „ 24—27 μ;
„ oospor. 19—21 „ „ 15—18 „;
„ cell. spermog. $7\frac{1}{2}$—$8\frac{1}{2}$ „ „ 6—10 „.

Hab. in Suecia.

73. Oe. cardiacum (Hass.) Wittr.

Dispos. Oed. Suec. p. 135. *Vesiculifera cardiaca* Hass. Hist. of Brit. Freshw. Alg. p. 203, t. 51, f. 4. *Vesiculifera pulchella* Hass. [1]) l. c. p. 199, t. 50, f. 3.

Oe. oogoniis singulis, obcordiformi-globosis, poro paullum supra medium sito apertis; oosporis globosis, oogonia non complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 2—10cellularibus, spermatozoidis binis; cellula terminali obtusa;

crassit. cell. veg. plant. fem. 18—30 μ, altit. 2—7plo major;
„ „ „ „ masc. 15—25 „ „ „—6 „ „ ;
„ oogon. 50—70 „ „ 58—86 μ;
„ oospor. 42—60 „ „ 42—60 „;
„ cell. spermog. 15—21 „ „ 10—13 „.

Hab. in Suecia, Dania, Germania, Britannia.

74. Oe. carbonicum Wittr. mscr.

Oe. oogoniis singulis vel binis, oboviformibus vel oboviformi-globosis, poro superiore apertis; oosporis ellipsoideo-globosis vel subglobosis, oogonia fere complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 2—5cellularibus, spermatozoidis binis; cellula terminali obtusa;

crassit. cell. veg. plant. fem. 16—30 μ, altit. 3—6plo major;
„ „ „ „ masc. 14—16 „ „ „ „ „ ;
„ oogon. 43—52 „ „ 50—72 μ;
„ oospor. 42—50 „ „ 46—56 „;
„ cell. spermog. 13—14 „ „ 12—14 „.

Hab. in Britannia, Dania.

75. Oe. Pringsheimii Cram.

in Hedw. n:o 3, p. 17, t. 1 C, f 1—4; Rabenh. Alg. Eur. n:o 790. *Oe. Nordstedtii* Wittr. Oedog. nov. p. 6, t. 1, f. 7 & 8.

Oe. oogoniis singulis vel 2—6 continuis, suboboviformi-globosis, operculo apertis, rima angustissima et perindistincta; oosporis globosis, oogonia non plane complentibus; plantis masculis paullulo gracilioribus quam femineis; spermogoniis 2—10cellularibus, his et cellulis vegetativis in parte fili superiore alternis; cellula terminali obtusa vel raro brevi-apiculata;

crassit. cell. veg. plant. fem. 12—20 μ, altit. 2—4plo major;
„ „ „ „ masc. 11—16 „ „ „ „ „ ;
„ oogon. 30—43 „ „ 36—45 μ;

[1]) Vidimus specimina originalia Hassalliana ex herbario cel. Professoris J. E. Areschoug.

crassit. oospor. 28—35 μ, altit. 28—34 μ;
„ cell. spermog. 10—15 „ „ 6—9 „. Tab. I, fig. 16 et 17.
Hab. in Suecia, Norvegia, Dania, Germania, Austria, Helvetia, Britannia.

76. Oe. punctato-striatum De Bar.

Ueb. Oedog. u. Bolb. p. 47, t. 2, f. 15 & 16; Rabenh. Alg. Sachs. n:o 214, Alg. Eur. n:o 2276.

Oe. oogoniis singulis, depresso-globosis, in medio manifeste circumscissis, poro in circumscissione sito apertis; oosporis depresso-globosis, oogonia prope complentibus; plantis masculis paullulo gracilioribus quam femineis; spermogoniis 3—7cellularibus; spermatozoidis singulis; membrana cellularum vegetativarum et oogoniorum punctulis spiraliter dispositis ornata; cellula basali depresso-globosa, membrana verticaliter plicata;

crassit. cell. veg. plant. fem. 18—22 μ, altit. 2—6plo major;
„ „ „ „ masc. 16—19 „ „ „ „ „ ;
„ oogon. 48—55 „ „ 38—48 μ;
„ oospor. 44—51 „ „ 35—43 „;
„ cell. spermog. 16—18 „ „ 6—10 „;
„ „ basal. 28—30 „ „ 23—25 „.
Hab. in Suecia, Norvegia, Germania, Groenlandia, Brasilia.

$\beta\beta$. Species oosporis ellipsoideis vel oviformibus.

77. Oe. rhodosporum (Welw.) Wittr.

(Welw. Crypt. lusit. n:o 277).

Oe. oogoniis singulis, rarius binis vel ternis, oboviformibus ad oboviformi-globosis, operculo apertis, rima angustissima; oosporis oboviformibus ad oboviformi-globosis, oogonia complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 2—6cellularibus;

crassit. cell. veg. plant. fem. 16—22 μ, altit. $1^1/_4$—$2^1/_2$plo major;
„ „ „ „ masc. 15—19 „ „ $1^3/_4$— „ „ ;
„ oogon. 39—43 „ „ 45—54 μ;
„ oospor. 37—39 „ „ 40—48 „;
„ cell. spermog. 16—18 „ „ 9—12 „.
Hab. in Lusitania, Gallia.

78. Oe. Boscii (Le Cl.) Wittr.

Dispos. Oedog. Suec. p. 136. Aresch. Alg. Scand. exsic. ser. nov. fasc. 7 & 8, n:o 351; Rabenh. Alg. Eur. n:ris 2198 et 2369. *Prolifera Boscii* Le Clerc Sur gen. Prolif. p. 474, t. 23, f. 5.

Oe. oogoniis singulis, raro binis, oblongo-ellipsoideis, poro superiore apertis; oosporis ellipsoideis, oogonia longe non complentibus, membrana in latere interiore longitudinaliter costata; plantis masculis eadem prope crassitudine ac femineis; spermogoniis 3—6cellularibus, spermatozoidis binis; cellula terminali gracili et subhyalina:

crassit. cell. veget. 14—20 μ, altit. 4—6plo major;
„ oogon. 40—45 „ „ 80—100 μ;
„ oospor. 36—40 „ „ 60—65 „;
„ cell. spermog. 13—14 „ „ 6—9 „.

Hab. in Suecia, Norvegia, Austria, Gallia, Groenlandia.

79. Oe. tumidulum Kütz.

Dec. Alg. n:o 60; *non* Pringsh.

Oe. oogoniis singulis, ellipsoideo-oboviformibus, poro superiore apertis; oosporis ellipsoideis, oogonia fere complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 6—45cellularibus, spermatozoidis binis.

crassit. cell. veg. plant. fem. 18—25 μ, altit. $3^1/_2$—5plo major;
„ „ „ „ masc. 15—18 „ „ 4— „ „ ;
„ oogon. 56—58 „ „ 78—90 μ;
„ oospor. 49—54 „ „ 61—68 „;
„ cell. spermog. 15—17 „ „ 9—12 „.

Hab. in Germania.

80. Oe. Landsboroughi (Hass.) Wittr.

Vesiculifera Landsboroughi Ann. of Nat. Hist. vol. X, p. 389[1]); Hist. of Brit. Freshw. Alg. p. 197, t. 51, f. 2. *Oe. gemelliparum* Hantzsch in Rabenh. Alg. Eur. n:o 1118; *Oe. gemelliparum* var. *major* Wittr. Dispos. Oedog. Suec. p. 137.

Oe. oogoniis singulis, raro binis, oboviformibus, poro superiore apertis; oosporis oboviformibus, oogonia complentibus (raro oogonia non complentibus, ellipsoideis); plantis masculis paullulo gracilioribus quam femineis; spermogoniis 5—25cellularibus; spermatozoidis binis, divisione verticali ortis; cellula terminali obtusa;

crassit. cell. veg. plant. fem. 33—36 μ, altit. 4—6plo major;
„ „ „ „ masc. 31—33 „ „ „ „ „ ;
„ oogon. 63—75 „ „ 105—120 μ;
„ oospor. 59—70 „ „ 75—102 „;
„ cell. spermog. 27—30 „ „ 9—12 „.

Hab. in Suecia, Germania, Gallia, Britannia.

[1]) Vidimus exemplaria originalia Hassalliana ex herbario cel. Professoris J. E. Areschoug.

β. *Gemelliparum* Pringsh.

Oedogonium gemelliparum Pringsh. Beitr. z. Morph. d. Alg. I, p. 71, t. 5, f. 10.

Var. minor, oogoniis oviformibus, oosporis oogonia complentibus; cellulis terminalibus longissimis, subhyalinis;

crassit. cell. veget. plant. fem. $20^1/_2$—27 μ, altit. 3—5—8plo major;
„ oogon. 55—57 „ „ 75—80 μ;
„ oospor. 49—51 „ „ 65—69 „.

Hab. in Germania (sec. cel. Pringsheim).

81. Oe. rivulare (Le Cl.) Al. Br.

Ueb. Chytrid. p. 23, t. 1, f. 1—10. (?)*Prolifera rivularis* Le Clerc Sur. gen. Prolif. p. 472, t. 23, f. 1.

Oe. oogoniis singulis vel 2—7 continuis, oboviformibus, poro superiore apertis; oosporis oboviformibus, rarius ellipsoideis vel subglobosis, oogonia longe non complentibus; plantis masculis paullo gracilioribus quam femineis; spermogoniis 3—9cellularibus, spermatozoidis binis;

crassit. cell. veg. plant. fem. 35—45 μ, altit. 3—8plo major;
„ „ „ „ masc. 30—36 „ „ 4— „ „ ;
„ oogon. 70—85 „ „ 130—160 μ;
„ oospor. 55—70 „ „ 65—100 „;
„ cell. spermog. 21—28 „ „ 14—16 „.

Hab. in Dania, Germania.

Species, quarum organa fructificationis non satis nota nobis sunt.

82. Oe. tapeinosporum Wittr. mscr.

Oe. oogoniis singulis, depresso-globosis, adversus mitram sensim angustatis; oosporis depresso-globosis (vel potius transverse ellipsoideis), oogonia non complentibus;

crassit. cell. veget. $2^2/_3$—4 μ, altit. 5—8plo major;
„ oogon. 15—16 „ „ 18—20 μ;
„ oospor. 13—14 „ „ 9—10 „.

Hab. in Brasilia.

83. Oe. spetsbergense Wittr. mscr.

Oe. monoicum; oogoniis singulis, depresso-globosis; spermogoniis unicellularibus, spermatozoidis binis; cellula terminali obtusa;

crassit. cell. veget. 6—7 μ, altit. 3—5plo major;
„ oogon. 21—23 „ „ 19—26 μ;
„ cell. spermog. 5 „ „ 10 „.

Hab. in Insulis Spetsbergensibus.

Cum. *Oe. cryptoporo* Wittr. comparandum.

84. Oe. Gunnii Wittr. mscr.

Oe. monoicum(?); oogoniis 2—4 continuis, rarius singulis, depresso-globosis, poro mediano apertis; oosporis depresso-globosis, oogonia complentibus; spermogoniis(?) subepigynis;

crassit. cell. veget. 6—7 μ, altit. 7—10plo major;
„ oogon. 23—27 „ „ 19—27 μ;
„ oospor. 22—26 „ „ 17—21 „;
„ cell. spermog.(?) 6 „ „ 12 „.

Hab. in Tasmania.

Species hæc de *Oe. cryptoporo β vulgari* Wittr. admonet.

85. Oe. nanum Wittr. mscr.

Oe. tumidulum Hohen. Alg. mar. sicc. n:o 404, ex parte.

Oe. oogoniis singulis, globosis, operculo apertis, rima lata; oosporis globosis, oogonia complentibus;

crassit. cell. veget. 6—9 μ, altit. $1^1/_2$—3plo major;
„ oogon. 24—28 „ „ 30—33 μ;
„ oospor. 23—27 „ „ 25—26 „.

Hab. in India cis Gangem.

86. Oe. calvum Wittr. mscr.

Oe. vesicatum Zell. in Rabenh. Alg. Eur. n:o 2348.

Oe. oogoniis 2—3 continuis vel singulis, subdepresso-globosis (pars mitralis deest), in medio circumscissis, poro in circumscissione sito apertis;

crassit. cell. veget. 7—9 μ, altit. 3—$4^1/_2$plo major;
„ oogon. 27—28 „ „ 25—27 μ.

Hab. in India trans Gangem

87. Oe. longatum Kütz.

Spec. Alg. p. 364; Tab. Phyc. III, t. 33. f. 6.

Oe. oogoniis singulis (sæpe solitariis, terminalibus), rarius 2—3 continuis, ellipsoideis, operculo apertis, rima angusta; oosporis globoso-ellipsoideis, oogonia fere complentibus; cellula terminali obtusa;

crassit. cell. veget. 5—6 μ, altit. 2—3plo major;
„ oogon. 16—17 „ „ 21—24 μ;
„ oospor. 15—16 „ „ $17\frac{1}{2}$—$18\frac{1}{2}$ „.

Hab. in Suecia, Britannia.

Species hæc ad *Oe. crispulum* Wittr. et Nordst. proxima nobis videtur.

88. Oe. Lagerstedtii Wittr. mscr.

Oe. oogoniis 2—4 continuis vel singulis, depresso-oboviformibus, in medio processibus 8—11, rotundatis, verticillatis instructis, infra medium circumscissis, poro in circumscissione sito apertis, a vertice visis orbicularibus, margine undulata, undulis 8—11 (plerumque 8); oosporis subglobosis, oogonia non plane complentibus;

crassit. cell. veget. 5—6 μ, altit. 2—3plo major;
„ oogon. 17—19 „ „ 14—16 μ;
„ oospor. 13—15 „ „ 12—13 „.

Hab. in Suecia.

Cum *Oe. platygyno* Wittr. comparandum.

89. Oe. plicatulum Wittr. mscr.

Oe. Rothii Zell. in Rabenh. Alg. Eur. n:o 2347.

Oe. oogoniis singulis vel 2—3 continuis, globoso-pyriformibus, plicis longitudinalibus, minimis, circa 12 in medio ornatis, operculo apertis, rima angustissima; oosporis subdepresso-globosis, oogonia prope complentibus; filis hinc illic spiraliter contortis;

crassit. cell. veget. 7—8 μ, altit. 2—$5\frac{1}{2}$plo major;
„ oogon. 23—26 „ „ 24—25 μ;
„ oospor. 21—23 „ „ 21—22 „.

Hab. in India trans Gangem.

90. Oe. Magnusii Wittr. mscr.

Oe. trioicum(?), oogoniis singulis vel binis (raro ternis), depresso-globosis, poro mediano apertis; oosporis oogonia complentibus, membrana undulato-verruculosa; spermogoniis 3—8cellularibus;

crassit. cell. veget. 7—9 μ, altit. $1^1/_2$—4plo major;
„ oogon. 24—27 „ „ 21—26 μ;
„ oospor. 22—25 „ „ 18—22 „;
„ cell. spermog. 8—9 „ „ 5—8 „.
Hab. in Germania.

91. Oe. londinense Wittr. mscr.

Oe. monoicum?, oogoniis binis vel singulis, globosis, in medio circumscissis, poro in circumscissione sito apertis; oosporis globosis, oogonia fere complentibus; spermogoniis? (an androsporangiis?) 1—2cellularibus, hypogynis;

crassit. cell. veget. 10—15 μ, altit. $1^1/_2$—5plo major;
„ oogon. 33—35 „ „ 33—43 μ;
„ oospor. 27—32 „ „ 27—32 „;
„ cell. spermog.? 26—27 „ „ 27—29 „.
Hab. in Britannia.

Oedogonio Areschougii Wittr. affine esse videtur.

92. Oe. Euganeorum Wittr. mscr.

Oe. oogoniis singulis, raro binis, pyriformi-oboviformibus, operculo apertis, rima angusta; oosporis globosis vel subglobosis, oogonia non plane complentibus;

crassit. cell. veget. 10—13 μ, altit. 3—4plo major;
„ oogon. 29—31 „ „ 33—35 μ;
„ oospor. 26—27 „ „ 27—29 „.
Hab. in Italia.

Oe. crispo (Hass.) Wittr. verisimiliter proximum.

93. Oe. pyriforme Wittr. mscr.

Oe. monoicum, oogoniis singulis, pyriformibus; spermogoniis 2—3cellularibus, subepigynis vel hypogynis;

crassit. cell. veget. 13—14 μ, altit. $3^1/_2$—6plo major;
„ oogon. 40—42 „ „ 54—60 μ;
„ cell. spermog. 10—12 „ „ 9—12 „.
Hab. in Tasmania.

94. Oe. vesicatum (Lyngb.) Wittr.

Conferva vesicata Lyngb. Hydroph. Dan. p. 144, t. 47, f. D 1 [1]).

Oe. oogoniis singulis, ellipsoideo-globosis, operculo apertis, rima angusta, oosporis ellipsoideo-globosis, oogonia fere complentibus;

[1]) Exemplaria originalia, a cel. Professore C. Jessen benevole communicata, examinavimus.

crassit. cell. veget. 17—21 μ, altit. $1^1/_4$—3plo major;
„ oogon. 43—45 „ „ 51—60 μ;
„ oospor. 37—38 „ „ 41—42 „.
Hab. in Dania.

95. Oe. moniliforme Wittr. mscr.

Oe. oogoniis 2—5 continuis vel singulis, oboviformi-globosis, poro superiore apertis; oosporis globosis ad oboviformi-globosis, oogonia prope complentibus; membrana oosporarum crassa, episporio indistincte verruculoso;
crassit. cell. veget. 9—11 μ, altit. 3—5plo major;
„ oogon. 24—28 „ „ 28—35 μ;
„ oospor. 23—27 „ „ 23—26 „;
„ membr. oospor. $2^3/_4$ „.
Hab. in Suecia.

Ad speciem sequentem vertens.

96. Oe. Monile Berk. et Harv.

in Hook. Flor. Tasm. p. 342, t. 196, f. B. [1])

Oe. oogoniis 2—3 continuis, rarius singulis, subglobosis, poro superiore apertis; oosporis subglobosis, oogonia complentibus, episporio verruculoso; cellulis suffultoriis tumidis; cellula terminali (quæ sæpius est oogonium) obtusa;
crassit. cell. veget. 11—15 μ, altit. 5—7plo major;
„ „ suffult. 25—29 „ „ 42—45 μ;
„ oogon. inferior. e plur. contin. 36 „ „ 33—36 „;
„ „ singul. vel suprem. etc. 39 „ „ 48—54 „;
„ oospor. 35—38 „ „ 32—36 „.
Hab. in Tasmania.

97. Oe. Sancti Thomæ Wittr. et Clev.

Oe. oogoniis singulis vel 2—3 continuis, pyriformibus, poro superiore apertis; oosporis pyriformi-oboviformibus, oogonia non plane complentibus; cellulis terminalibus gracillimis, subhyalinis;
crassit. cell. veget. 7—15 μ, altit. 2—6plo major;
„ „ terminal. 2—4 „ „ 5—7 „ „ ;
„ oogon. 28—33 „ „ 36—48 μ;
„ oospor. 25—30 „ „ 28—35 „.
Hab. in India occidentali.

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[1]) Exemplaria originalia in herbario Kewensi examinavimus.

98. Oe. obtruncatum Wittr. mscr.

Oe. tumidulum Hohen. Alg. mar. sicc. n:o 404, ex parte.

Oe. oogoniis 2—3 continuis vel singulis, ellipsoideis vel globoso-ellipsoideis, membrana crassa, operculo apicali deciduo apertis (oogoniis itaque superioribus oogoniorum continuorum etiam deciduis); oosporis oogonia plane complentibus;

crassit. cell. veget. 18—22 μ, altit. 3—5plo major;
„ oogon. 48—55 „ „ 56—64 μ;
„ oospor. 46—52 „ „ 52—57 „.

Hab. in India cis Gangem.

Hæc species de *Oe. acrosporo* De Bar. admonet.

99. Oe. Fonticola Al. Br.

in Kütz. Spec. Alg. p. 368 [1]).

Oe. oogoniis singulis (raro binis), globoso-oboviformibus, poro superiore apertis; cellulis vegetativis in superiore parte fili longioribus quam in inferiore;

crassit. cell. veget. 16—26 μ, altit. par ad 2plo (raro 3plo) major;
„ oogon. 36—40 „ „ 44—55 μ.

Hab. in Germania.

100. Oe. Montagnei Fior. Mazz.

De nov. microph. p. 259, t. 1, f. 1, 2, 5; Erb. critt. Ital. n:o 582.

Oe. oogoniis singulis, rarius binis, globoso-oboviformibus, operculo apertis, rima foecundationis angustissima; oosporis oogonia complentibus;

crassit. cell. veget. 20—30 μ, altit. par ad 4plo major;
„ oogon. 47—52 „ „ 53—57 μ;
„ oospor. 43—46 „ „ 46—50 „.

Hab. in Italia.

Fortasse varietas major *Oe. rhodospori* (Welw.) Wittr.

101. Oe. plagiostomum Wittr. mscr.

Oe. spec. Wittr. Gotl. o. Öl. Sötv. Alg. p. 24, t. 1, f. 11.

Oe. oogoniis singulis, oboviformi-globosis, poro rimæformi, obliquo, superius sito apertis; oosporis subglobosis, oogonia fere complentibus, membrana crassa;

[1]) Specimina originalia in herbario cel. Professoris Al. Braun vidimus.

crassit. cell. veget. 25—27 μ, altit. 3—4plo major;
„ oogon. 45—47 „ „ 53—60 μ;
„ oospor. 43—45 „ „ 47—49 „;
„ membr. oospor. 4—4½ „.
Hab. in Suecia, Dania.

102. Oe. Hutchinsiæ Wittr. mscr.

Oe. oogoniis singulis, subdepresso- ad suboviformi-globosis, poro superiore apertis; oosporis oogonia complentibus, episporio punctato-verruculoso; cellulis suffultoriis tumidis;

crassit. cell. veget. 30—35 μ, altit. 4—6plo major;
„ oogon. 62—75 „ „ 65—95 μ;
„ oospor. 60—73 „ „ 55—72 „;
„ cell. suffult. 40—50 „ „ 2—4plo major.
Hab. in Hibernia.

103. Oe. princeps (Hass.) Wittr.

Vesiculifera princeps Hass. Obs. on. Freshw. Conf. p. 388, ex p. [1]). *V. capillaris* Hass. Hist. o. Brit. Freshw. Alg. p. 195, t. 50, f. 1 et 2. *Oe.* spec. Welw. Phycot. Lusit. n:o 109.

Oe. oogoniis singulis, suboboviformi-globosis, poro superiore apertis; oosporis globosis, oogonia non plane complentibus;

crassit. cell. veget. 37—45 μ, altit. 1¼—2¼plo major;
„ oogon. 61—75 „ „ 68—80 μ;
„ oospor. 58—66 „ „ 60—65 „.
Hab. in Britannia, Lusitania.

104. Oe. giganteum Kütz.

Phyc. Germ. p. 200; Tab. Phyc. III, t. 37, f. 2. ?*Vesiculifera lacustris* Hass. Hist. of Brit. Freshw. Alg. p. 198, t. 52, f. 1.

Oe. oogoniis singulis, paullum tumidis, cylindrico-oboviformibus, poro superiore apertis; oosporis cylindrico-ellipsoideis, oogonia fere complentibus (interdum lageniformibus, brevicollibus et tunc oogonia plane explentibus); episporio subtiliter scrobiculato; cellulis suffultoriis subtumidis;

crassit. cell. veget. 30—42 μ, altit. 2—4½plo major;
„ „ suffult. 54—60 „ „ 1½—1¾ „ „ ;

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[1]) Vidimus specimina originalia Hassalliana e herbario cel. Professoris J. E. Areschoug.

crassit. oogon. 57—69 μ, altit. 78—106 μ;
„ oospor. 54—65 „ „ 75—103 „.
Hab. in Suecia, Dania, Germania.
Oedogonio capilliformi Kütz. proximum.

105. Oe. crassum (Hass.) Wittr.

Gotl. o. Öl. Sötv. Alg. p. 20, t. 1, f. 4—6. (?) *Vesiculifera crassa* Hass. Obs. on Freshw. Conf. p. 389.

Oe. oogoniis singulis (raro binis), oboviformi-ellipsoideis, paullum tumidis, poro superiore apertis; oosporis ellipsoideis, oogonia non complentibus;
crassit. cell. veget. 33—55 μ, altit. 2—5plo major;
„ oogon. 65—70 „ „ 100—125 μ;
„ oospor. 60—66 „ „ 80—110 „.
Hab. in Suecia, Germania.

GENUS II. BULBOCHÆTE Ag.

Syn. Alg. Scand. p. XXIX; Pringsh. Beitr. z. Morph. d. Alg. I, p. 71; char. amplif.

SECTIO I.

Species oogoniis globosis vel subglobosis, patentibus [1]); oosporis eadem forma ac oogoniis eaque complentibus; cellula suffultoria superiore in sectione optica verticali pentagona [1]), inferiore tetragona.

Species omnes (adhuc cognitæ) hujus sectionis dioicæ nannandræ et (Bulb. polyandra Clev. excepta) gynandrosporæ sunt.

A. Species nannandribus unicellularibus.

1. B. elachistandra Wittr. mscr.

B. intermedia Pringsh. Beitr. z. Morph. d. Alg. I, p. 56, t. 4, f. 15—23; *non* De Bar.

B. oogoniis globosis, sub androsporangiis sitis; dissepimento cellularum suffultoriarum fere mediano; episporio lævi; androsporangiis epigynis

[1]) Apud *B. Brébissonii* Kütz. solam oogonia semper sunt erecta, apud *B. elatiorem* Pringsh. organa hæc vel erecta vel patentia sunt. Oogoniis erectis utraque cellula suffultoria est tetragona.

vel sparsis; nannandribus minimis, oboviformibus inferne acuminatis, rectis, in oogoniis sedentibus;

crassit. cell. veget. 20—24 μ, altit. $1^1/_2$—$2^1/_4$plo major;
„ „ androsp. 14—16 „ „ 10—11 μ;
„ oogon. 46 „ „ 44 „;
„ nannandr. 9 „ „ 19 „.

Hab. in Germania (sec. cel. Pringsheim).

B. Species nannandribus bicellularibus, spermogonio interiore.

α. Species cellula basali (stipite) nannandrium breviore quam spermogonio.

2. B. intermedia De Bar.

Ueb. Oedog. u. Bolb. p. 72 et sequ. t. 4, f. 1—7; *non* Pringsh.

B. oogoniis subdepresso-globosis, sub androsporangiis sitis; dissepimento cellularum suffultoriarum fere mediano; episporio subtilissime crenulato (raro lævi); androsporangiis 1—2cellularibus, epigynis, raro sparsis; nannandribus in oogoniis sedentibus, stipite paullum curvato;

crassit. cell. veget. 17—19 μ, altit. $1^1/_2$—3plo major;
„ „ androsp. 13 „ „ 10 μ;
„ oogon. 40—48 „ „ 31—40 „;
„ nannandr. 9—10 „ „ 24—26 „.

Hab. in Suecia, Norvegia, Germania, Britannia, Hibernia, Groenlandia.

β. depressa Wittr. mscr.

Var. oogoniis depresso-globosis; dissepimento cellularum suffultoriarum paullum supra medium posito; episporio lævi crassoque; androsporangiis epigynis;

crassit. cell. veget. 14—18 μ, altit. $2^1/_2$—$4^1/_2$plo major;
„ „ androsp. 11—13 „ „ 10—11 μ;
„ oogon. 42—46 „ „ 30—34 „;
„ nannandr. 9—11 „ „ 22—25 „. Tab. I, fig. 18.

Hab. in Suecia, Norvegia.

3. B. Nordstedtii Wittr. mscr.

B. oogoniis parvis, subquadrangulari- vel subobcordiformi-globosis, sub androsporangiis sitis; dissepimento cellularum suffultoriarum superiore (raro

submediano); episporio subtilissime crenulato vel sublævi; androsporangiis 1-cellularibus, epigynis; nannandribus in oogoniis sedentibus, stipite paullulum curvato;

crassit. cell. veget. 15—17 μ, altit. 2—4plo major;
„ „ androsp. 10—11 „ „ 11 μ;
„ oogon. 36—42 „ „ 29—35 „;
„ nannandr. 9—10 „ „ 23—25 „.

Hab. in Suecia.

4. B. quadrata Wittr.

Gotl. o. Öl. Sötv. Alg. p. 19, t. 1, f. 3.

B. oogoniis biconico-globosis, apicibus truncatis (in sectione optica verticali fere quadratis), sub androsporangiis sitis; dissepimento cellularum suffultoriarum paullum infra medium posito vel rarius submediano; episporio subtilissime crenulato; androsporangiis 2cellularibus, epigynis vel raro sparsis; nannandribus in oogoniis sedentibus, stipite curvato;

crassit. cell. veget. 20—25 μ, altit. $1^1/_2$—2plo major;
„ „ androsp. 15—16 „ „ 11—12 μ;
„ oogon. 40—50 „ „ 40—45 „;
„ nannandr. 11—12 „ „ 30—32 „.

Hab. in Suecia.

5. B. angulosa Wittr. et Lund. mscr.

B. oogoniis biconico-globosis, apicibus truncatis, lateribus coni superioris paullulum retusis, (in sectione optica verticali subquadratis), sub setis terminalibus, rarius sub androsporangiis sitis; dissepimento cellularum suffultoriarum paullum supra medium posito; episporio lævi; androsporangiis 2—3cellularibus, sparsis vel epigynis; nannandribus in oogoniis sedentibus, stipite paullulum curvato;

crassit. cell. veget. 13—18 μ, altit. $1^1/_2$—$2^1/_2$plo major;
„ „ androsp. 10 „ „ 9 μ;
„ oogon. 36—41 „ „ 33—38 „;
„ nannandr. 8—9 „ „ 18—21 „.

Hab. in Suecia.

6. B. crenulata Pringsh.

Beitr. z. Morph. d. Alg. I, p. 72, t. 6, f. 4.

B. oogonis depresso-globosis, sub setis terminalibus, rarius sub androsporangiis vel sub cellulis vegetativis sitis; dissepimento cellularum suffultoriarum mediano vel paullum infra medium posito; episporio distincte

crenulato; androsporangiis sparsis, rarius epigynis; nannandribus in oogoniis vel prope ea sedentibus, stipite subrecto;

crassit. cell. veget 16—19 μ, altit. 2—3½plo major;
„ „ androsp. 12 „ „ 10 μ;
„ oogon. 45—48 „ „ 35—38 „;
„ nannandr. 9 „ „ 26 „.

Hab. in Suecia, Norvegia, Germania (sec. cel. Pringsheim), Gallia.

7. B. polyandra Clev.

in Wittr. Dispos. Oed. Suec. p. 140.

B. idioandrospora; oogoniis subdepresso-globosis, sub setis terminalibus vel sub cellulis vegetativis sitis; dissepimento cellularum suffultoriarum superiore vel rarius submediano; episporio subtilissime crenulato vel sublævi; androsporangiis 4—10cellularibus; nannandribus in oogoniis sedentibus, stipite paullum curvato;

crassit. cell. veget. 15—20 μ, altit. 3—5plo major;
„ oogon. 35—46 „ „ 32—38 μ;
„ cell. androsp. 13—14 „ „ 12—15 „;
„ nannandr. 8—9 „ „ 23 „. Tab. I, fig. 19, 20.

Hab. in Suecia, Norvegia, Germania, Gallia, Britannia, Hibernia, Groenlandia.

8. B. borealis Wittr

Dispos. Oed. Suec. p. 138.

B. oogoniis subdepresso-globosis, in medio profunde circumscissis (poro foecundationis in circumscissione?), sub cellulis vegetativis sitis; dissepimento cellularum suffultoriarum superiore; episporio lævi; androsporangiis 2—3cellularibus, subepigynis, patentibus: nannandribus in oogoniis sedentibus, stipite paullum curvato;

crassit. cell. veget. 18—21 μ, altit. 1¼—2plo major;
„ „ androsp. 12—13 „ „ 8—9 μ;
„ oogon. 40—48 „ „ 35—40 „;
„ nannandr. 9—10 „ „ 18—21 „.

Hab. in Suecia, Norvegia.

9. B. Brébissonii Kütz.

Tab. Phyc. IV, p. 19 t. 86, f. B et. f. Rabenh. Alg. Eur. n:o 1055. — *B. tumida* Wittr. Dispos. Oedog. Suec. p. 139.

B. oogoniis obcordiformi-globosis, infra truncatis, erectis, sub setis terminalibus vel sub androsporangiis sitis; dissepimento cellularum suffultoriarum

infimo; episporio subtiliter crenulato; androsporangiis 2—3cellularibus, sparsis vel epigynis; nannandribus in oogoniis, raro prope ea sedentibus, stipite recto vel subrecto;

crassit. cell. veget. 17—20 μ, altit. 3—4½plo major;
„ „ androsp. 15 „ „ 15—18 μ;
„ oogon. 42—50 „ „ 37—45 „;
„ nannandr. 10—12 „ „ 28—33 „.

Hab. in Suecia, Norvegia, Hollandia, Gallia, Hibernia.

10. B. sessilis Wittr.

Gotl. o. Öl. Sötv. Alg. p. 18, t. 1, f. 2.

B. oogoniis subquadrangulari-globosis, sub setis terminalibus (raro sub cellulis vegetativis) sitis; dissepimento cellularum suffultoriarum superiore; episporio crenulato; androsporangiis sparsis, pluricellularibus; nannandribus in oogoniis sedentibus, stipite subrecto;

crassit. cell. veget. 19—21 μ, altit. 2—2½plo major;
„ „ androsp. 11—12 „ „ 10—12 „;
„ oogon. 53—56 „ „ 47—50 μ;
„ nannandr. 9—11 „ „ 26—27 „.

Hab. in Suecia, Gallia.

11. B. setigera (Roth) Ag.

Syn. Alg. Scand. p. 71; Pringsh. Beitr. z. Morph. d. Alg. I, p. 72, t. 6, f. 3; Rabenh. Alg. Sachs. n:o 158, Alg. Eur. n:o 1670. *Conferva setigera* Roth Catal. III, p. 283, t. 8, f. 1.

B. oogoniis depresse subquadrangulari-globosis, sub setis terminalibus vel sub androsporangiis sitis; membrana oogoniorum post foecundationem incrassata; dissepimento cellularum suffultoriarum paullum supra medium posito vel mediano; episporio granulato; androsporangiis sparsis vel epigynis, bicellularibus; nannandribus in oogoniis vel prope ea sedentibus, stipite recto;

crassit. cell. veget. 25—28 μ, altit. 2½—5plo major;
„ „ androsp. 18—20 „ „ 14—18 μ;
„ oogon. 75—80 „ „ 60—65 „;
„ nannandr. 12½—13½ „ „ 34—36 „.

Hab. in Suecia, Norvegia, Germania, Gallia, Hibernia.

b. Species cellula basali (stipite) nannandrium longiore quam spermogonio.

12. B. gigantea Pringsh.

Beitr. z. Morph. d. Alg. I, p. 71, t. 6, f. 1.

B. idioandrospora?; oogoniis subdepresse obcordiformi-globosis, sub setis terminalibus, raro sub cellulis vegetativis sitis; dissepimento cellularum suffultoriarum mediano vel paullum supra medium posito; episporio verrucoso; nannandribus paullo longioribus quam oogoniis, in his sedentibus, stipite duplo fere longiore quam spermogonio, arcuato;

crassit. cell. veget. 24—27 μ [1]), altit. 2—3plo major;
„ oogon. 62—66 „ „ 51—58 μ [1]);
„ stip. nannandr. 11—12 „ „ 40—45 „ ;
„ cell. spermog. 13—14 „ „ 20—22 „ .

Hab. in Suecia, Germania (sec. cel. Pringsheim), Gallia.

13. B. valida Wittr.

Gotl. o. Öl. Sötv. Alg. p. 17, t. 1, f. 1.

B. oogoniis depresso-globosis vel depresse obcordiformi-globosis, sub setis terminalibus sitis; dissepimento cellularum suffultoriarum fere mediano (sæpe paullum infra medium posito); episporio granulato; androsporangiis sparsis, 3cellularibus; nannandribus paullo brevioribus quam oogoniis, in his sedentibus, stipite paullulo longiore quam spermogonio, curvato;

crassit. cell. veget. 25—27 μ, altit. 2—3½plo major;
„ „ androsp. 18—19 „ „ 10—12 μ;
„ oogon. 59—66 „ „ 48—56 „;
„ stip. nannandr. 9½—10 „ „ 23—29 „;
„ cell. spermog. 11—11½ „ „ 21—22 „.

Hab. in Suecia.

C. Species nannandribus bicellularibus, spermogonio exteriore.

14. B. crassa Pringsh.

Beitr. z. Morph. d. Alg. I, p. 72, t. 6, f. 2.

B. oogoniis subdepresso-globosis, sub setis terminalibus (rarissime sub cellulis vegetativis) sitis; dissepimento cellularum suffultoriarum mediano vel paullum supra medium posito; episporio non verrucoso; androsporangiis

[1]) Mensuræ a cel. Pringsheim l. c. editæ paullo majores sunt.

sparsis, pluricellularibus; nannandribus paullo longioribus quam oogoniis, in his sedentibus, stipite duplo longiore quam spermogonio, arcuato;

crassit. cell. veget. 22—25½ μ, altit. 2—2½plo major;
„ „ androsp. 16 „ „ 13—16 μ;
„ oogon. 52—60 „ „ 42—51 „;
„ stip. nannandr. 10 „ „ 44 „;
„ cell. spermog. 8 „ „ 21 „.

Hab. in Germania (sec. cel. Pringsheim).

15. B. elatior Pringsh.

Beitr. z. Morph. d. Alg. I, p. 73, t. 6, f. 5. — Inter *Oe. Rothii* Zell. in Rabenh. Alg. Eur. n:o 2347.

B. oogoniis erectis vel patentibus, depresse obcordiformi-globosis, sub androsporangiis sitis; dissepimento cellularum suffultoriarum infimo; membrana oogoniorum post foecundationem incrassata; episporio lævi; androsporangiis epigynis (raro sparsis), 2cellularibus; nannandribus paullo brevioribus quam oogoniis, in cellulis suffultoriis (raro in cellulis vegetativis trivialibus vel in oogoniis) sedentibus, stipite vix duplo longiore quam spermogonio, recto;

crassit. cell. veget. 13—18 μ, altit. 2—3½plo major;
„ „ androsp. 10—13 „ „ 8—11 μ;
„ oogon. 34—44 „ „ 31—38 „;
„ stip. nannandr. 8—10 „ „ 18—24 „;
„ cell. spermog. 6½—8 „ „ 10—11 „.

Hab. in Suecia, Germania (sec. cel. Pringsheim), India trans Gangem.

SECTIO II.

Species oogoniis ellipsoideis vel subellipsoideis; oosporis eadem forma ac oogoniis eaque complentibus, episporio longitudinaliter costato, costis plus minusve crenulatis [1]); dissepimento cellularum suffultoriarum [2]) in suprema parte earum posito; cellula suffultoria superiore (in sectione optica verticali) tetragona, inferiore pentagona, rarius (oogoniorum erectorum) tetragona.

[1]) De sculptura episporii confer tab. 1, fig. 22 *oo'*, 23, 24.

[2]) Apud. *B. pygmæam* Pringsh. Wittr. dissepimentum in cellulis suffultoriis deesse videtur.

SUBSECTIO I. Species monoicæ.

16. **B. Monile** Wittr. et Lund. mscr.

B. oogoniis ellipsoideis, erectis (raro patentibus), sub cellulis vegetativis sitis; spermogoniis 2—?cellularibus, patentibus vel erectis, subepigynis vel sparsis; cellulis vegetativis subglobosis;

crassit. cell. veget. 13½—16 μ, altit. vix par;
„ oogon. 21½—23½ „ „ 30—33 μ;
„ cell. spermog. 8—10 „ „ 7—8 „.

Hab. in Suecia.

17. **B. nana** Wittr.

Oedog. nov. p. 7, t. 1, f. 9.

B. oogoniis ellipsoideis, patentibus, sub setis terminalibus vel sub cellulis vegetativis sitis; spermogoniis 1—2cellularibus, erectis, subepigynis vel sparsis;

crassit. cell. veget. 12—15 μ, altit. par—1⅓ major;
„ oogon. 20—25 „ „ 33—36 μ;
„ cell. spermog. 8—9 „ „ 6—9 „.

Hab. in Suecia, Austria, Groenlandia.

β subbasispora Wittr. mscr.

Var. oogoniis sub cellulis vegetativis, spermogoniiferis sitis; spermogoniis 2cellularibus, erectis (raro patentibus), subepigynis;

crassit. cell. veget. 13—15 μ, altit. par—1¼plo major;
„ oogon. 20—24 „ „ 32—35 μ;
„ cell. spermog. 7—8 „ „ 5—6 „.

Hab. in Norvegia.

18. **B. basispora** Wittr. et Lund. mscr.

B. oogoniis ellipsoideis, patentibus vel rarius erectis, sub cellulis vegetativis sitis, plerumque in ramis pluricellularibus basalibus; spermogoniis 1—?cellularibus, patentibus (raro erectis), subepigynis vel sparsis;

crassit. cell. veget. 16—19 μ, altit. par—1¼plo major;
„ oogon. 25—28 „ „ 42—45 μ;
„ cell. spermog. 9—11 „ „ 6—8 „.

Hab. in Suecia.

19. **B. mirabilis** Wittr.

Dispos. Oedog. Suec. p. 137, t. 1, f. 8, 9.

B. oogoniis ellipsoideis vel suboblongo-ellipsoideis, patentibus vel rarius erectis, sub setis terminalibus vel sub cellulis vegetativis

sitis; spermogoniis 2—4cellularibus, erectis (raro patentibus), subepigynis vel sparsis; (*Obs.* Divisio horizontalis cellularum vegetativarum non rara est.)

crassit. cell. veget. 16—20 μ, altit. $1^1/_4$—$1^2/_3$plo major;
„ oogon. 27—35 „ „ 46—56 μ;
„ cell. spermog. 10—12 „ „ 7—9 „.

Hab. in Suecia, Norvegia, Britannia, Hibernia, Lapponia rossica, Groenlandia.

β lapponica Wittr. et Lund. mscr.

Var minor, oogoniis patentibus (raro erectis); spermogoniis 1—2cellularibus, patentibus vel rarius erectis;

crassit. cell. veget. 15—19 μ, altit. $1^1/_4$—2plo major;
„ oogon. 25—27 „ „ 40—45 μ;
„ cell. spermog. 8—9 „ „ 6—7 „.

Hab. in Suecia.

γ immersa Wittr. mscr.

Var. minor, oogoniis sub cellulis vegetativis (raro sub setis terminalibus) sitis, sæpe in ramis pluricellularibus basalibus; spermogoniis 1—2cellularibus, erectis vel rarius patentibus, subepigynis vel rarius sparsis;

crassit. cell. veget. $13^1/_2$—15 μ, altit. $1^1/_4$—$1^3/_4$plo major;
„ oogon. 25—33 „ „ 40—48 μ;
„ cell. spermog. $7^1/_2$—9 „ „ $6^1/_2$—8 „.

Hab. in Norvegia.

20. **B. De Baryana** [1]) Wittr. et Lund. mscr.

B. oogoniis ellipsoideis, patentibus, sub cellulis vegetativis vel sub setis terminalibus sitis; spermogoniis patentibus (raro erectis), subepigynis vel sparsis; (*Obs.* Divisio horizontalis cellularum vegetativarum non rara est.)

crassit. cell. veget. 20—22 μ, altit. $1^1/_3$—$1^1/_2$plo major;
„ oogon. 28—32 „ „ 46—$49^1/_2$ μ;
„ cell. spermog. 11—12 „ „ $8^1/_2$—9 „.

Hab. in Suecia.

21. **B. megastoma** Wittr. et Lund. mscr.

B. oogoniis subanguloso-ellipsoideis, poro foecundationis magno, tubo foecundationis excedente, patentibus (raro erectis), sub setis terminalibus vel rarius sub cellulis vegetativis sitis; spermogoniis 1—2cellularibus, erectis

[1]) Cel. Professor A. De Bary speciem monoicam generis *Bulbochætes* primus observavit; vide Bot. Zeit. 1858, Bericht üb. d. Fortschr. d. Algenkunde p. 82.

(raro patentibus), sparsis vel subepigynis; (*Obs* Divisio horizontalis cellularum vegetativarum non rara est.)

crassit. cell. veget 19½—24 μ, altit. par—1½plo major;
„ oogon. 34—36 „ „ 48—54 μ;
„ cell. spermog. 12—13 „ „ 8—9 „. Tab. I, fig. 21.
Hab. in Suecia.

SUBSECTIO II. Species dioicæ, nannandræ, et (quantum scimus) gynandrosporæ; nannandribus rectis, stipitatis, spermogonio exteriore, 2—5cellulari.

A. Species cellulis suffultoriis dissepimento carentibus.

22. B. pygmæa Pringsh.; Wittr.

Dispos. Oedog. Suec. p. 141. *B. pygmæa b. minor* Pringsh. Beitr. z. Morph. d. Alg. I, p. 74, t. 6, f. 10.

B. oogoniis ellipsoideis, patentibus, sub setis terminalibus, vel sub cellulis vegetativis sitis, (in sectione longitudinali optica) subtetragonis; androsporangiis sparsis; nannandribus prope oogonia sedentibus; (Planta filo primario curvato, brevi.)

crassit. cell. veget. 12—15 μ, altit. tertia parte minor — par;
„ oogon. 23—25 „ „ 34—40 μ;
„ stip. nannandr. 11—12 „ „ 15—18 „;
„ cell. spermog. 7—7½ „ „ 7—7½ „.
Hab. in Suecia, Norvegia, Germania (sec. cel. Pringsheim), Hibernia.

B. Species cellulis suffultoriis dissepimento præditis.

23. B. subsimplex Wittr.

Dispos. Oedog. Suec. p. 142, *B. pygmæa a major* Pringsh. Beitr. z. Morph. d. Alg. I, p. 74, t. 6, f. 11.

B. oogoniis ellipsoideis patentibus, sub androsporangiis vel sub setis terminalibns sitis; androsporangiis epigynis; nannandribus in oogoniis vel prope hæc sedentibus; (Planta erecta, ramis evolutis paucissimis.)

crassit. cell. veget. 15—16 μ, altit. par —1½plo major;
„ oogon. 26—28 „ „ 39—42 μ;
„ cell. androsp. 11—12 „ „ 12—16 „;
„ stip. nannandr. 10½ „ „ 15 „;
„ cell. spermog. 7½ „ „ 7 „.
Hab. in Suecia, Germania (sec. cel. Pringsheim).

24. B. rhadinospora Wittr. mscr.

B. oogoniis oblongo-ellipsoideis, patentibus, sub androsporangiis vel sub setis terminalibus sitis; androsporangiis epigynis; nannandribus prope oogonia vel in iis sedentibus;

crassit. cell. veget. 15—21 μ, altit. $1^1/_2$—2plo major;
„ oogon. 29—35 „ „ 64—80 μ;
„ cell. androsp. 13—15 „ „ 16—23 „;
„ stip. nannandr. 16 „ „ 23—25 „;
„ cell. spermog. 8 „ „ 7—8 „.

Hab. in Suecia, Tasmania.

25. B. varians Wittr.

Dispos. Oed. Suec. p. 143.

B. oogoniis ellipsoideis, erectis vel patentibus, sub setis terminalibus vel sub androsporangiis sitis; androsporangiis sparsis vel epigynis; nannandribus prope oogonia (rarissime in iis) sedentibus;

crassit. cell. veget. 18—22 μ, altit. $1^1/_4$—$^1/_2$plo major;
„ oogon. 30—34 „ „ 46—54 μ;
„ cell. androsp. 16 „ „ 18 „;
„ stip. nannandr. 15—16 „ „ 26—27 „;
„ cell. spermog. 8—9 „ „ 6—7 „.

Hab. in Suecia, Norvegia, Dania.

β alpina Wittr. et Lund. mscr.

Var. minor, oogoniis plerumque erectis, sub setis terminalibus vel sub androsporangiis (rarissime sub cellulis vegetativis) sitis;

crassit. cell. veget. 14—17 μ, altit. $1^1/_4$—$1^1/_2$plo major;
„ oogon. 27—28 „ „ 41—46 μ;
„ cell. androsp. 10—11 „ „ 14—15 „;
„ stip. nannandr. 12 „ „ 20 „;
„ cell. spermog. 7—8 „ „ 5 „.

Hab. in Suecia.

26. B. lagoensis Wittr. mscr.

B. oogoniis ellipsoideis, erectis, sub cellulis vegetativis sitis (sæpe ramos pluricellulares ferentibus); cellulis suffultoriis paullo brevioribus quam cellulis vegetativis trivialibus; nannandribus prope oogonia sedentibus;

crassit. cell. veget. 21—22 μ, altit. $1^1/_4$—$1^1/_2$plo major;
„ „ suffult. 20—23 „ „ 24—28 μ;
„ oogon. 39—41 „ „ 61—63 „;
„ stip. nannandr. 16—$16^1/_2$ „ „ 24 „;
„ cell. spermog. $9^1/_2$—10 „ „ 7—$7^1/_2$ „.
Hab. in Brasilia.

27. B. minor Al. Br.

in Kütz. Spec. Alg. p. 422; Pringsh. Beitr. z. Morph. d. Alg. I, p. 74, t. 6, f. 8.

B. oogoniis suboblongo-ellipsoideis, erectis, (raro patentibus), sub setis terminalibus vel sub androsporangiis vel sub cellulis vegetativis sitis; androsporangiis epigynis vel sparsis; nannandribus prope oogonia sedentibus;
crassit. cell. veget. 20—25 μ, altit. $1^1/_2$—2plo major;
„ oogon. 33—42 „ „ 60—69 „;
„ cell. androsp. 15—16 „ „ 17—21 μ;
„ stip. nannandr. 13 „ „ 22 „;
„ cell. spermog. 6 „ „ 7 „.
Hab. in Suecia, Germania.

28. B. denticulata Wittr. mscr.

B. oogoniis brevi-ellipsoideis, patentibus, sub setis terminalibus (rarissime sub cellulis vegetativis), sitis; episporio manifestissime denticulato; nannandribus in oogoniis vel prope ea sedentibus; (Divisio horizontalis cellularum vegetativarum non rara est.)
crassit. cell. veget. 27—30 μ, altit. $1^1/_4$—$1^1/_2$plo major;
„ oogon. 50—54 „ „ 69—75 μ;
„ stip. nannandr. 20—21 „ „ 30—31 „;
„ cell. spermog. 13—14 „ „ 8—9 „.
Hab. in Norvegia.

29. B. imperialis Wittr. mscr.

B. oogoniis brevi-ellipsoideis, erectis, sub setis terminalibus sitis; androsporangiis sparsis; nannandribus in oogoniis vel prope ea sedentibus;
crassit. cell. veget. 27—30 μ, altit. $2^1/_4$—$3^1/_2$plo major;
„ oogon. 81—83 „ „ 104—106 μ;
„ cell. androsp. 18 „ „ 28 „;
„ stip. nannandr. 21 „ „ 33 „;
„ cell. spermog. 13—16 „ „ 9 „.
Hab. in Brasilia.

β regalis Wittr. mscr.

Var. minor, cellulis vegetativis longioribus;
crassit. cell. veget. 24—26 μ, altit. 3—4½plo major;
„ oogon. 69 „ „ 90 μ.
Hab. in Brasilia.

30. B. repanda Wittr. mscr.

B. oogoniis suboblongo-ellipsoideis, patentibus vel rarius erectis, sub cellulis vegetativis vel sub setis terminaliabus vel sub androsporangiis sitis; androsporangiis epigynis; nannandribus prope oogonia vel in iis sedentibus;
crassit. cell. veget. 12—15 μ, altit. 2—3½plo major;
„ oogon. 26—30 „ „ 45—58 μ;
„ cell. androsp. 13—14 „ „ 16—21 „;
„ stip. nannandr. 11—14 „ „ 21—22 „;
„ cell. spermog. 7½—9 „ „ 5—6 „.
Hab. in Suecia, Groenlandia.

31. B. insignis Pringsh.

Beitr. z. Morph. d. Alg. I, p. 72, t. 6, f. 7. *B. speciosa* Wittr. Dispos. Oedog. Suec. 143. (?)*B. Pringsheimiana* Arch. Proc. Dubl. Micr. Cl. p. 38, t. 4.

B. oogoniis ellipsoideis, patentibus vel erectis, sub androsporangiis vel sub setis terminalibus sitis; episporio subtilissime transverse striato; androsporangiis epigynis vel rarius sparsis; nannandribus prope oogonia vel in iis sedentibus;
crassit. cell. veget. 20—25 μ, altit. 2½—4½plo major;
„ oogon. 46—60 „ „ 70—100 μ;
„ cell. androsp. 16—20 „ „ 20—25 „;
„ stip. nannandr. 17—19 „ „ 29—31 „;
„ cell. spermog, 10—11 „ „ 7½—8 „.
Hab. in Suecia, Germania.

32. B. rectangularis Wittr.

Dispos. Oed. Suec. p. 142.

B. oogoniis ellipsoideis, patentibus vel rarius erectis, sub setis terminalibus vel sub androsporangiis vel rarius sub cellulis vegetativis sitis; androsporangiis sparsis vel epigynis; cellulis vegetativis (in sectione longitudinali optica) subrectangularibus; (Divisio horizontalis cellularum vegetativarum sæpe occurrit.) ramis plantæ paucis, longissimis; nannandribus prope oogonia vel in iis sedentibus;

crassit. cell. veget. 19—23 μ, altit. $1^1/_4$—2plo major;
„ oogon. 33—39 „ „ 48—55 μ;
„ cell. androsp. 15—$16^1/_2$ „ „ 16—27 „;
„ stip. nannandr. 15—18 „ „ 22—27 „;
„ cell. spermog. 8—$9^1/_2$ „ „ $5^1/_2$—$6^1/_2$ „. Tab. I, fig. 22—24.
Hab. in Suecia, Norvegia, Hibernia.

β norvegica Wittr. mscr.

Var. minor, oogoniis erectis vel rarius patentibus; androsporangiis epigynis; nannandribus prope oogonia sedentibus;
crassit. cell. veget. 15—17 μ, altit. $1^1/_2$—2plo major;
„ oogon. 26—27 „ „ 43—45 μ;
„ cell. androsp. 13—14 „ „ 16—20 „;
„ stip. nannandr. 14 „ „ 24 „;
„ cell. spermog. 9 „.
Hab. in Norvegia.

γ tenuis Wittr. mscr.

Var. tenuior, oogoniis erectis vel patentibus; androsporangiis epigynis vel rarius sparsis; nannandribus prope oogonia sedentibus;
crassit. cell. veget. 13—16 μ, altit. 2—$2^1/_2$plo major;
„ oogon. 22—25 „ „ 42—45 μ;
„ cell. androsp. 12 „ „ 13 „;
„ stip. nannandr. 14 „ „ 18 „;
„ cell. spermog. 8 „.
Hab. in Suecia.

δ Lundellii Wittr. mscr.

Var. oogoniis erectis vel patentibus; androsporangiis epigynis (rarissime sparsis); cellulis vegetativis longioribus;
crassit. cell. veget. 14—16 μ, altit. 2—3plo major;
„ oogon. 31—36 „ „ 49—58 μ;
„ cell. androsp. 14—15 „ „ 16—20 „;
„ stip. nannandr. 15—16 „ „ 25—27 „;
„ cell. spermog. $8^1/_2$—10 „ „ 6—7 „.
Hab. in Suecia.

33. B. anomala Pringsh.

Beitr. z. Morph. d. Alg. I, p. 73, t. 6, f. 6.

B. oogoniis ellipsoideis, erectis, sub cellulis vegetativis sitis; nannandribus prope oogonia sedentibus; (Divisio horizontalis cellularum vegetativarum sæpe occurrit.)

crassit. cell. veget. 27 μ, altit. 2plo major;
„ oogon. 49—51 „ „ 75—80 μ;
„ stip. nannandr. 18 „ „ 37 „;
„ cell. spermog. 13 „ „ 13 „.
Hab. in Germania (sec. cel. Pringsheim).

Species, quarum organa fructificationis non satis nota nobis sunt.

34. B. gracilis Pringsh.

Beitr. z. Morph. d. Alg. I, p. 74, t. 6, f. 9.

B. monoica(?); oogoniis oblongo-ellipsoideis, patentibus vel rarius erectis, sub cellulis vegetativis sitis, in ramis pluricellularibus basalibus; cellulis suffultoriis sine dissepimento(?);
crassit. cell. veget. $13^1/_2$—14 μ, altit. $1^1/_4$—$1^1/_2$plo major;
„ oogon. 21—24 „ „ 49—54 μ.
Hab. in Suecia, Germania (sec. cel. Pringsheim).

35. B. brevifulta Wittr. mscr.

B. dioica nannandra?; oogoniis erectis, brevi-ellipsoideis, sub setis terminalibus sitis; cellulis suffultoriis brevibus tumidisque;
crassit. cell. veget. 19—24 μ, altit. $2^1/_4$—3plo major;
„ „ suffult. 22—26 „ „ 27—30 μ;
„ oogon. 38—42 „ „ 50—53 „.
Hab. in Brasilia.

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EXPLICATIO ABBREVIATIONUM.

Ag. Syn. Alg. Scand. = C. A. Agardh. Synopsis Algarum Scandinaviæ. Lundæ 1817.

Al. Br. Ueb. Chytrid. = Al. Braun. Ueber Chytridium, eine Gattung einzelliger Schmarotzergewächse auf Algen und Infusorien. (Abhandlungen der Königl. Akademie der Wissenschaften zu Berlin 1855. Berlin 1855.)

Arch. Proc. Dubl. Micr. Cl. = W. Archer in Proceedings of the Dublin Microscopical Club. Part I, vol. 1.

Aresch. Alg. Scand. exsicc. = J. E. Areschoug. Algæ Scandinavicæ exsiccatæ. Series nova. Upsaliæ 1861—1872.

Auersw. = Auerswald.

Berk. & Harv. = J. Berkeley et W. H. Harvey.

Bréb. = A. de Brébisson.

Clev. = P. T. Cleve.

Cram. in Hedw. = C. Cramer in Hedwigia, n:o 3. Dresden 1859.

De Bar. Fortschr. d. Algenk. = A. De Bary. Bericht über die Fortschritte der Algenkunde in den Jahren 1855, 1856 und 1857. (Betanische Zeitung. 16ter Jahrg. 1858. Halle 1858.)

„ „ Ueb. geschl. Zeug. bei Alg. = A. De Bary. Ueber geschlechtlichen Zeugungsprozess bei den Algen. (Berichte über die Verhandlungen der Gesellschaft für Beförderung der Naturwissenschaften zu Freiburg i. B. 1856. Freiburg i. B. 1856.)

„ „ Ueb. Oedog. u. Bolb. = A. De Bary. Ueber die Algengattungen Oedogonium und Bolbochæte. (Abhandlungen der Senkenbergischen Gesellschaft. Band I. Frankfurt a. M. 1854.)

Erb. critt. ital = Erbario crittogamico italiano.

Fior.-Mazz. De nov. microph = Elisabeth Fiorini-Mazzanti. De novis microphyceis. (Atti dell' Accademia Pontificia de' nuovi Lincei. Anno XIII. Roma 1860.)

Hass. Descr. on Freshw. Conf. = A. H. Hassall. Description of British Freshwater Confervæ. (The Annals and Magazine of Natural History. Vol. XI. London 1843.)

„ Hist. of Brit. Freshw. Alg. = A. H. Hassall. A. History of the British Freshwater Algæ. London 1845.

„ Obs. on Freshw. Conf. = A. H. Hassall. Observations on a new Group, Genus and Subgenus of Freshwater Confervæ. (The Ann. and Mag. of Nat. Hist. Vol. X. London 1842.)

Hohen. Alg. mar. sicc. = R. F. Hohenacker. Algæ marinæ siccatæ. 9:te und 10:te Liefer. Kirchheim u. T. 1860, 1862.

Hook. Flor. Tasm. = J. D. Hooker. Flora Tasmaniæ Vol. II. London 1860. (The Botany of the Antarctic voyage of H. M. Disovery Ships Erebus and Terror in the years 1839—1843, by J. D. Hooker.)

Jur. Beitr. z. Morph. d. Oedog. = L. Juranyi. Beitrag zur Morphologie der Oedogonien. (Jahrbücher für wissenschaftliche Botanik. Band. 9. Heft 1. Leipzig 1873.)

Kütz. Dec. Alg. = F. T. Kützing. Algarum aquæ dulcis germanicarum Decas I—XVI. Halis Saxonum 1833—1836.

„ Phyc. gener. = F. T. Kützing. Phycologica generalis oder Anatomie, Fysiologie und Systemkunde der Tange. Leipzig 1843.

„ Phyc. germ. = F. T. Kützing. Phycologia germanica, d. i. Deutschlands Algen. in bündigen Beschreibungen. Nordhausen 1845.

„ Spec. Alg. = F. T. Kützing. Species Algarum. Lipsiæ 1849.

„ Tab. Phyc. = F. T. Kützing. Tabulæ phycologicæ oder Abbildungen der Tange. Band III, IV. Nordhausen 1853, 1854.

Le Cl. Sur. gen. Prolif. = L. Le Clerc. Sur la fructification du genre Prolifére de M. Vaucher. (Mémoires du museum d'histoire naturelle. Tome 3, Paris 1817.)

Link Epist. d. Alg. = H. F. Link. Epistola de Algis aquaticis in genera disponendis. (C. G. Nees ab Esenbeck. Horæ physicæ berolinenses. Bonnæ 1820.)

Lin. Spec. Plant. = Caroli Linnæi Species plantarum. Tom. II. Holmiæ 1753.

Lund. = P. M. Lundell.

Lyngb. Hydroph. Dan. = H. C. Lyngbye. Tentamen hydrophytologiæ danicæ. Hafniæ 1819.

Nordst. = C. F. O. Nordstedt.

Pringsh. Beitr. z. Morph. d. Alg I. = N. Pringsheim. Beiträge zur Morphologie und Systematik der Algen. I. (Jahrbücher für wissenschaftliche Botanik. Band. I. Berlin 1858.)

Rabenh. Alg. Eur. = L. Rabenhorst. Die Algen Europa's. Dresden 1861—1873.

„ Alg. Sachs. = L. Rabenhorst. Die Algen Sachsens. Dresden. 1850—1860.

Roth Catal. Bot. III. = A. G. Roth. Catalecta botanica. Fasc. III. Lipsiæ 1806.

Vaup. Bidr. t. Oedog. Morph. = C. Vaupell. Bidrag till Oedogoniernes Morphologie. (Oversigt over det Kongl. danske Videnskabernes Selskabs Forhandlinger 1861. Kjöbenhavn 1861.)

„ Iagt. öv. Oedog. = C. Vaupell. Iagttagelser over Befrugtningen hos en art af slægten Oedogonium. Kjöbenhavn 1859.

Welw. Crypt. Lusit. = F. Welwitsch. Cryptotheca Lusitanica.

„ Phycot. Lusit. = F. Welwitsch. Phycothæca Lusitanica.

Wittr. Dispos. Oedog. Suec. = V. B. Wittrock. Dispositio Oedogoniacearum suecicarum. (Öfversigt af Kgl. Vetenskaps-Akademiens Förhandlingar. 27:de årgången 1870. N:o 3. Stockholm 1870.)

„ Gotl. o. Öl. Sötv. Alg. = V. B. Wittrock. Om Gotlands och Ölands sötvattensalger. (Bihang till Kgl. Svenska Vetenskaps-Akademiens Handlingar. Band. I. N:o 1. Stockholm 1872.)

„ Oedog. nov. = V. B. Wittrock. Oedogoniaceæ novæ, in Suecia lectæ. (C. F. O. Nordstedt. Botaniska notiser 1872. Lund 1872.)

Zell. = G. Zeller.

EXPLICATIO FIGURARUM.

Obs. Omnes figuræ 300ies amplificatæ sunt.

Fig. 1. **Oedogonium fragile** Wittr. *oo* oogonium poro superiore apertum, foecundatum, oosporam includens. *sp* spermogonium bicellulare, hypogynum, cujus utraque cellula spermatozoidas binas etiamnunc includet. Foecundatione oogonii vicini jam peracta, individuum hoc proterogynum dici potest.

Fig. 2, 3. **Oe. nodulosum** Wittr. *o* oogonium non foecundatum, rima foecundationis nondum formata. *oo* oogonium operculo apertum, foecundatum, oosporam includens. *sp* spermogonium unicellulare, terminale, subepigynum, rima transversa apertum, alteram spermatozoidarum binarum etiamnunc includens. *sp'* spermogonium tricellulare, hypogynum, spermatozoidis emissis.

Fig. 4. **Oe. upsaliense** Wittr. *oo* oogonium oosporam includens. *sp* spermogonium bicellulare, subhypogynum.

Fig. 5—9. **Oe. platygynum** Wittr. *o* oogonium nondum apertum. *oo* oogonium infra medium circumscissum, poro in circumscissione sito apertum, foecundatum, oosporam includens. *n* nannaner unicellularis, operculo deciduo apertus, spermatozoidis binis emissis. 7, 8, 9 oogonia a vertice visa.

Fig. 10, 11. **Oe. Areschougii** Wittr. *o* oogonium nondum apertum. *oo* oogonium in medio circumscissum, poro in circumscissione sito apertum, foecundatum, oosporam includens. *a* androsporangium bicellulare, androsporis nondum formatis. *v* cellula vegetativa capitellata. *n* nannaner unicellularis, operculo apertus, spermatozoida altera emissa, altera etiamnunc inclusa. *n'* nannaner evacuatus.

Fig. 12—14. **Oe. flavescens** (Hass.) Wittr. *a* androsporangium, cujus cellula infima androsporam nondum emisit. *a'* androsporangium quadricellulare, evacuatum. *Obs.* Androsporangia hujus speciei in exemplaribus propriis posita sunt. *oo* oogonium oosporam includens, poro mediano apertum. *n* nannaner cellulæ suffultoriæ stipite unicellulari affixus; spermogonio exteriore, unicellulari, spermatozoidis jam emissis.

Fig. 15. **Oe. stellatum** Wittr. *oo* oogonium foecundatum, oosporam episporio echinato includens. *a* androsporangium bicellulare, androspora cellulæ superioris emissa, inferioris nondum formata. *n* nannaner cellulæ suffultoriæ stipite unicellulari affixus, spermogonio exteriore, bicellulari; cellula superior spermogonii est evacuata, inferior spermatozoidas binas etiamnunc includet.

Fig. 16, 17. **Oe. Pringsheimii** Cram. *sp* spermogonium quinquecellulare, cellulis superioribus evacuatis, inferioribus spermatozoidas binas includentibus. *oo* oogonium foecundatum, oosporam includens. *o* oogonium rima ac poro apertum, ad foecundationem paratum.

Fig. 18. **Bulbochæte intermedia** De Bar. *β depressa* Wittr. *oo* oogonium patens, foecundatum. *a* androsporangium epigynum, evacuatum. *st* seta terminalis. *ss* cellula suffultoria superior. *si* cellula suffultoria inferior. *n* nannaner bicellularís, spermogonio interiore, jam aperto, evacuato.

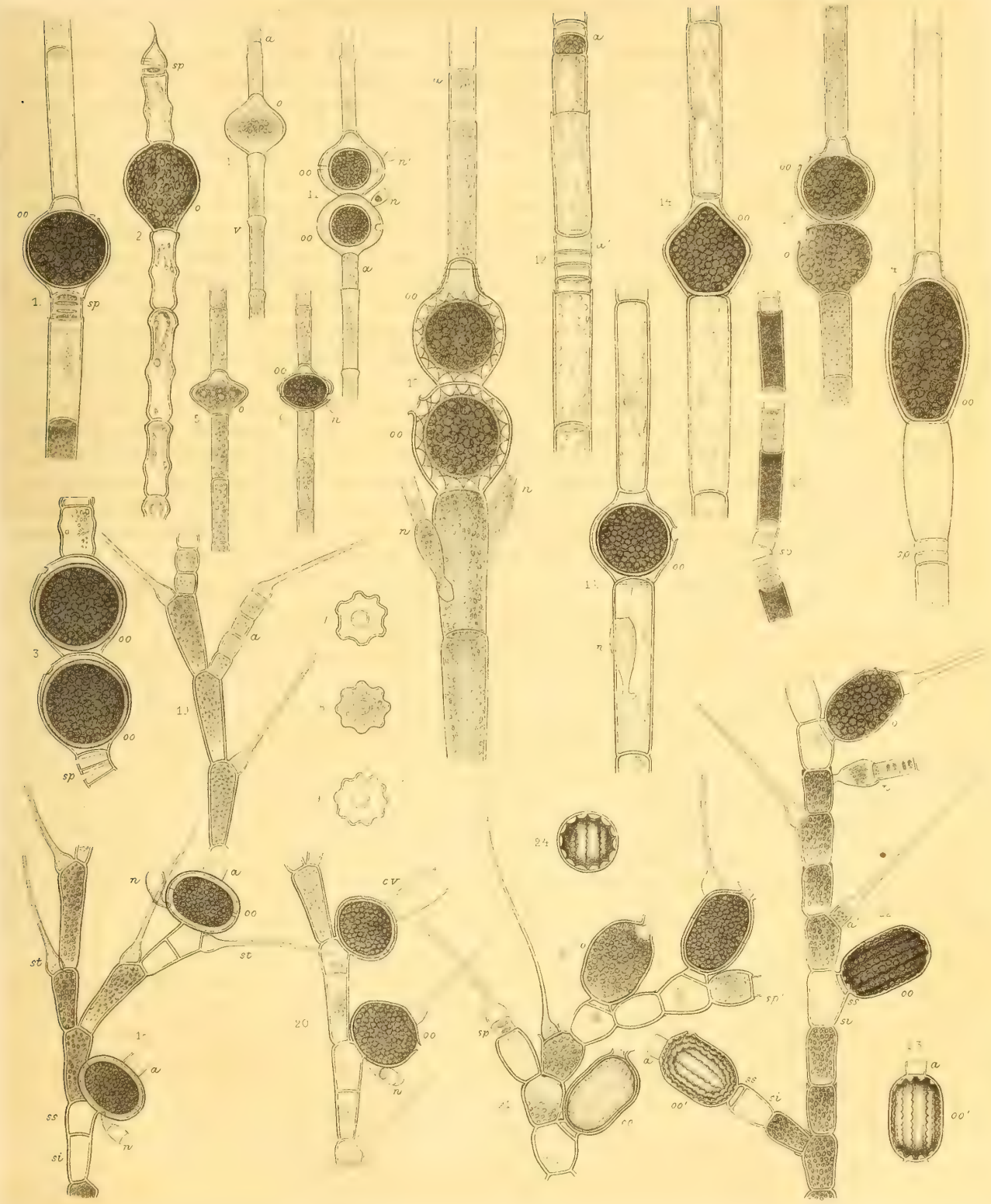
Fig. 19, 20. **B. polyandra** Clev. *oo* oogonium poro superiore apertum, foecundatum. *a* androsporangium quadricellulare, androspora cellulæ supremæ jam formata. *Obs.* Androsporangia hujus speciei in exemplaribus propriis posita sunt. *n* nannaner spermogonio interiore, operculo aperto; spermatozoida altera egressa, altera etiamnunc inclusa. *cv* cellula vegetativa, quæ zoosporam trivialem peperit.

Fig. 21. **B. megastoma** Wittr. & Lund. *o* oogonium poro supremo tuboque foecundationis apertum, nondum foecundatum. *oo* oogonium foecundatum, oosporam inanem includens, episporio longitudinaliter costato. *sp* spermogonium erectum, unicellulare, spermatozoidis binis inclusis. *sp'* spermogonium erectum, evacuatum.

Fig. 22—24. **B. rectangularis** Wittr. *o* oogonium patens, poro superiore apertum, nondum foecundatum. *oo* oogonium patens, foecundatum, oosporam includens. Fig. 22, *oo'* oogonium erectum, oosporam evacuatam includens, costis denticulatis episporii manifestis. Fig. 23, *oo*, oogonium idem circa axem longitudinalem 90° rotatum. Fig. 24 oogonium idem a vertice visum. *a* androsporangium epigynum, evacuatum. *a'* androsporangium unicellulare, androspora nondum emissa. *ss* cellula suffultoria superior. *si* cellula suffultoria inferior. *n* nannaner stipite unicellulari, spermogonio exteriore, spermatozoidis cellularum mediarum jam formatis.

ADDENDA ET CORRIGENDA.

Pag.	1,	lin.	3	*pro*	monographia	*lege* monographiam
»	4,	»	22	»	102	» 101
»	»	»	»	*post*	Norvegia 29,	*adde* Rossia 1,
»	»	»	»	*pro*	46	*lege* 47
»	»	»	23	»	5	» 9
»	»	»	»	»	22	» 21
«	5,	»	2	*post*	Bolivia 1,	*adde* Ægypto 1,
»	»	»	4	*pro*	141	*lege* 140
»	6.	»	21	*delendum est*	Hibernia	
»	7.	»	29	*post*	Germania,	*adde* Gallia, Hibernia,
»	8,	»	30	»	Gallia.	» Austria.
»	12,	»	8	»	singulis,	» raro binis,
»	15,	»	25	»	Dania.	» Austria.
»	36,	»	2	*pro*	Gemelliparum Pringsh.	*lege* gemelliparum (Pringsh.) Wittr.



Auctor del

Gen. Stab. Lit. Anst.

Nylén.

1 Oedogonium fragile. 2,3 Oe. nodulosum. 4 Oe. upsaliense. 5–9 Oe. platygynum. 10,11 Oe. Areschougii. 12–14 Oe. flavescens. 15 Oe. stellatum. 16,17 Oe. Pringsheimii. 18 Bulbochæte intermedia β. 19,20 B. polyandra. 21 B. megastoma. 22–24 B. rectangularis.

UNTERSUCHUNGEN

ÜBER

DIE MOLEKÜLARVOLUMINA

EINIGER REIHEN VON ISOMORPHEN SALZEN

VON

OTTO PETTERSSON.

(Überliefert der K. Societät der Wissenschaften zu Upsala d. 6 Dec. 1873.)

UPSALA 1873,
DRUCK DER AKADEMISCHEN BUCHDRUCKEREI.
ED. BERLING.

§. 1.

Nachdem die Chemische Forschung von der Beobachtung der Eigenschaften der Körper und der Reaktionen bei ihrer Einwirkung auf einander auf die Ermittlung der quantitativen Verhältnissen, die bei diesen Reaktionen eingehalten werden, und der Gewichtsmengen, welche jedesmal in Wechselwirkung treten, gekommen war, musste geraume Zeit vergehen, ehe man mit der Erforschung der Grundgesetzen dieses Gegenstandes so weit vorgerückt hatte, dass man die Aufmerksamkeit der zweiten inhærenten Eigenschaft der Körper, der Raumerfüllung derselben, zuwenden konnte. Denn Gesetzmässigkeiten in der letzteren Beziehung existiren eigentlich nur in Zusammenhang mit und bezogen auf die relativen Gewichtsmengen, die den chemischen Umsetzungen zu Grunde liegen d. h. die Aeqvivalente, und kommen sonst nicht zum Vorschein.

Die Verhältnisse, nach welchen die gasförmigen Körper, ihrem Volume nach, zu Verbindungeu zusammentreten, müssten natürlich ihrer Einfachheit wegen zuerst die Aufmerksamkeit der Forscher auf sich lenken. In der That treten diese Verhältnisse so auffallend hervor z. B. bei der Synthese des Wassers, die man wohl nicht ohne Grund als das Fundamentalexperiment der neueren Chemie und als den ersten unumstösslichen Beweis der Anschauung, welche LAVOISIER derselben zu Grunde gelegt hat, betrachten kann, und ferner bei der Elektrolyse desselben, dass sie noch einfacher erscheinen als die Gewichtsverhältnisse und sich von selbst dem Beobachter aufdringen müssen. Im Jahre 1815 bestätigt GAY-LUSSAC die Einfachheit der volumetrischen Verhältnisse bei seinen Untersuchungen über die Blausäure, das Cyan u. s. w. und beruft sich auf dieselbe, um die bezweifelte Richtigkeit der Analyse des Alkohols zu beweisen. Wie dieser Gegenstand nachher in der Theorie von BERZELIUS Berücksichtigung gefunden und in seiner späteren Entwicklung zu der Entdeckung des Gesetzes von AVOGADRO und dem wichtigsten Fortschritte der neueren Chemie, der Fest-

stellung der Molekülargrösse der gasförmigen Körper, geführt hat, ist zu gut bekannt, um weiter erwähnt zu werden.

Über die Dichtigkeits- und Volumverhältnisse der festen Körper hat KUFFER [1]) im Jahre 1824 eine Arbeit veröffentlicht, nachher hat DUMAS die Aufmerksamkeit auf die wichtige Thatsache gelenkt, dass das specifische Volum isomorpher Körper annähernd dasselbe ist; dann hat BOULLAY wie auch PERSOZ und BAUDRIEMONT verschiedene Ansichten über diesen Gegenstand aufgestellt, aber erst in den Jahren 1839—1844 ist derselbe in ausführlicher Weise durch die Arbeiten KOPPS [2]) und SCHRÖDERS [3]) untersucht worden, denn zu dieser Zeit hatte sich durch zahlreiche specifische Gewichtsbestimmungen (v. KARSTEN, KOPP u. A.) ein Material angehäuft, das jetzt von der Speculation in Anspruch genommen werden konnte.

Jede Speculation in volumtheoretischer Hinsicht fusst auf die experimentelle Ermittelung der Räume, die von denjenigen Gewichtsmengen der einfachen und zusammengesetzten Körper ausgefüllt werden, welche in chemischer Hinsicht entsprechend sind, als Grundlage. Als entsprechend muss man hier natürlich die relativen Grössen betrachten, welche die Gewichtsverhältnisse der kleinsten im freien Zustande existirenden Mengen der Körper ausdrücken (d. h. dieselben, welche die neuere Chemie Moleküle nennt), denn über den Raum, der innerhalb des Moleküls von jedem Einzelnen der konstituirenden Atomen eingenommen wird, geben natürlich die Experimente direkt keinen Aufschluss.

Das Molekülargewicht dividirt durch das Gewicht der Volumseinheit (das heisst das specifische Gewicht) eines Körpers ist das Molekülar-volum desselben. Das Molekülar-volum ist also eine Function von zwei Grössen: specifischem Gewicht und Molekülargewicht. Fehler in der Bestimmung eines von diesen beiden werden deshalb auf die Bestimmung des Molekülarvolums Einfluss ausüben. Das specifische Gewicht ist schwierig tadelfrei zu bestimmen, welches leicht daraus ersichtlich wird, dass die Angaben für einen und denselben Körper bei verschiedenen Beobachtern sehr abweichen. Auch darüber, welche Zahlen als die Molekülargewichte der chemischen Verbindungen zu betrachten sind, haben die Chemiker verschiedene Ansichten gehabt, und auch jetzt ist man nur bei den Körpern einig, deren Dampf-

[1]) Annales de Chem. et de Phys. XXV. p. 337.

[2]) Pogg. Ann. Bd. 37 s. 133. Bd. 52. s. 446 und besonders: Über das specif. Gew. d. Chem. Verbindungen, Frankfurt a. M. 1841 und: Bemerkungen zur Volumtheorie. Braunschweig 1844.

[3]) besonders: Die Molekülar-Volumina der Chem. Verbind. Mannheim 1843.

dichte bestimmt ist; für die Körper im festen Zustande können mehrere Annahmen zulässig sein. Fehlerhafte Messung des specifischen Gewichtes wird die Bestimmung der Molekülarvolumina falsch ausfallen lassen, und die Regelmässigkeiten, die zwischen denselben obwalten, gar nicht zum Vorschein bringen; fehlerhafte Anschauung über die Molekülargewichte der betreffenden Körper werden die besprochenen Regelmässigkeiten zwar nicht austilgen, aber doch das Gesetz, das ihnen zu Grunde liegt, verschleiert hervortreten lassen.

In diesem Theil vorliegenden Werkes werden wir die Resultate, die aus den experimentellen Untersuchungen hervorgegangen, in zwei verschiedenen Weisen darlegen.

Erstens werden wir von den Aequivalentzahlen ausgehend, die Formeln der Verbindungen so schreiben, dass das Gewicht eines zusammengesetzten Körpers durch die Summe der Aeqvivalentzahlen der eingehenden Bestandtheile ausgedrückt wird, also in der einfachst möglichen Weise. Mit der Ausnahme, dass wir keine Brüche der Aeqvivalentzahlen anwenden, also keine halbe Aeqvivalente u. s. w. in den Formeln eingehen lassen, schliesst diese Schreibweise keine vorausgefasste Ansicht ein, sondern wird von uns nur angewandt, um die Zusammensetzung der Körper so einfach wie möglich auszudrücken. Den Quotienten aus dieser Zahl und dem specifischen Gewicht nennen wir *Aeqvivalent-volum.*

Nachher wollen wir die Formeln der Körper so schreiben, wie es die moderne Chemie, auf mehrfachen Gründen gestützt, jetzt allgemein thut und nachsehen, ob die Quotienten der Division von diesen Zahlen mit den specifischen Gewichten, die *Molekülar-volumina*, in der Sprache der modernen Chemie, die unter ihnen herrschenden Regelmässigkeiten leichter ausdrücken und einfacher erklären lassen oder nicht.

Die Aeqvivalent- und Atomzahlen der einfachen Körper, die diesen Berechnungen zu Grunde gelegt sind, werden wir weiter unten anführen.

Ausserdem kommen in wissenschaftlichen Arbeiten über diesen Gegenstand noch einige andere Ausdrücke vor, wie specifisches Volum, Atomvolum und Atomzahl. *Specifisches Volum* wird oft in demselben Sinne wie Molekülarvolum und Atomvolum gebraucht, eigentlich bedeutet es aber: das Volum der Gewichtseinheit; was unter Atomvolum überall, wo es in diesem Aufsatz vorkommt, verstanden werden soll, muss näher besprochen werden. Zwar ist das Wort *Atomvolum* oft in viel mehr umfassender Bedeutung gebraucht worden, aber Folgendes scheint doch als Conseqvenz der Begriffsbestimmung von Molekül und Atom mit unabweisbarer Nothwendigkeit hervorzugehen. Da die neuere chemische Theorie einen bestimmten Unterschied

zwischen den Begriffen Atom und Molekül gemacht, muss diess auch bei den einfachen Körpern durchgeführt werden. So ist das Atomgewicht des Sauerstoffs 16, das Molekülargewicht 32. In einzelnen Fällen können diese Grössen zusammenfallen wie z. B. bei dem Quecksilber, wo das Atomgewicht gleich dem Molekülargewicht = 200 ist. Aber bei den meisten Elementen kennt man nicht sicher, wie gross das Molekülargewicht anzunehmen ist, weil das einzig völlig entscheidende Kriterium, die Bestimmung der Dampfdichte, hier fehlt.

Hier kann also vom Molekülarvolumen keine Rede sein. Dagegen ist man über die Grösse des Atomgewichtes eines Elementes nicht im Dunkeln. Die bekannte Atomgewichtszahl dividirt durch das für den Körper im freien Zustande gefundene specifische Gewicht ist es nun, was wir das *ursprüngliche Atomvolum* nennen wollen.

Die Volumtheorie, besonders die der festen Körper, mit denen wir in diesem Aufsatze allein zu thun haben, hat vom Anfang her auf die Lösung eines Problems hingearbeitet: aus den gekannten Volumina der Elemente im freien Zustande zu den Volumina zu schliessen, welche sie innerhalb der chemischen Verbindungen einnehmen. Bei der Vereinigung der Elemente zu zusammengesetzten Verbindungen tritt in der Regel Condensation ein, und nur bei den festen und flüssigen Körpern findet ausnahmsweise Ausdehnung statt, sonst immer Zusammenziehung, denn bei diesen Körpern geschieht es im Allgemeinen nicht, was bei den gasförmigen Körpern sehr gewöhnlich ist, dass das Volum der entstandenen Verbindung gleich der Summe der Volumina der Componenten ist.

Wenn dem so ist, so können die einzelnen Bestandtheile innerhalb des Moleküls eines zusammengesetzten Körpers nicht mit ihrem ursprünglichen Atomvolum enthalten sein, so ist z. B. für die meisten Oxyde und Salze der leichten Metalle das Volum des Oxydes oder des Salzes kleiner, als das ursprüngliche Atomvolum des Metalles. Angenommen nun, man wäre für irgend eine Reihe z. B. von Oxyden berechtigt anzunehmen, der Sauerstoff fülle in allen einen gewissen, bestimmten Raum aus, so würde man durch Subtraction desselben von dem Molekülarvolum der ganzen Verbindung das *kondensirte Atomvolum* des beziehenden Metalls als Rest erhalten.

SCHRÖDER hat zuerst die Aufmerksamkeit der Forscher darauf gelenkt, dass man sich die Bestandtheile einer chemischen Verbindung in einem anderen Zustand der Raumerfüllung darin enthalten denken muss als im isolirten Zustande und ihnen also ein anderes Atomvolum zuzuschreiben nöthig hat. Zugleich glaubte er nachweisen zu können, dass die Atomvolumina, womit Bestandtheile in Verbindungen eingehen, zu denen, welche sie im iso-

lirten Zustande haben, im einfachen Verhältnisse ständen. Er war auf diese Idee gekommen durch eine Beobachtung, die er gemacht hatte, dass wenn man von den Molekülarvolumina analoger Verbindungen einer Reihe von Salzen der schweren Metalle mit einer und derselben Säure die ursprünglichen Atomvolumina der entsprechenden Bestandtheile subtrahirt, für das Volum des gemeinsamen Bestandtheiles in vielen Fällen ein gleicher Rest bleibt. Diese Thatsache ist seither der Ausgangspunkt gewesen für die weitere Forschung in der Volumtheorie und als Ziel hat man sich die Erklärung der Beziehungen zwischen den ursprünglichen und den condensirten Atomvolumina und was damit im engsten Zusammenhang steht, die Erklärung der Dichtigkeit der Verbindungen aus den Dichten der Bestandtheile durch ein einfaches Gesetz gestellt. Die Entwicklung dieses Strebens ist zugleich die Geschichte der Volumtheorie der festen Körper. Da wir, wenigstens in diesem Abschnitt des vorligenden Arbeites, nicht auf das hypothetische Gebiet der condensirten Atomvolumina einzugehen beabsichtigen, sondern uns zu den Schlussfolgerungen beschränken wollen, welche ungezwungen aus den Experimenten hervorgehen, brauchen wir hier nicht diese Entwickelung zu verfolgen und weder auf die Ansichten Schröders, wonach die Condensation der Atomvolumina durch die Verhältnisse der ganzen Zahlen 1, 2, 3 u. s. w. zu einander auszudrücken wären, oder auf die Ansichten über die Relation von Dichte und Aeqvivalent von BÖDEKER, PLAYFAIR, NORDENSKIÖLD u. A. einzugehen, noch die eigenthümlichen Anschauungen von PLAYFAIR und JOULE[1]) zu besprechen, wonach die Volume der festen Körper in dem Verhältnisse zu einander stehen sollten, dass sie Multipla von dem sechszehnten Theil des Molekülarvolums des Eises wären: Die *Atomzahl* ist nichts als der umgekehrte Werth des Atomvolums.

Die vorliegende Arbeit ist eine Untersuchung über die specifischen Gewichte und die daraus berechneten Molekülarvolumina einiger Reihen von Salzen, besonders der Schwefelsäure und der Selensäure. Wir haben mit der Bestimmung des Molekülarvolumina der Alaunarten den Anfang gemacht, und durch die Resultate, die daraus hervorgegangen, Veranlassung gefunden, die Reihe der schwefelsauren und selensauren Salzen sowie auch die übrigen Reihen von hier angeführten Salzen, die damit in engerem oder entfernterem Zusammenhang stehen, näher auf ihr Molekularvolum zu untersuchen.

[1]) Später, wenn wir die Molekülarvolumina der wasserhaltigen Salzen besprechen, müssen wir die Resultate dieser Forscher vorübergehend erwähnen.

Wir werden die Experimente und Resultate in der Ordnung darstellen, wie sie angestellt worden und folglich erst die Alaunarten besprechen und dann zu den einfachen schwefel- und selensauren Salzen und den übrigen übergehen.

Die Untersuchungen sind zu Upsala in dem Laboratorium des Herrn Professor SVANBERG im Sommer und Herbste dieses Jahres ausgeführt worden.

§ 2.

Die Volumtheorie der festen Körper hat den Erwartungen, die sie anfangs erregt, nicht entsprochen. Die Regelmässigkeiten hinsichtlich der Atomvolumina der Bestandtheile und der Molekülarvolumina der daraus gebildeten Verbindungen oder die Beziehungen zwischen den Dichten eines Zusammengesetzen Körpers und der einzelnen Componenten desselben, worin SCHRÖDER zuerst und seitdem viele Andere durchgehende Gesetzmässigkeiten zu finden glaubten, haben sich nachher bei der genauen Kritik von KOPP [1]), LIEBIG [2]), FILHOL [3]) u. Mehreren nicht bewährt oder wenigstens die allgemeine Gültigkeit verloren, die die Erfinder derselben ihnen zuerkennen wollten, weil die aus den Theorien berechneten Resultate in vielen Fällen zu sehr von den experimentell gefundenen abweichen, oder vielmehr weil diese letzteren zu wenig übereinstimmend sind, dass man darauf eine Theorie, die etwas mehr als einen Schein von Wahrscheinlichkeit haben soll, darauf bauen könnte. Da das Molekülarvolum eine Function von dem Molekülargewicht und dem specifischen Gewicht ist, und das erstere seinem Zahlenwerthe nach gewöhnlich sehr scharf bestimmt ist, so beruht die Unsicherheit des Molekülarvolums beinahe ausschliesslich auf ungenauer Bestimmung des specifischen Gewichtes. In der That hat auch KOPP [4]) sehr schön gezeigt, dass man bei der grossen Menge von specifischen Gewichtsbestimmungen, die grossentheils sehr variiren, wenn man unter ihnen nach Belieben wählen darf, leicht Gründe für irgend eine Theorie ausfinden kann. KOPP hat diesen Mangel an Genauigkeit abzuhelfen versucht, zum Theil durch eigene sehr sorgfältige Messungen, zum Theil durch Prüfung der Angaben von anderen Beobachtern und dadurch, dass er aus den an-

[1]) Bemerkungen zur Volumtheorie 1844.

[2]) Jahresber. für 1847—1848. S. 41. I.

[3]) Etudes sur le Rapport qui existe entre le poids atomique, la force cristalline et la densite des corps. 1847.

[4]) Bemerkungen u. s. w. S. 88.

nehmbaren von diesen die Mittelzahlen berechnete. Auf dem allerdings sehr beschränkten Gebiet der vorliegenden Untersuchungen haben wir uns als Hauptaufgabe gestellt: möglichst zuverlässige Bestimmung der specifischen Gewichte. Wir haben zu diesem Zweck uns der Methode bedient, das specifische Gewicht durch die Gewichtsverlust zu bestimmen, welche die Substanzen bei der Wägung in Benzin erleiden, also denselben Weg genommen, welchen die meisten Beobachter vorher eingeschlagen, und nur in der Ausführung der Wägungen ein Vorsichtsmaas getroffen, das man, so viel wir wissen, vorher ausser Acht gelassen, wenigstens bei den Reihen von Salzen, die Gegenstand unsrer Untersuchung gewesen.

Wir wollen jetzt die Weise, worauf wir diese Messungen gemacht, eingehend besprechen, was um so mehr nöthig sein dürfte, da unsre Bestimmungen bisweilen bedeutend von den übrigens sehr sorgfältigen Bestimmungen mehreren Forscher wie Schiff, Topsöe u. A. abweichen, und diess einzig und allein in der Verschiedenheit des Verfahrens liegen muss.

Unsre Resultate machen keineswegs Anspruch auf ausserordentliche Genauigkeit, aber wir werden versuchen zu zeigen, dass die Fehlerquellen so weit wie möglich ausser Einfluss gestellt und die Versuchsdifferenzen innerhalb gewissen, auf der Methode beruhenden, Grenzen liegen, und dass die Bestimmungen, welche alle in ganz derselben Weise gemacht sind, mit einander vergleichbar sein müssen.

Ungenauigkeiteu der specifischen Gewichtsbestimmung können von zwei Ursachen herrühren: Unreinheit der Substanz und fehlerhafter Messung. Der Vorsichtsmaasregeln, die wir getroffen, um jede Substanz, so rein wie möglich darzustellen, werden wir bei der Beschreibung eines jeden Salzes erwähnen, der Beschreibung der Art und Weise, wie die Messungen geschehen, widmen wir den folgenden Paragraphen.

§ 3.

Die Substanzen wurden zur Bestimmung ihres specifischen Gewichtes in einem kleinen Glasgefäss (dem unteren Theil eines abgesprengten Proberöhrchens) gewogen, das mittelst eines Haares an den Wagebalken einer feinen anlytischen Wage gehängt werden konnte. Nach dem Tariren des Glasgefässes wurde das Gewicht der Substanz bestimmt, erst durch Wägung in der Luft, und zweitens nach dem das Röhrchen mit seinem Inhalt in Benzin gesenkt worden war. Es sei s das specifische Gewicht der Substanz, m das Gewicht derselben in der Luft, g die besonders ermittelte

Gewichtsabnahme des Glasgefässes in Benzin, μ die Abnahme des Gefässes und der Substanz zusammen in Benzin und δ das specifische Gewicht des letzteren; so ist

$$s = \frac{m}{\mu - g}\delta.$$

Die Mengen Substanz die zu jedem Versuch gebraucht worden, sind jedesmal in dem Folgenden angegeben. Die Grösse von m hat im allgemeinen von 1,5 bis 3 Gr. variirt. Einzelne Bestimmungen überschreiten diese Grenzen. Die Fehler in der Bestimmung von m, welche dadurch entstehen, dass die Wägungen in der Luft geschehen, sind nicht berüksichtigt worden.

g und δ sind durch zahlreiche Reihen von Versuchen bestimmt. Auf die Werthen dieser Qvantitäten hat die Temperatur einen sehr bemerkenswerthen Einfluss ausgeübt. Besprechen wir erst die Bestimmungen von δ. Das Benzin (des reinste der Schwedischen Pharmacopé) war vorher umdestillirt worden. Es war dann eine rein wasserhelle Flüssigkeit, die ohne sichtbaren Rest verdampfte, und dessen specifisches Gewicht mit der Temperatur wechselte. So war δ z. B. bei 16,8° C. 0,8839 und bei 21° C. = 0,8794. Nachdem die verschiedenen Werthen von δ für die dazwischen liegenden Temperaturen einmal durch Versuche bestimmt waren, wurde später täglich wenigstens eine Bestimmung von δ gemacht, um zu sehen; ob das Benzin mit der Zeit eine Veränderung seines specifischen Gewichtes erleide. Es hat sich dann gezeigt, dass diess nicht der Fall ist, und die jedesmal gefundenen Werthen von δ weichen von den zuerst gefundenen immer nur in der letzten Decimalstelle ab. Das specifische Gewicht des Benzins ist durch Ermittlung der Gewichtsabnahmen gefunden, welche eine hohle, zugeschmolzene Glaskugel erleidet, wenn sie einmal in Benzin und nachher in reinem destillirten Wasser von derselben Temperatur gewogen wird.

Auch die Gewichtsabnahme des Gefässes g ist durch eine Reihe vorläufiger Bestimmungen ermittelt, sie hat zwischen den besprochenen Temperaturgrenzen von 17°—21° von 1,3978 bis 1,3940 Gr. variirt. Da diese Variationen einen sehr störenden Einfluss auf die Resultate ausüben konnten, haben wir uns nicht begnügt mit der einmaligen Bestimmung der Gewichtsabnahme, die jeder Temperatur entspreche, sondern bei den Messungen, sobald sich die Temperatur erhöhte oder erniedrigte, neu den Werth von g bestimmt.

Es ist jetzt übrig, etwas über die Bestimmung von der Quantität μ mitzutheilen. μ ist die Gewichsabnahme des Gefässes und der Substanz zusammen, wenn sie in Benzin gewogen werden. Aber jede Substanz, mag

sie pulverförmig sein oder aus Krystallen bestehen, hält auf ihrer Oberfläche und in ihren Poren eine Quantität Luft kondensirt. Wird diese Luftmenge nicht vor der Wägung in der Flüssigkeit entfernt, so wird die Substanz ein grösseres Volum der Flüssigkeit verdrängen, als sie sonst thun sollte, und das specifische Gewicht wird zu niedrig ausfallen. Bei der specifischen Gewichtsbestimmung von Mineralien hat man diesen Übelstand zu verhüten gesucht dadurch, dass man die in dem Piknometer eingeschlossene Substanz, sammt mit der Flüssigkeit vorher bis zum Kochen der letzteren erhitzt, wobei die Luft von der Substanz entweicht. Salze und dergleichen, welche wegen Wasserverlust oder sonst aus irgend einem Grunde nicht das Kochen vertragen, rührt man gewöhnlich unter der Flüssigkeit mit einem Platindraht um, um die Luftblasen zu entfernen [1]).

Als wir zuerst nach dieser Methode arbeiteten und die specifischen Gewichte der Alaune zu bestimmen hatten, bei welchen die Bestimmungen auf die zweite Decimalstelle stimmen müssen, damit die daraus berechneten Molekülarvolumina nicht um mehr als eine ganze Einheit differiren sollen, zeigte es sich, trots aller Vorsicht und dem sorgfältigsten Umrühren, vollkommen unmöglich, die gewünschte Übereinstimmung zu erreichen. Besonders fielen die Resultate bedeutend ungleich aus, wenn man die Menge der Substanz variirte, oder je nachdem man ganze Krystalle oder gröbere und feinere Splitter von Krystallen oder die Substanz pulverförmig anwendete. So z. B. war alles was aus mehr als 20 Bestimmungen des Molekülarvolums des gewöhnlichen Alauns geschlossen werden konnte, dass dies irgendwo zwischen den Zahlen 271 bis 274 liegen musste.

Da dieses unzweifelhaft auf mangelhaftes Wegschaffen der an der Oberfläche der Substanz anhaftenden Luftschicht beruhen musste, haben wir diese Fehlerquelle bei den Bestimmungen folgendermassen zu vermeiden gesucht:

In das Gefäss, worin die abgewogene Substanz sich befand, wurde Benzin gefüllt, bis dass es zwei bis drei Linien über die Oberfläche derselben stand. Nachher wurde das Gefäss unter der Glocke eines ausgezeichneten Luftpumpes (von DELEUIL in Paris) aufgehängt und die Luft vorsichtig evacuirt, damit die aus der Substanz entweichenden Luftblasen nicht Umherspritzen verursachen sollten. Wenn die Verdünnung allmählig so weit gegangen, dass der Manometer nur 1 bis 1 $^1/_2$ Zoll Qwecksilberdruck in dem Recipienten [2]) zeigte, und keine Luftblasen mehr aus der Substanz

[1]) siehe Schrauf: Lehrbuch d. Phys. Mineralogie Bd. II. S. 13.

[2]) länger lässt sich die Verdünnung nicht treiben zufolge der Spannkraft des gesättigten Benzindampfes.

entwichen, wurde das Gefäss herausgenommen, an die Wage gehängt und, in einem geräumigen Gefäss mit Benzin schwebend, gewogen; dies jedoch erst nach halbstündigem Stehen in der Wage, weil durch das Abdampfen des Benzins während des Auspumpens der Luft die Temperatur beträchtlich gesunken war, und dies erst ausgeglichen werden musste, ehe das Resultat der Wägung konstant werden konnte.

Vollkommen wird der Fehler natürlich nicht durch diese Operation beseitigt, denn die entfernte Luftmenge ist nur gleich der Differenz zwischen der Quantität, welche das Salz bei gewöhnlichem Luftdruck kondensirt hält und derjenigen, welche es bei dem genannten kleinen Druck, von 1 Zoll Qwecksilberhöhe im Manometer, zurückzuhalten vermag. Wenn der Fehler dadurch nicht verschwindend klein wird, so ist es doch einleuchtend, dass er auf ein Minimum reducirt wird. Dies wird auch bestätigt durch die Resultate, welche in der Regel sehr übereinstimmend sind, und ganz wie man es erwarten sollte, das specifische Gewicht der Substanzen so gut wie ohne Ausnahme höher ausfallen lassen als die vorher angestellten Versuche.

Als Beispeil wollen wir für einige Alaune die von SCHIFF[1]) beobachteten specifischen Gewichte und Aeqvivalent-volumina derselben mit den von uns mit Hülfe der angegebenen Methode gefundenen vergleichend anführen:

	nach SCHIFF		nach unsren Bestimmungen.	
	Aeqvivalent-Volum.	Specifisches Gew.	Specif. Gew.	Aeqvivalent Vol.
$KOSO_3 + Al_2O_3 3SO_3 + 24H0$	275,6	1,722	1,755	270,4.
$NaOSO_3 + Al_2O_3 3SO_3 + 24H0$	279,2	1,641	1,693	270,8.
$NH_4OSO_3 + Al_2O_3 3SO_3 + 24H0$	279,6	1,621	1,647	275,2.
$KOSO_3 + Cr_2O_3 3SO_3 + 24H0$	271,4	1,845	1,841	271,3.
$NH_4OSO_3 + Cr_2O_3 3SO_3 + 24H0$	276,2	1,736	1,728	276,8.
$NH_4OSO_3 + Fe_2O_3 3SO_3 + 24H0$	281,4	1,712	1,723	279,6.

Die von uns angegebenen Werthe der beiden letzteren Reihen sind keine Mittelzahlen, sondern es ist nur den im Folgenden angeführten Messungen jedesmal die zuerst angegebene Bestimmung als Beispiel entnommen. Der Beweis, dass die von uns beobachteten Resultate als die richtigen zu betrachten sind, wird in dem Folgenden gegeben. Übrigens werden wir da auch zeigen, dass zwischen den Aeqvivalentvolumina der Alaune nach unserer

[1]) Ann. d. chem. u. Pharm. CVII S. 81. Die in der ersten Spalte aufgenommenen Zahlen sind doppelt so gross wie die in der Originalabhandlung, weil die von SCHIFF gebrauchte Aeqvivalentzahl nur halb so gross ist als die unsrige.

Bestimmung eine Gesetzmässigkeit herrscht, die den Beobachtungen von SCHIFF abgeht. Dass der Unterschied zwischen unsren Bestimmungen und den von SCHIFF angestellten einzig der Methode zuzuschreiben ist, hat sich sehr klar dadurch gezeigt, dass wir, wenn wir auf die gewöhnliche Weise die Messungen gemacht, Resultate erhalten haben, die allerdings sehr viel von einander abweichen, aber doch oft mit den von SCHIFF erhaltenen übereinstimmen. So haben wir in mehreren Bestimmungen das specifische Gewicht des schwefelsauren Kali-Thonerde-Alauns zu 1,738 gefunden, also sehr übereinstimmend mit der Bestimmung von SCHIFF u. s. w.

Die Fähigkeit der Substanzen, Luft auf sich zu kondensiren, ist sehr verschieden (wie es sich deutlich durch das Verhalten derselben unter dem Luftpump gezeigt hat), so hält gewöhnlicher Chrom-Kali- und Chrom-Ammoniakalaun sehr wenig Luft zurück, (deshalb sind die Bestimmungen SCHIFFS für diese Salze auch mit den richtigen übereinstimmend).

Wäre nicht dieser Umstand, würde man z. B. die sonst sehr sorgfältig gemachten Bestimmungen der Molekülarvolumina der selensauren Salze von TOPSOE [1]), welche durchgehend zu gross sind, nur als mit einem konstanten Fehler behaftet und übrigens mit einander vergleichbar betrachten können, was aber jetzt nicht der Fall ist.

Als ein anderes Beispiel wollen wir das wasserhaltige schwefelsaure Kupferoxyd anführen:

1) Pulverisirte Substanz 2,9688 gr. bei 20° C. (die Luft durch Auspumpen entfernt) gab specifisches Gew. = 2,292, Molekülarvolum = 108,7.

2) Grosse Krystalle 2,7570 gr. bei 19,4° C. (unter dem Luftpump behandelt) Specifisches Gew. = 2,286, Molekülarvolum 109.0.

3) Krystalle 3,3304 gr. bei 19,6° C. (die Luft nur durch Umrühren entfernt) specifisches Gew. = 2,266, Molekülarvolum = 110,0. [2])

Eine fernere Stütze für die Zuverlässigkeit der von uns angewandten Methode ist der Umstand, dass die Bestimmungen nach derselben Übereinstimmung zeigen, mag die Substanz als feines Pulver, als Krystallsplitter oder als ganze Krystalle angewandt worden sein.

Dies steht allerdings im Widerspruch mit den bisher gehegten Ansichten. SCHRAUF (Lehrbuch der Phys. Mineralogie II. Bd. S. 19.) äussert hierüber:

[1]) Krystallograf. Kemiske Undersøgelser over de Selensure-Salte. Kopenhagen 1870.

[2]) In der erwähnten Arbeit von Topsoe: Krystallografisk-Kemiske Undersögelser etc. wird S. 58 das Molekülarvolum des schwefelsauren Kupferoxydes (nach Kopp) zu 110,1 angegeben, also völlig übereinstimmend mit diesem Resultat.

"Um immer vergleichbare Resultate zu erzielen, muss auch das benutzte Material ein gleiches und zweckmässig ausgewähltes sein."

"Sieht man ab von den Variationen der Dichte, die hervorgebracht sind durch chemische Beimengungen, so wird man zur Bestimmung nur kleine Krystalle oder gröbliches Pulver verwenden dürfen."

"Allzu feines Pulver zu verwenden, ist nicht räthlich, denn nach G. Rose (Pogg. Ann. 73) steigt die Dichte mit der Feinheit des Pulvers. Er fand für die Dichte des Baryts in Krystallen 4,475, gepulvert 4,48, als chemischer Niederschlag 4,52."

"Der Grund dieser Erscheinung kann nur darin gesucht werden, dass ein fein vertheiltes Pulver auf das Wasser, analog wie dies für Gase bekannt ist, verdichtend einwirkt. Man wägt somit den Körper nicht in Wasser von normaler Dichte, sondern mit einer Hülle von specifisch schwererer Flüssigkeit und erhält somit einen geringeren Gewichtsverlust als denjenigen, welcher der Dichte wahrhaft entspricht."

Diese Anschauung ist unzweifelhaft irrig, denn erstens ist es schwierig, wenn man sich der aus der Physik bekannten Thatsache erinnert, dass die Flüssigkeiten ihr Volum und also ihr specifisches Gewicht bei gesteigertem Druck fast gar nicht ändern, sich die ungeheure Kraft vorzustellen, welche die Theilchen einer festen Substanz auf die umgebende Flüssigkeit ausüben müssten, um sie bemerkenswerth zu verdichten, und zweitens ist es leicht, sich zu überzeugen, dass Krystallsplitter, grobes oder feines Pulver ganz dasselbe specifische Gewicht geben, wenn man nur Sorge trägt, die anhaftende Luft genügend wegzuschaffen. Was SCHRAUF für die Folge einer Kondensation der Flüssigkeit hält, beruht also auf die verschiedene Fähigkeit der Substanzen, Luft auf sich zu kondensiren, je nach dem sie gröber oder feiner vertheilt sind. Der grösste Theil der folgenden Bestimmungen ist mit fein zerriebenen Substanzen gemacht, und wir haben bei dem Auspumpen der Luft bemerkt, dass kleine Krystalle oder Theile von Krystallen die Luft oft weit hartnäckiger zurückhalten als feines Pulver.

Unter vielen wollen wir nur folgendes Beispiel anführen.

1) Natürlicher Gyps (durchsichtig, und sehr rein) 2,1591 Gr. fein zerriebenes Pulver bei 18,2° C. (die Luft ausgepumpt) gab: specifisches Gewicht = 2,3228, Molekülarvolum = 74,0.

2) Gyps in Splittern 2,8649 Gr. bei 18° C. (die Luft ausgepumpt) gab specifisches Gew. = 2,3223, Molekülarvolum = 74,0.

3) Gyps gepulvert 2,2137 Gr. bei 19,4° C. (die Luft nur durch Umrühren entfernt) gab specifisches Gew. = 2,2745, Molekülarvolum = 75,6.

4) Gyps in Splittern 2,5120 gr. bei 18° C. (die Luft durch Umrühren weggeschafft) specifisches Gew. = 2,3086, Molekülar-Volum = 74,5.

Die specifischen Gewichtsbestimmungen sind alle bei Temperaturen von 17° bis 21° C. gemacht. Nur äusserst selten sind diese Grenzen überschritten worden. *Dies ist dieselbe Temperatur, wobei die Salze auskrystallisirt sind.* Da die Salze nicht denselben Ausdehnungskoefficient wie das Wasser haben, ist es einleuchtend, dass das specifische Gewicht eine Function der Temperatur, wobei die Messung geschah, sein muss. Über die Grösse des Fehlers, der durch ein Nichtbeachten dieses Umstands zwischen den angegebenen Grenzen 17°—21° C. begangen sein kann, können wir uns durch die Untersuchung Playfairs und Joules[1]) über die Ausdehnung von Salzen mit steigender Temperatur annähernder Weise eine Vorstellung machen. Wir entnehmen der angeführten Arbeit folgende Angaben für einige Salze, deren specifisches Gewicht im Folgenden bestimmt ist.

		Kubische Ausdehnung von 0° bis 100°.
	Schwefelsaures Kali	0,010697
	Schwefelsaures Ammoniumoxyd . . .	0,010934
Wasserhalt:	Schwefelsaures Kupferoxyd	0,009525.
	Kali-Chrom-Alaun	0,005242.
	Kali-Thonerde-Alaun	0,003682.

Der mittlere Ausdehnungskoefficient des Wassers für jeden Temperaturgrad bei gewöhnlicher Temperatur ist nach Desprez = 0,00017. Man sieht hieraus, dass allerdings die Salze sich mit der Temperatur weniger ausdehnen als das Wasser, dass aber der Unterschied so gering ist, dass eine Korrektion für einen Unterschied von etwa 4° C. verschwindend klein sein würde. Dagegen wollen wir nicht behaupten, dass die im Folgenden angeführten Bestimmungen streng genommen ohne weiteres mit Messungen, die sich auf eine sehr verschiedene Temperatur beziehen, (z. B. 0° C oder + 4° C.) vergleichbar wären.

Nach unserer Meinung muss bei jeder specifischen Gewichtsbestimmung, die Vertrauen verdienen soll, zugleich angegeben sein:

I. die Menge und Darstellungsweise der Substanz.

II. die Temperatur, wobei die Messung geschehen.

III. die Vorsichtsmassen, die man getroffen, um die anhängende Luft zu entfernen.

Zuletzt müssen wir noch einer Frage einige Aufmerksamkeit widmen.

Die in dem Folgenden angeführten specifischen Gewichtsbestimmungen beziehen sich auf Substanzen, deren Dichten sehr verschieden sind.

[1]) Qu. I. of Chem. Soc. I. 121.

Schwefelsaurer Natron-Thonerdealaun hat z. B. das specifische Gewicht von 1,692 und selensaures Thalliumoxyd 7,067. Bei so weit verschiedenen Substanzen erscheint es annehmbar, dass die Fehlergrenzen, innerhalb welchen die Bestimmungen variiren dürfen, nicht dieselben sein können.

In der That verhält es sich auch so.

Eine Differentiation der Gleichung:

$$s = \frac{m}{\mu - g}\delta$$

wird über diese Frage Aufklärung geben

Angenommen m, δ und g wären tadellos bestimmt, wollen wir nachsehen, welchen Einfluss eine Variation in dem Werthe von μ, etwa so gross, wie man Grund hat zu vermuthen, dass die Versuchsfehler sein können, auf den Werth von s ausübt

Durch logarithmische Differentiation der Gleichung erhalten wir

$$Log\, s = Log\, m + Log\, \delta - Log\,(\mu - g)$$

$$\frac{ds}{s} = -\frac{d\mu}{\mu - g}$$

$$ds = -\frac{s d\mu}{\mu - g}$$

Wir wollen diese Formel auf 4 von den in dem Folgenden angeführten Bestimmungen anwenden. Betrachten wir zuerst den schwefelsauren Kali-Thonerdealaun und zwar die erste Bestimmung davon. Hier ist $s = 1{,}755$. Der Verlust des Salzes in Benzin war $1{,}3087 = \mu - g$.

Der Wägungsfehler bei der Bestimmung von $(\mu - g)$ kann, obschon die Wägung in Benzin statt fand und einer Korrektion für die Temperatur (siehe oben) unterworfen werden musste, mit keinem Schein von Wahrscheinlichkeit 0,002 gr. übersteigen. Nehmen wie an, dies wäre eben die Grösse des Versuchsfehlers gewesen, also $d\mu = 0{,}0020$. Da das — Zeichen nur andeutet, dass die Variationen in ds und $d\mu$ in entgegengesetzter Richtung statt finden, wollen wir es bei der Berechnung ausser Acht lassen. Durch Einsetzung der Werthe von δ und $\mu - g$ in die angegebene Formel erhalten wir den numer: Werth v. $ds = 0{,}0026\ldots$

Also: angenommen, der grösste Wägungsfehler, der nach irgend einer Wahrscheinlichkeit bei der Bestimmung begangen sein kann (d. h. $(\mu - g)$ wäre um 0,002 Gr. zu hoch oder zu niedrig gefunden), hätte hier statt gefunden, so würde dadurch der Werth von s um 0,0026 zu niedrig oder zu hoch ausfallen.

Also: wären nur bei der Bestimmung Versuchsfehler begangen, so würde bei den verschiedenen Messungen (vorausgesetzt, dass sie mit ungefähr ähnlichen Quantitäten vorgenommen wären, was auch der Fall ist) die Resultate in der zweiten Decimalstelle völlig konstant sein und in der Dritten nicht mehr als höchstens um 3 Einheiten von einander abweichen.

Jetzt zeigt es sich aber, dass bei sämmtlichen Bestimmungen der Alaunarten, welche sehr viele sind, trotz aller Sorgfalt die Resultate nur bis auf die zweite Decimalstelle völlig übereinstimmen aber nicht in der Dritten. Die Schwankungen in den Resultaten sind also unvermeidlich bis auf drei mal so gross, als es erlaubt wäre, wenn man auf die möglichen Versuchsfehler allein Rücksicht nimmt.

Was zeigt dies an?

Messungsfehler in der Bestimmung der übrigen Quantitäten m, δ, g können nicht gern die Ursache sein. Denn sie können allerdings die gefundenen Resultate von dem wirklich wahren Werthe entfernen, aber da alle Messungen unter beinahe vollkommen gleichen Umständen stattfanden, so müssen sich diese Fehler bei dem Vergleich der Bestimmungen mit einander grösstentheils eliminiren.

Die Ursache muss in der Methode liegen.

Dann kann sie aber nirgends als in der Bestimmung von μ gesucht werden, und es ist also sehr wahrscheinlich, dass

die anhängende Luft auch nicht durch Auspumpen unter dem Luftpump vollständig entfernt werden kann, sondern dass ein Theil zurückbleibt, und zwar scheint dieser Fehler, der ganz und gar an der Methode liegt, etwa drei mal so gross zu sein als die Fehler, welche durch Ungenauigkeiten in der Messung entstehen können.

Nehmen wir jetzt als Beispiel die Bestimmung des wasserhaltigen schwefelsauren Lithions und zwar wie gewöhnlich die in dem Folgenden zuerst angeführte. Das Salz war spektroskopisch auf seine Reinheit geprüft, und der Einwurf, den man gegen die Bestimmungen der Alaunarten erheben kann, die Differenzen zwischen denselben rührten von der Auspressung her, findet hier nicht statt, denn eine Wasserbestimmung an dem angewandten Material ergab:

	gefunden	berechnet
HO	14,07 %	14,06 %

$s = 2{,}0562$, $(\mu - g) = 1{,}1962$. Angenommen wie vorher $d\mu = 0{,}0020$ gr.

Die Rechnung ergiebt:

Num. W. v. $ds = 0{,}0034$..

Nehmen wir an, auch hier wäre der auf die Methode beruhende Fehler un-

gefähr dreimal so gross, so würden Schwankungen in den Resultaten entstehen, wodurch die einzelnen Bestimmungen um 0,0102 von einander abweichen könnten.

Von den beiden im Folgenden angeführten Messungen ist die Erste: Spec. Gew. = 2,0562, die Zweite: Spec. Gew. = 2,0669, Diff = 0,0107.

Drittens nehmen wir die Bestimmung an Rubidiumplatinchlorid.

Die zuerst angeführte Bestimmung gab: $s = 3{,}9434$, $(\mu - g) = 0{,}5233$, $d\mu = 0{,}0020$ gr.

Num. W. v. $ds = 0{,}0150$.

Wenn hier der auf die Methode beruhende Fehler dreimal so gross anzunehmen wäre, so würden die Resultate um 0,0450 abweichen dürfen. Ihre wirkliche Differenz ist in diesem Fall jedoch nur 0,0163. Die beiden im Folgenden angeführten Bestimmungen sind:

[1]) Spec. Gew = 3,0834 [2]) Spec. Gew = 3,9597.

Zuletzt betrachten wir die Bestimmung des selensauren Thallions. $s = 7{,}029$, $\mu - g = 0{,}3200$, $d\mu$ wie gewöhnlich $= 0{,}0020$ gr.

Durch Rechnung erhält man:

Num. W. v. $ds = 0{,}0438$.

Die in dieser Arbeit aufgenommen Bestimmungen geben für das selensaure Thalliumoxyd:

[1]) Spec. Gew. = 7,0194, [2]) Spec. Gew. = 7,0679, Diff. = 0,0485.

Wir sehen, dass der unvermeidliche, der Methode zuzuschreibende Fehler, den wir bei den specifisch leichteren Substanzen als etwa drei mal so gross, wie die möglichen Messungsfehler anzunehmen genöthigt waren, bei den specifisch schwereren Körpern immer kleiner wird und zuletzt neben dem Beobachtungsfehler wenig bemerkbar ist.

Auf diese Thatsache gestützt, wagen wir die folgende Vermuthung aufzustellen:

Die specifisch schwereren Salze scheinen im Allgemeinen nicht in dem selben Grade Luft auf sich zu kondensiren wie die specifisch leichteren oder wenigstens diese Luft weniger hartnäckig unter vermindertem Luftdruck zurückzuhalten.

Im Zusammenhang hiermit bemerken wir, dass die von uns erhaltenen Resultate, sofern sie sich auf Substanzen vou einem specifischen Gewichte von 1,3... bis 3,00 beziehen, im Allgemeinen durchaus nicht mit den Resultaten anderer Beobachter, welche die anhängende Luft nur durch

Umrühren [1]) beseitigt, stimmen, sondern durchgehend grösser sind, dass unsre Bestimmungen bei Substanzen von grösserem specifischen Gewicht (von 3,0—8,0) aber oft sich sehr wenig von den sorgfältigen Bestimmungen von KOPP, TOPSOE u. A. unterscheiden.

Da wir für unsre Resultate keine grössere Genauigkeit in Anspruch nehmen wollen, als ihnen in der Wirklichkeit zukommt, wollen wir die specifischen Gewichte der leichteren Salze mit drei Decimalen und die der schwereren nur mit zwei anführen (obschon wir bei den Rechnungen immer mit vier operirt haben) und im Voraus ausdrücklich bemerken, dass die letzte Ziffer unsicher ist.

Wir haben diesen Gegenstand vielleicht zu ausführlich behandelt, aber bei der Unmasse von einander widersprechenden specifischen Gewichtsbestimmungen, die veröffentlicht sind und theilweise als Grund für Theorien gedient haben, muss man einmal eine genaue Kritik der Methode und der Resultate anstellen, um in den wichtigen Verhältnissen der Molekülarvolumina endlich zur Klarheit zu kommen.

	Aeqv.gew.	Atomgew.		Aeqv.gew.	Atomgew.
Ka	39,11	39,11	*Cr*	26,24	52,48
Na	23,00	23,00	*Cu*	31,70	63,40
Rb	85,40	85,40	*Pb*	103,50	207,00
Cs	133,00	133,00	*Ag*	108,00	108,00
Li	7,00	7,00	*Pt*	98,94	197,88
Tl	204,00	204,00	*Cl*	35,46	35,46
Ba	68,50	137,00	*Br*	80,00	80,00
Sr	43,75	87,50	*I*	127,00	127,00
Ca	20,00	40,00	*O*	8,00	16,00
Fe	28,00	56,00	*S*	16,00	32,00
Al	13,75	27,50	*Se*	39,50	79,00

[1]) Schiff hat sich allerdings bei seinen Bestimmungen einer von der üblichen abweichenden Methode bedient, aber auch bei dem von ihm adoptirten Verfahren muss die an den Substanzen adhärirende Luft die Resultate falsch ausfallen lassen. Er bemerkt ausdrücklich, dass er sie nur durch Anklopfen an der Röhre oder durch Umrühren weggeschafft hat. Siehe Ann. d. Chem. u. Pharm. Bd CVII s. 60.

§ 4.

DIE ALAUNE.

Die folgende Untersuchung umfasst (mit 1 oder 2 Ausnahmen [1]) alle bisher dargestellten Alaunarten [2]) der Schwefelsäure und Selensäure und noch einige andere, worüber wir in der Litteratur keine Angaben gefunden, deren Existenz jedoch leicht vorauszusehen war. Die Reihe der schwefelsauren Alaunarten ist grösstentheils längst bekannt, und auch die Dichten und Molekülarvolumina sind bei einem Theil derselben vorher studirt worden (wir heben unter den Beobachtern dieses Gegenstandes besonders KOPP und SCHIFF hervor).

Von den Alaunarten der Selensäure hat WOHLWILL einige dargestellt und untersucht, nachher hat der Verfasser von dieser Arbeit eine Reihe von Analysen der Selensäure-Alaunarten mit Thonerde und Chromoxyd veröffentlicht, neulich hat GERICHTEN Alaune dargestellt, worin zugleich Schwefelsäure und Selensäure eingehen und von diesen den schwefel-selensauren- und den selen-schwefelsauren-Thonerde-Kalialaun ($KaOSeO_3 + Al_2O_3 3SO_3 + 24aq$ und $KaOSO_3 + Al_2O_3 3SeO_3 + 24aq$) analysirt.

Über die specifischen Gewichte dieser Alaunarten finden sich bei den genannten Verfassern keine Angabe.

Die besonderen Vorsichtsmassen, die wir getroffen, um jedes Salz in dem Zustande vollkommener Reinheit zu erhalten, werden wir bei der Beschreibung jedes einzelnen Alaunartes erwähnen; jetzt wollen wir einige Angaben über die Art und Weise, worauf wir die zu unserem Zwecke nöthige Selensäure und die schwefel- und selensauren Salze von Thonerde, Eisenoxyd und Chromoxyd dargestellt haben, vorausschicken.

Als Rohmaterial zur Darstellung der Selensäure diente Selenschlamm aus Fahlun. Die Reindarstellung des Selens aus diesem Material haben wir in einem anderen Aufsatz eingehend besprochen, hier können wir uns

[1]) Wir gedenken hier namentlich des Schwefelsäure-Chromoxyd-Natron- und des Schwefelsäure-Indiumoxyd-Kali-alauns. Den ersten dieser Alaune haben wir nicht nöthig gehabt darzustellen, da es uns nicht gelungen ist, das entsprechende Selensäuresalz krystallisirt zu erhalten. Den zweiten haben wir nicht aus Mangel an Material darstellen können.

[2]) Wir meinen hier nur Alaune mit anorganischen Basen. Für die Untersuchung der Raumerfüllung der intressanten Alaunarten mit organischen Basen wie Chinin (v. Wills 1842), Methylamin (v. Alth 1854) und Glykokoll, Tyrosin, Leucin (von Svanberg 1862) hoffen wir einmal später Gelegenheit zu finden.

deshalb damit begnügen, zu erwähnen, dass das Selen mit Cyankalium ausgezogen und mit Salzsäure aus der Lösung ausgefällt wurde, dass es nachher mit Salpetersäure zu Seleniger Säure oxydirt, und diese letztere durch wiederholte Sublimation in Glasröhren gereinigt wurde. Durch diese Operationen erhielten wir eine Selenige Säure von vollkommener Reinheit, die sich vollständig in Wasser lösen, und ohne Rückstand sublimiren liess. Aus diesem Præparat und reinem salpetersauren Silberoxyd[1]) wurde selenigsaures Silberoxyd bereitet. Dieses Salz wurde nach dem Auswaschen nach der Methode von THOMSON[2]) in Wasser suspendirt, mit Brom[3]) geschüttelt, und die erhaltene wässrige Selensäure durch Abdampfen concentrirt und von überschüssigem Brom befreit.

Die Thonerde wurde aus eisenfreiem Römischen Alaun bereitet. Wir hatten vorher Gelegenheit gehabt zu bemerken, dass auch durch dreimal wiederholtes Fällen mit Ammoniak und Auswaschen das Thonerdehydrat sich jedoch nicht schwefelsäurefrei erhalten lässt. Deshalb wurde zuerst die Schwefelsäure durch Chlorbarium ausgefällt und die abfiltrirte Lösung zweimal durch Ammoniak, beziehungsweise kohlensaures Kali oder Natron (wenn die Thonerde zur Darstellung von den Alaunarten der letzteren Basen dienen sollte) gefällt und ausgewaschen. Die schwefelsaure- und selensaure Thonerde wurde durch Einwirkung von reiner Schwefelsäure und Selensäure auf das so dargestellte Hydrat erhalten. Bisweilen liess sich die Lösung des Thonerdehydrats in Selensäure schwierig werkstellen; in dem Falle wurde dasselbe, mit der nöthigen Menge der Säure in einem zugeschmolzenen Glasrohr eingeschlossen, auf 200° – 300° erhitzt; eine Operation, die immer zu dem erzielten Zwecke führte.

Schwefelsaures Eisenoxyd wurde durch Auflösen von Klavierdraht in reiner überschüssiger Schwefelsäure, Oxydation des Oxydulsalzes mittelst Salpetersäure und Eindampfen als Krystallmasse erhalten, die nachher ausgepresst wurde.

Einige der Chrom-Alaunarten sind aus Chromoxydhydrat gemacht, das in Schwefelsäure oder Selensäure gelöst war. Es ist jedoch sehr schwierig eine Lösung desselben in den genannten Säuren, besonders in der Selensäure zu erhalten, die wirklich krystallisirbares Salz enthält, weil das Chromoxydhydrat bei dem Auswaschen leicht in die Modifikation übergeht,

[1]) Das Silber war nach der Methode von Stas ausgefällt, und die Salpetersäure zweimal umdestillirt worden.

[2]) Tidsskrift for Physik o. Chemie 1870.

[3]) Das Brom war vorher umdestillirt worden.

die mit Säuren grüne Lösungen giebt. Die grüne Lösung des schwefelsauren Chromoxydes geht allerdings nach langer Zeit in die violette Modifikation über, aber mit dem selensauren Chromoxyd geschieht dies nicht. Deshalb haben wir den folgenden, ungemein leichteren Weg [1]) eingeschlagen.

Reines Chromoxydhydrat wurde in der folgenden Weise bereitet. Zweifach chromsaures Kali, mehrmals umkrystallisirt, wurde in Wasser gelöst, mit Salzsäure und Alkohol reducirt, aus der grünen Chromchloridlösung Chromoxydhydrat mit Ammon gefällt, nachher in Salzsäure gelöst und wieder gefällt. Zuletzt wurde es nach sorgfältigem Auswaschen in reiner Salpetersäure gelöst, die Lösung durch Eindampfen sehr koncentrirt, Schwefelsäure, beziehungsweise Selensäure und Alkohol zugesetzt, wobei sich das schwefelsaure oder selensaure Chromoxyd als eine schöne grauviolette Salzmasse ausschied, die mit Alkohol sich auswaschen liess. Sie wurde bis zum Trocknen derselben auf einer porösen Porzellanplatte liegen gelassen und dann zwischen Papier sorgfältig ausgepresst.

Zuletzt wollen wir etwas mittheilen über die Auspressung der von uns untersuchten krystallisirbaren Salze, welche in der Regel zu den specifischen Gewichtsbestimmungen in ziemlich fein zerriebenem Zustand angewendet wurden. (Bisweilen sind, wo dies ausdrücklich bemerkt worden ist, Krystallsplitter oder ganze Krystalle angewendet worden). Da bei dem Auspressen pulverförmiger Körper zwischen gewöhnliches Fliesspapier leicht Fasern desselben an der Substanz heften bleiben, so haben wir zu diesem Zweck feines Schwedisches Filtrirpapier gebraucht, das vorher durch starkes Pressen zwischen Buchdruckerwalzen vollkommen glatt geworden war. Von diesem Papier hefteten keine Theilchen an der Substanz auch bei sehr starkem Auspressen.

Zwischen Blättern von diesem Papier ist jedes Salz drei bis viermal ausgepresst worden.

Die Aeqvivalentformel der Alaune schreiben wir $KOSO_3 + Al_2O_3 3SO_3 + 24HO$, wie wir es auch in dem Vorigen gethan; als Molekülarformel nehmen wir unter mehreren, die vorgeschlagen sind und zulässig sein können, die folgende als die gebräuchlichste an:

$$\begin{matrix} K \\ Al \end{matrix}\Big\} 2SO_4 + 12H_2O$$

Alle in dem Folgenden angeführten Alaunarte sind vor den specifischen Gewichtsbestimmungen mehrmals umkrystallisirt worden. Nach die-

[1]) Derselbe ist von Lœwig (Ann. d. Chem. et Phys. XIV. 239) für die Darstellung des schwefelsauren Salzes vorgeschlagen.

ser Annahme werden die Zahlenwerthe, welche das Aeqvivalentgewicht und das Molekülargewicht eines Alauns ausdrücken, dieselben sein, und folglich Aeqvivalent- und Molekülarvolum zusammenfallen.

Kali-Thonerde-Schwefelsäure-Alaun

Aeqv. Gew. = 474,61 = Mol. Gew.

Saures weinsaures Kali wurde einige Mal umkrystallisirt und geglüht, und das gebildete kohlensaure Kali mit Wasser ausgelaugt, die Lösung mit reiner Schwefelsäure[1]) gesättigt, eingedampft und zur Krystallisation gebracht. Aus dem so erhaltenen reinen schwefelsauren Kali und der im Vorigen erwähnten Lösung von schwefelsaurer Thonerde wurde die Lösung bereitet, woraus der Alaun krystallisirte.

1) 2,6178 Gr. bei 20,5° C. spec. Gew. = 1,755. Aeq. Vol. = Mol. Vol. = 270,4.
2) 2,7150 Gr. bei 21,0° C. spec. Gew. = 1,753. Aeq. Vol. = Mol. Vol. = 270,6.
3) 1,2628 Gr. bei 21,0° C. spec. Gew. = 1,749. Aeq. Vol. = Mol. Vol. = 271,3.
Ganze Krystalle, vollkommen durchsichtig, wohl getrocknet.
4) 3,1430 Gr. bei 21,0° C. spec. Gew. = 1,750. Aeq. Vol. = Mol Vol. = 271,1.
Gewöhnlicher Alaun zu glasglänzenden Splittern zerstossen, die durch Sieben durch ein Metallnetz von dem feineren Pulver getrennt waren.

Mittleres Aeqvivalent- und Molekularvolum = 270,8.

Specifisches Gewicht nach früheren Bestimmungen = 1,722 v. Schiff; 1,724 v. Kopp. 1,731 v. Playfair und Ioule.

Kali-Chromoxyd-Schwefelsäure-Alaun

Aeqv. Gew. = 499,59 = Mol. Gew.

Aus reinem schwefelsauren Kali und schwefelsauren Chromoxyd bereitet.

1) 1,4186 Gr. bei 20,2° C. spec. Gew. = 1,841. Aeq. Vol. = Mol. Vol. = 271,3.
2) 1,3700 Gr. bei 21,0° C. spec. Gew. = 1,849. Aeq. Vol. = Mol. Vol. = 270,1
3) 1,4565 Gr. bei 21,0° C. spec. Gew. = 1,840. Aeq. Vol. = Mol. Vol. = 271,4.
4) 1,9105 Gr. bei 21,0° C. spec. Gew. = 1,839. Aeq. Vol. = Mol. Vol. = 271,6.

Mittleres Aeqvivalent und Molekularvolum = 271,1.

Nach früheren Bestimmungen = 1,845 (v. Schiff); 1,848 (v. Kopp); 1,843 (v. Playfair und Ioule).

Kali-Eisenoxyd-Schwefelsäure-Alaun

Aeqv. Gew. = 503,11 = Mol. Gew.

Aus schwefelsaurem Kali und schwefelsaurem Eisenoxyd dargestellt. Da der Alaun nicht ausgepresst werden kann ohne zersetzt zu werden, wurde

[1]) bei dem selensauren Kali-Alaun mit Selensäure u. s. w.

er nur zwischen Fliesspapier getrocknet und nachher 5 bis 10 Minuten an der Luft liegen gelassen. Aus diesem Grunde stimmen die Bestimmungen des specifischen Gewichts nicht völlig mit einander.

1) 1,4070 Gr. bei 17,0° C. spec. Gew. = 1,831. Aeq. Vol. = Mol. Vol. = 274,5.
2) 2,3129 Gr. bei 16,8° C. spec. Gew. = 1,819. Aeq. Vol. = Mol. Vol. = 276,4.
Das Material für diese Bestimmung war Splitter von Krystallen.
3) 2,0988 Gr. bei 17,5° C. spec. Gew. = 1,822. Aeq. Vol. = Mol. Vol. = 276,1.
Mitleres Aeqvivalent- und Molekularvolum = 275,7.

Kali-Thonerde-Selensäure-Alaun.

Aeqv. Gew. = 568,61 = Mol. Gew.

Aus selensaurem Kali und selensaurer Thonerde.

1) 2,3531 Gr. bei 21,0° C. spec. Gew. = 1,998. Aeq. Vol. = Mol. Vol. = 284,4.
2) 2,3121 Gr. bei 20,1° C. spec. Gew. = 2,004. Aeq. Vol. = Mol. Vol. = 283,7.
Mittleres Aeqvivalent- und Molekularvolum = 284,0. [1])

Kali-Chromoxyd-Selensäure-Alaun.

Aeqv. Gew. = 593,59 = Mol. Gew.

Aus selensaurem Kali und selensaurem Chromoxyd.

1) 1,4054 Gr. bei 17,6° C. spec. Gew. = 2,076. Aeq. Vol. = Mol. Vol. = 285,8.
2) 1,3921 Gr. bei 17,2° C. spec. Gew. = 2,081. Aeq. Vol. = Mol. Vol. = 285,2.
3) 1,1374 Gr. bei 17,0° C. spec. Gew. = 2,077. Aeq. Vol. = Mol. Vol. = 285,6.
Mittleres Aeqvivalent- und Molekularvolum = 285,5.

Natron-Thonerde-Schwefelsäure-Alaun.

Aeq. Gew. = 458,50 = Mol. Gew.

Doppelt kohlensaures Natron, mehrfach umkrystallisirt, wurde geglüht und in Wasser gelöst, die Lösung mit reinem, durch Fällung bereitetem kohlensauren Baryt mehrere Stunden in der Wärme digerirt und nachher abfiltrirt. Das kohlensaure Natron erwies sich dann bei Prüfung vollkommen rein von Schwefelsäure, wovon es früher Spuren enthielt. Durch Zusatz von reiner Schwefelsäure und Stehen der Lösung bis zum Auskrystallisiren des Salzes wurde reines schwefelsaures Natron erhalten. Aus dem so bereiteten Salz und aus schwefelsaurer Thonerde war der Alaun, der zu den folgenden Versuchen angewandt wurde, bereitet. Das Thonerdehydrat, das zur Darstellung desselben diente, war in der oben genannten Weise

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[1]) Für die selensauren Alaunarten haben wir in der Litteratur keine frühere Angaben des specifischen Gewichts gefunden.

bereitet, mit der Ausnahme, dass es zweimal nach einander mit kohlensaurem Natron statt Ammoniak ausgefällt war.

1) 2,1730 Gr. bei 18,0° C. spec. Gew. = 1,693. Aeq. Vol. = Mol. Vol. = 270,8.
2) 2,0809 Gr. bei 18,0° C. spec. Gew. = 1,686. Aeq. Vol. = Mol. Vol. = 271,9.
3) 1,9614 Gr. bei 18,2° C. spec. Gew. = 1,694. Aeq. Vol. = Mol. Vol. = 270,6.

Mittleres Aeqvivalent- und Molekularvolum = 271,1.

Nach früheren Bestimmungen = 1,641 (v. Schiff).

Natron-Thonerde-Selensäure-Alaun.

Aeq. Gew. = 552,50 = Mol. Gew.

Dargestellt aus selensaurem Natron und selensaurer Thonerde. Es wurde dazu kohlensaures Natron und Thonerdehydrat, von der bei dem vorigen Alaune erwähnten Bereitung, angewendet. Da dieser Alaun in gewisser Hinsicht sich sehr eigenthümlich verhält, und in der Reihe der Molekülarvolumina der Alaunarten eine anomale Stellung einnimmt, können folgende Notizen über seine Zusammensetzung und über die Art und Weise, auf welche er auskrystallisirt, einige Aufmerksamkeit verdienen. Er ist überaus leicht in Wasser löslich und krystallisirt daraus nur bei sehr langsamen Verdunsten im offenen Gefäss bei gewöhnlicher Temperatur. Er wird dann bisweilen in sehr grossen Krystallen erhalten, die zwar dem regulären System anzuhören scheinen aber gewöhnlich doch in ihrem Aussehen sich sehr von den Formen unterscheiden, worin die übrigen Alaune auskrystallisiren. Bisweilen schiesst er in kleinen, scharf ausgebildeten Krystallen an, die nach einer Richtung ausgezogen erscheinen und wie kleine zugespitze Prismen aussehen, die sich in Gruppen strahlenweise um ein Centrum her geordnet haben. Er scheint dann seinem Forme nach gar nichts mit den übrigen Alaunarten und mit den Krystallformen des regulären Systemes im allgemeinen gemeinsam zu haben.

Dass dies aber wirklich der Fall ist, wagen wir doch durchaus nicht zu behaupten; es ist ja z. B. bekannt, dass der Salmiak in Formen auskrystallisirt, die dem regulären System gar nicht anzuhören scheinen, und doch lassen sie sich nach Rammelsbergs Untersuchungen darauf zurückführen. Wir bedauern, dass wir nicht im Stande gewesen, durch krystallografische Messung dieses Verhältniss aufzuklären. Nur soviel wollen wir bemerken, dass *dem Alaun, in welcher Form er auch auskrystallisirt, dasselbe specifische Gewicht zukommt.*

Da der Verdacht entstehen konnte, es wäre dieser Alaun nicht nach denselben qvantitativen Verhältnissen zusammengesetzt, wie die übrigen Alaune, haben wir ihn wiederholt analysirt:[1])

2,5908 Gr. von dem Alaun gab NaO = 0,1444 gr.; SeO_3 = 1,1842 gr. (bestimmt als selensaures Baryt); Al_2O_3 = 0,2488 gr.

	gefunden	berechnet nach der gewöhnlichen Formel für die Alaune.
SeO_3	45,70 %	45,98 %
Al_2O_3	9,60 %	9,32 %
NaO	5,57 %	5,61 %
HO (Verlust)	39,13 %	39,09 %
	100,00.	100,00.

Wir haben hier also mit einem wirklichen Alaun zu thun.

1) 2,5908 Gr. bei 21,0° C. spec. Gew. = 2,061 Aeq. Vol. = Mol. Vol. = 268,0. Dieselbe Probe, welche nachher zur Analyse verwendet wurde.

2) 1,8334 Gr. bei 20,8° C. spec. Gew. = 2,069 Aeq. Vol. = Mol. Vol. = 267,0.

3) 1,7954 Gr. bei 20,8° C. spec. Gew. = 2,071 Aeq. Vol. = Mol. Vol. = 266,8.

Mittleres Aeqvivalent- und Molekularvolum = 267,2.

Ammoniak-Thonerde-Schwefelsäure-Alaun.

Aeq. Gew. = 453,50 = Mol. Gew.

Aus schwefelsaurem Ammon und schwefelsaurer Thonerde. Das erste war durch Sättigung reiner Schwefelsäure mit Ammon bereitet. Einige Bestimmungen sind mit gewöhnlichem Ammoniakalaun aus dem Vorrath des Laboratoriums gemacht.

1). 2,4155 Gr. bei 18,2° C. spec. Gew. = 1,647 Aeq. Vol. = Mol. Vol. = 275,2. Gewöhnlicher Alaun in Splittern.

2). 2,3207 Gr. bei 19,5° C. spec. Gew. = 1,642 Aeq. Vol. = Mol. Vol. = 276,1.

3). 1,4178 Gr. bei 19,2° C. spec. Gew. = 1,641 Aeq. Vol. = Mol. Vol. = 276,3.

4). 2 8720 Gr. bei 19,0° C. spec. Gew. = 1,638 Aeq. Vol. = Mol. Vol. = 276,8.

Ganze durchsichtige Krystalle, die an der Luft 2 Stunden vor der Bestimmung gelegen.

Mittleres Aeqvivalent- und Molekularvolum = 276,1.

Frühere Bestimmungen: 1,621 (v. Schiff) 1,626 (v. Kopp) 1,625 (v. Playfair u. Ioule).

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[1]) In einem 1872 gedrucktem Aufsatz: Über die selensauren Alaunarten u. s. w. haben wir ausserdem eine Analyse desselben Alaunes mitgetheilt.

0,4164 gr. der Substanz gab Al_2O_3 = 0,0388 gr. SeO_3 = =0,1917 gr. NaO = 0,0589 gr.

	gefuuden	bevehnet
SeO_3	46,03	45,98
Al^2O_3	9,31	9,32
NaO	6,17	5,61
HO	38,49	39,09
	100,00.	100,00.

Ammoniak-Chromoxyd-Schwefelsäure-Alaun

Aeq. Gew. = 478,48 = Mol. Gew.

Aus schwefelsaurem Ammon und schwefelsaurem Chromoxyd.

[1]) 1,1596 Gr. bei 20,0° C. spec. Gew. = 1,728 Aeq. Vol. = Mol. Vol. = 276,8.

Aeqvivalent- und Molekularvolum = 276,8.

Frühere Bestimmung 1,736 v. Schrötter.

Ammoniak-Eisenoxyd-Schwefelsäure-Alaun.

Aeq. Gew. = 482,00 = Mol. Gew.

In analoger Weise bereitet wie das entsprechende Kaliumsalz.

[1]) 2,0728 Gr. bei 18,0° C. spec. Gew. = 1,723 Aeq. Vol. = Mol. Vol. = 279,6.

[2]) 2,2579 Gr. bei 18,2° C. spec. Gew. = 1,720 Aeq. Vol. = Mol. Vol. = 280,2.

[3]) 1,7069 Gr. bei 17,0° C. spec. Gew. = 1,725 Aeq. Vol. = Mol. Vol. = 279,3.

Mittleres Aeqvivalent- und Molekularvolum = 279,7.

Dieser Alaun lässt sich beim Erhitzen auf 100° vollständig entwässern.

4,2288 Gr. ausgepresstes Salz verloren bei 100° 1,8628 Gr. Wasser = 44,05 %; nach Rechnung sollten sie 44,80 % verlieren.

Auf dem in dieser Weise entwässerten Salze wurde die folgende Bestimmung gemacht.

2,1363 Gr. bei 16,8° C. spec. Gew. = 2,540 Aeq. Vol. = 104,7.

Frühere Dichtigkeitsbestimmungen des wasserhaltigen Alauns 1,712 (v. Kopp) 1,715 (v. Playfair u. Ioule.)

Ammoniak-Thonerde-Selensäure-Alaun.

Aeq. Gew. = 547,50 = Mol. Gew.

Aus selensaurem Ammon und selensaurer Thonerde.

1) 1,9647 Gr. bei 19,0° C. spec. Gew. = 1,689 Aeq. Vol. = Mol. Vol = 289,7.
Substanz pulverförmig.

2) 2,4300 Gr. bei 19,0° C. spec. Gew = 1,890 Aeq. Vol. = Mol. Vol. = 289,6.
Die Substanz bestand aus groben Krystallsplittern.

3) 3,7213 Gr. bei 17,0° C. spec. Gew. = 1,894 Aeq. Vol. = Mol. Vol. = 289,0.

4) 2,1708 Gr. bei 20,5° C. spec. Gew. = 1,895 Aeq. Vol. = Mol. Vol. = 288,9.

Mittleres Aeqvivalent- und Molekularvolum = 289,3.

Ammoniak-Chromoxyd-Selensäure-Alaun.

Aeq. Gew. = 572,48 = Mol. Gew.

Aus selensauren Ammon und selensaurem Chromoxyd.

1) 1,9691 Gr. bei 20,0° C. spec. Gew. = 1,984 Aeq. Vol. = Mol. Vol. = 288,5.
Substanz gepulvert.

2) 0,8591 Gr. bei 20,0° C. spec. Gew. = 1,980 Aeq. Vol. = Mol. Vol. = 289,0. Kleine Krystalle.

Mittleres Aeqvivalent- und Molekularvolum = 288,7.

Rubidion-Thonerde-Schwefelsäure-Alaun.

Aeq. Gew. = 520,90 = Mol. Gew.

30 Grammen Rubidionalaun, der sich vor dem Spektroskop als Cæsiumfrei erwiesen[1]) hatte, wurden 15 Mal umkrystallisirt in der Weise, dass jedes mal das beim Erkalten der Lösung auskrystallisirte Salz von neuem in kochendem Wasser gelöst, und das Gefäss mit der Lösung an einem kühlen Platz stehen lassen wurde. Das zuletzt auskrystallisirte Salz wurde zu den folgenden Bestimmungen angewendet.

1) 2,7792 Gr. bei 20,0° C. spec. Gew. = 1,890 Aeq. Vol. = Mol. Vol. = 275,5.
2) 1,8276 Gr. bei 20,0° C. spec. Gew. = 1,891 Aeq. Vol. = Mol. Vol. = 275,5.
3) 0,9939 Gr. bei 20,0° C. spec Gew. = 1,890 Aeq. Vol. = Mol. Vol. = 275,5. Krystalle, die 12 Stunden an der Luft gelegen.
4) 2,8590 Gr. bei 19,2° C. spec. Gew. = 1,890 Aeq. Vol. = Mol. Vol. = 225,5. Grössere Krystalle.

Mittleres Aeqvivalent- und Molekularvolum = 275,5.

Für die folgenden Alaunarten haben wir in der Litteratur keine Angaben des specifischen Gewichtes gefunden.

Rubidion-Chromoxyd-Schwefelsäure-Alaun[2])

Aeq. Gew. = 545,88 = Mol. Gew.

Aus schwefelsaurem Rubidion und schwefelsaurem Chromoxyd. Das schwefelsaure Rubidion wurde folgendermassen dargestellt. Chlorrubidium (Caesiumfrei) wurde mit Platinchlorid verbunden und das gebildete Rubidium-Platin-Chlorid mit heissem Wasser lange ausgewaschen. Nach dem Trocknen wurde es in einer Glasröhre eingewogen und durch Wasserstoffgas bei gelinder Glühhitze reducirt.

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[1]) Die Vorschriften der Lehrbücher, Rubidiumsalze auch spektralanalytisch auf einen Gehalt an Kalium zu prüfen, konnten nicht gefolgt werden, da bei Spektralapparaten, die kein ganz ausserordentliches Brechungs-vermögen besitzen, die Linien von Kalium und Rubidium so nahe an einander fallen, dass es vollkommen unmöglich ist, kleine Mengen von Kaliumsalz neben viel Rubidium zu entdecken.

[2]) Dieser Alaun ist sehr schwer in Wasser löslich, und kann nur aus grossen Mengen desselben umkrystallisirt erhalten werden. Beim Vermischen der Lösungen von schwefelsaurem Rubidion und Chromoxyd fiel er sogleich aus in der Gestalt von mikroskopischen Krystallen.

Aus 9,1838 Gr. $RbCl+PtCl_2$ wurde bei der Reduction 3,1161 Gr. Pt erhalten. Aus diesem Versuch berechnet sich das Aeqivalentgewicht des Salzes zu 291,5. Nach der Formel ist es 290,72. Da die Bestimmung des Aeqvivalentgewichtes des Chlorrubidium also ein wenig zu hoch ausgefallen, konnte es unmöglich von Chloriden mit geringerem Aeqvivalentgewicht wie Ka Cl, Am Cl verunreinigt sein.

Der Rückstand in der Röhre, der aus Platin und Chlorrubidium bestand, wurde mit Wasser ausgelaugt, die Lösung zum Trocknen verdampft und mit überschüssiger Schwefelsäure behandelt, und der Überschuss der letzteren durch Glühen verjagt unter Zusatz von kohlensaurem Ammon. In Wasser gelöst und mehrmals umkrystallisirt gab der Rückstand reines schwefelsaures Rubidion.

1) 0,9620 Gr. bei 16,8° C. spec. Gew. = 1,969 Aeq Vol. = Mol. Vol. = 277,1.
2) 1,8215 Gr. bei 16,8° C. spec. Gew. = 1,967 Aeq. Vol. = Mol. Vol. = 277,5.

Mittleres Aeqvivalent- und Molekularvolum = 277,3.

Rubidion-Thonerde-Selensäure-Alaun.

Aeq. Gew. = 614,90 = Mol. Gew.

Aus selensaurem Rubidion und selensaurer Thonerde dargestellt. Das Thonerdehydrat, welches zu der Bereitung des letzteren Salzes genommen wurde, war wie gewöhnlich zweimal nach einander aus schwefelsäurefreier Lösung gefällt und nach der letzten Fällung sehr lange ausgewaschen worden. Beim Kochen desselben mit Kalilauge entwickelte sich keine Spur von Ammoniak. Das selensaure Rubidion war aus reinem Chlor-rubidium durch Schütteln einer Lösung desselben mit selensaurem Silberoxyd und Eindampfen der von dem ausgeschiedenen Chlorsilber abfiltrirten Flüssigkeit bis zur Krystallisation bereitet worden.

1) 1,4668 Gr. bei 21,0° C. spec. Gew. = 2,134 Aeq. Vol. = Mol. Vol. = 288.0.
2) 1,6278 Gr. bei 17,0° C. spec. Gew. = 2,132 Aeq. Vol. = Mol. Vol. = 288,3.
3) 1,7126 Gr. bei 17,2° C. spec. Gew. = 2,135 Aeq. Vol. = Mol. Vol. = 288,0.

Mittleres Aeqvivalent- und Molekularvolum = 288,1.

Rubidion-Chromoxyd-Selensäure-Alaun [1]).

Aeq. Gew. = 639,88 = Mol. Gew.

Aus selensaurem Rubidion und selensaurem Chromoxyd.

1) 0,7484 Gr. bei 17,0 C. spec. Gew. = 2,223 Aeq. Vol. = Mol. Vol. = 287,8.
2) 1,1667 Gr. bei 18,8° C. spec. Gew. = 2,214 Aeq. Vol. = Mol Vol. = 288,9.

Mittleres Aeqvivalent- und Molekülarvolum = 288,4.

[1]) Unter den von uns dargestellten selensauren Alaunarten, die im Allgemeinen leichtlöslich sind, ist dieser der schwerlöslichste.

Cæsion-Thonerde-Schwefelsäure-Alaun.

Aeq. Gew. = 568,50 = Mol. Gew.

Die Quantität von Cæsium-Alaun, worüber wir zu verfügen hatten, war so gering, dass wir, um Verlust an Substanz zu vermeiden, uns damit begnügen mussten, sie einige Mal zu umkrystallisiren. Sie erwies sich doch bei spektralanalytischer Prüfung frei von Kali, Natron und Rubidion.

1) 2,2115 Gr. bei 20,0° C. spec. Gew. = 2,000. Aeq. Vol. = Mol. Vol. = 284,2.
2) 2,1506 Gr. bei 18,1° C. spec. Gew. = 1,994. Aeq. Vol. = Mol. Vol. = 285,0.

Mittleres Aeqvivalent- und Molekularvolum = 284,6.

Cæsion-Thonerde-Selensäure-Alaun.

Aeq. Gew. = 662,50 = Mol. Gew.

Dieser Alaun ist aus dem schwefelsauren in der Weise dargestellt, dass zuerst durch Chlorbarium die Schwefelsäure daraus entfernt, und nachher die von dem Schwefelsäureniederschlag abfiltrirte Lösung mit selensaurem Silberoxyd geschüttelt wurde. Aus dem Filtrat krystallisirte nachher der Selensäure-Alaun.

1) 2,1779 Gr. bei 19,8° C. spec. Gew. = 2,225. Aeq. Vol. = Mol. Vol. = 297,7.
2) 2,0535 Gr. bei 18,8° C. spec. Gew. = 2,223. Aeq. Vol. = Mol. Vol. = 298,0.

Mitleres Aeqvivalent- und Molekularvolum = 297,8.

Thallion-Thonerde-Schwefelsäure-Alaun.

Aeq. Gew. = 639,50 = Mol. Gew.

Aus schwefelsaurem Thallion und schwefelsaurer Thonerde. Eine Lösung von unreinem schwefelsauren Thallion wurde mit Schwefelwasserstoff behandelt, wobei eine kleine Menge von Schwefelmetallen (wahrscheinlich Schwefelblei u. s. w.) abgeschieden wurde, nachher wurde durch Jodkalium das Thallium als reines Jodthallium ausgefällt und abfiltrirt. Dieses diente als Material für die Darstellung der übrigen von uns untersuchten Thalliumverbindungen. Schwefelsaures Thallium wurde daraus durch Abdampfen mit Schwefelsäure und wiederholtes Umkrystallisiren bereitet.

Die Thallionalaune sind nicht leicht im reinen Zustand zu erhalten, und zwar ist dies nur möglich, wenn man einen Überschuss von dem Thonerde- beziehungsweise Chromoxyd- oder Eisenoxyd-salz anwendet. Aber auch dann scheidet sich bei dem Eindampfen der Lösungen in der Regel schwefelsaures oder selensaures Thallion aus, weil diese Salze ungleich schwerlöslicher sind als der Alaun. Lässt man aber die Lösung einige Tage stehen, so setzen sich die schon ausgeschiedenen Krystalle der frag-

lichen Salze mit dem Thonerdesalz um, nnd es bleiben nur klare, durchsichtige Alaunkrystalle zurück [1])

1) 2,0040 Gr. bei 21,0° C. spec. Gew. = 2,366. Aeq. Vol. = Mol. Vol. = 270,2.
2) 2,2353 Gr. bei 20,6° C. spec. Gew. = 2,368. Aeq. Vol. = Mol. Vol. = 270,0.
3) 2,8092 Gr. bei 15,8° C. spec. Gew. = 2,348. Aeq. Vol. = Mol. Vol. = 272,7.
4) 2,1995 Gr. bei 17,0° C. spec. Gew. = 2,384 Aeq. Vol. = Mol. Vol. = 268,2.
5) 2,2951 Gr. bei 17,0 C. spec. Gew. = 2,384. Aeq. Vol. = Mol. Vol. = 268,2.

Mittleres Aeqvivalent- und Molekularvolum. = 269,8.

Dieser Alaun verliert, bei 100° allmählig ungefähr 20 Aeq. Wasser.

4,3576 Gr. Alaun verloren bei 100° 1,2048 Gr. Wasser. entsprechend 27,64 % Berechnet 28,14 % Das Aeqvivalentgewicht des getrockneten Alauns war demnach 462,7. Eine specifische Gewichtsbestimmung an derselben Probe ergab:

3,1192 Gr. bei 17,0° C. spec. Gew. = 3,645 Aeq. Vol. = 126,9.

Thallion-Chromoxyd-Schwefelsäure-Alaun.

Aeq. Gew. = 664,48 = Mol Gew.

Aus schwefelsaurem Thallion und schwefelsaurem Chromoxyd.

1) 1,3355 Gr. bei 18,0° C. spec. Gew. = 2,402. Aeq. Vol. = Mol. Vol. = 276,5.
2) 1,6278 Gr. bei 15,0° C. spec. Gew = 2,392. Aeq. Vol. = Mol. Vol. = 277,7.

Mittleres Aeqvivalent- und Molekularvolum = 277,1.

Thallion-Eisenoxyd-Schwefelsäure-Alaun.

Aeq. Gew. = 668,00 = Mol. Gew.

Aus schwefelsaurem Thallion und schwefelsaurem Eisenoxyd. Krystallisirte in klaren, durchsichtigen Krystallen, die nicht mit überschüssigem Thallionsalz gemengt waren.

1) 3,1367 Gr. bei 15° C. spec. Gew. = 2,351. Aeq. Vol. = Mol. Vol. = 284,1.

Aeqvivalent- und Molekularvolum = 284,1.

Thallion-Thonerde-Selensäure-Alaun.

Aeq. Gew. = 733.50 = Mol. Gew.

Aus selensaurem Thallion und selensaurer Thonerde. Durch Kochen einer schwefelsauren Thallion-Lösung mit reinem, durch Fällung bereiteten kohlensaurem Baryt, konnten wir kohlensaures Thallion nicht frei von Schwefelsäure bereiten [2]). Wir haben deshalb das gelöste schwefelsaure Thallion

[1]) Der Umstand, dass die verschiedenen Bestimmungen bei den Thallionalaunarten nicht so gut mit einander übereinstimmen, kommt ohne Zweifel davon her, dass diese Umsetzung in einzelnen Fällen nicht ganz vollständig gewesen.

[2]) Diese Methode ist in dem Lehrbuch d. Chem. von Graham Otto empfohlen.

mit reinem Barythydrat gekocht und in die abfiltrirte Lösung Kohlensäure eingeleitet. Nach dem Abfiltriren von dem ausgeschiedenen kohlensauren Baryt und Zusatz von Selensäure gab die Lösung Krystalle von selensaurem Thallion.

Der Alaun ist sehr leichtlöslich und sehr schwierig, von selensaurem Thallion rein zu erhalten Durch Umkrystallisation konnte dieser Übelstand nicht abgeholfen werden; denn versucht man, die Alaunkrystalle in wenig Wasser zu lösen, so scheidet sich immer selensaures Thallion aus. Wir bemerken deshalb ausdrücklich, dass die beiden folgenden Bestimmungen nicht zuverlässig sind.

1) 1,0809 Gr. bei 17,5° C. spec. Gew. = 2,492. Aeq. Vol. = Mol. Vol. = 294,3.
2) 1.4509 Gr. bei 17,0° C. spec. Gew. = 2,514. Aeq. Vol. = Mol. Vol = 291,7.

Mittleres Aqvivalent- und Molekularvolum = 293,0.

Thallion-Chromoxyd-Selensäure-Alaun.

Aeq. Gew. = 758,48 = Mol. Gew.

Aus selensaurem Thallion und selensaurem Chromoxyd. Sehr leichtlöslich. Die zu der Bestimmung angewandten Krystalle waren vollkommen klar und durchscheinend.

1) 1,7687 Gr. bei 20,0° C. spec. Gew. = 2,630. Aeq. Vol. = Mol. Vol. = 288,3.

Aeqvivalent- und Molekularvolum 288,3.

In der grossen Reihe der Alaune unterscheiden sich die einzelnen Glieder von einander nur durch zwei wesentliche Eigenschaften: verschiedene Raumerfüllung und verschiedene Löslichkeit in Wasser.

In der Reihe der Alaune mit Schwefelsäure und Thonerde ist die Löslichkeit am grössten bei den Alaunarten, die das kleinste Molekülarvolum haben (Natron-, Thallion-, Kalialaun) und nimmt mit steigendem Volum rasch ab.

In der Reihe der Chromoxyd-Schwefelsäure- und Eisenoxyd-Schwefelsäurealaune tritt diese Gesetzmässigkeit schon weit undeutlicher hervor, und in der Reihe der Selensäurealaunarten kann sie kaum mehr wahrgenommen werden.

§ 5.

SCHWEFELSAURE UND SELENSAURE SALZE DER ALKALIEN.

Da die Reindarstellung derselben schon bei den Alaunen ausführlich besprochen worden ist, brauchen wir hier nichts weiter darüber mitzutheilen.

Mit Ausnahme des schwefelsauren und selensauren Rubidions und Cæsions sind sie alle vorher hinsichtlich ihrer Raumerfüllung und ihrer krystallografischen Verhältnisse studirt worden. [1])

Die Bestimmungen beziehen sich nur auf die neutralen Salze der Alkalien.

Alle von uns untersuchten Salze waren vorher mehrmals umkrystallisirt worden

Schwefelsaures Kali.

Aeq. Gew. = 87,11. Mol. Gew. = 174,22.

1) 4,0446 Gr. 18,2° C. spec. Gew. = 2,667. Aeq. Vol. = 32,68. Mol. Vol. = 65,37.
2) 1.6974 Gr. 18,2° C. spec. Gew. = 2,669. Aeq Vol. = 32,63. Mol. Vol. = 65,27.
3) 3,4840 Gr. 17,1° C. spec. Gew = 2,660. Aeq. Vol. = 32,74 Mol. Vol. = 65,49.

Mittleres Aeqvivalentvolum = 32,68, *Molekularvolum* = 65,37.

Frühere Bestimmungen: 2,653 (v. Schiff); 2,631 (v. Karsten); 2,625 (v. Filhol.) u. A.

Selensaures Kali.

Aeq. Gew. = 110,61. Mol. Gew. = 221,22.

1) 1,2610 Gr. 19,0° C. spec. Gew. = 3,077. Aeq. Vol. = 35,95 = Mol. Vol. = 71,89.
2) 1,8880 Gr. 21,0° C. spec. Gew. = 3,077. Aeq. Vol. = 35,95 = Mol. Vol. = 71,89.
3) 3,6442 Gr. 18,0° C. spec. Gew. = 3,074. Aeq. Vol. = 35,98 = Mol. Vol. = 71,96.

Mittleres Aeqvivalent-Volum = 35,96. *Molekularvolum* = 71,91.

Früher bestimmt v. Topsoe 3,052.

Schwefelsaures Natron. (wasserfreies).

Aeq. Gew. = 71. Mol. Gew. = 142.

1) 1,6494 Gr. 17,0° C. spec. Gew. 2,677. Aeq. Vol. = 26,52. Mol. Vol. = 53,04.

Dargestellt durch Erhitzen von wasserhaltigem schwefelsaurem Natron bis zum gelinden Glühen.

2) 1,3291 Gr. 17,0° C. spec. Gew. 2,687. Aeq. Vol. = 26,41. Mol. Vol. = 52,83.

In derselben Weise bereitet wie die vorige Probe.

Mittleres Aeqvivalentvolum 26,46. *Molekularvolum* 52,93.

Frühere Bestimmungen: 2,46 (v. Mohs) 2,63 (v. Karsten).

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[1]) Siehe über die Dichten der Schwefelsäure-Salzen, Schiff: Über die specifischen Volume einiger Reihen anorganischer Verbindungen (Ann. d. Chem. u. Pharm. (1868) Bd. CVII s. 64 u. A.; über die Molekülarvolumina der selensauren Salze, Topsöe: Krystallografisk-kemiske Undersögelser, Kopenhagen 1872 seite 58.

Schwefelsaures Natron (wasserhaltiges)

Aeq. Gew. 161. Mol. Gew. 322.

Das schwefelsaure Natron mit 10 Aeq. Wasser wurde in grossen Krystallen erhalten, die jedoch ihr Wasser sehr leicht verloren und deshalb sehr schwierig zu trocknen und auspressen waren. Die Bestimmungen können deshalb nicht unbedingtes Vertrauen beanspruchen.

1) 1,7602 Gr. bei 20,0° C. spec. Gew. = 1,492. Aeq. Vol. = 107,9. Mol. Vol. = 215,8.
2) 2,3206 Gr. bei 19,0° C. spec. Gew. = 1,485. Aeq. Vol. = 108,4. Mol. Vol. = 216,8.

Mittleres Aeqvivalentvolum = 108,1. *Molekularvolum* = 216,3.

Frühere Bestimmung: 1,481 (wird von Topsoe gebraucht).

Selensaures Natron (wasserfreies).

Aeq. Gew. = 94,50. Mol. Gew. = 189,00.

Durch Erhitzen des wasserhaltigen Salzes dargestellt.

1) 2,2205 Gr. bei 17,2° C. spec. Gew. = 3,209. Aeq. Vol. = 29,45. Mol. Vol. = 58,89.
2) 1,9720 Gr. bei 17,6° C. spec. Gew. = 3,217. Aeq. Vol. = 29,35. Mol. Vol. = 58,71.

Mittleres Aeqvivalentvolum = 29,40. *Molekularvolum* = 58,80.

Selensaures Natron (wasserhaltiges).

Aeq. Gew. = 184,50. Mol. Gew. = 369,00.

Krystallisirt in grossen Krystallen, die schwierig ohne Wasserverlust zu trocknen sind. Die Bestimmungen gaben verschiedene Resultate, je nachdem sie mit grossen, unter Fliesspapier getrockneten Krystallsplittern oder mit zerriebenem und ausgepresstem Material gemacht wurden.

1) 1,6620 Gr. bei 18,6° C. spec. Gew. = 1,604. Aeq. Vol. = 115,0. Mol. Vol. = 230,0.
Krystallsplitter getrocknet, nicht ausgepresst.

2) 2,0000 Gr. bei 19,0° C. spec. Gew. = 1,603. Aeq. Vol. = 115,0. Mol. Vol. = 230,1.
Material dasselbe als bei der vorigen Bestimmung.

3) 1,7934 Gr. bei 18,0° C. spec. Gew. = 1,615. Aeq. Vol. = 114,2. Mol. Vol. = 228,4.
Ausgepresstes Pulver.

4) 1,5746 Gr. bei 18,0° C. spec. Gew. = 1,617. Aeq. Vol. = 114,0. Mol. Vol. = 228,0.
Pulverisirte Substanz.

5) 1,9390 Gr. bei 17,9° C. spec. Gew. = 1,621. Aeq. Vol. = 113,8. Mol. Vol. = 227,6.

Mittleres Aqvivalentvolum = 114,4. *Molekularvolum* = 228,8.

Frühere Bestimmung 1,584 (v. Topsoe.)

Schwefelsaures Ammoniumoxyd.

Aeq. Gew. = 66,00. Mol. Gew. = 132,00.

1) 1,5907 Gr. bei 18,6° C. spec. Gew. = 1,771. Aeq. Vol. = 37,26. Mol. Vol. = 74,52.
2) 2,1845 Gr. bei 18,3° C. spec. Gew. = 1,775. Aeq. Vol. = 37,18. Mol. Vol. = 74,37.

3) 2,5553 Gr. bei 17,9° C. spec. Gew. = 1,766. Aeq. Vol. = 37,35. Mol. Vol. = 74,71.
4) 0,6000 Gr. bei 18,5° C. spec. Gew. = 1,769. Aeq. Vol. = 37,29. Mol. Vol. = 74,58.
Mittleres Aeqvivalentvolum = 37,26. *Molekularvolum* = 74,56.
Frühere Bestimmungen: 1,628 (v. Schiff); 1,761 (v. Playfair u. Joule).

Selensaures Ammoniumoxyd.

Aeq. Gew. = 89,50. Mol. Gew. = 179,00.

1) 1,9188 Gr. bei 18,0° C. spec. Gew. = 2,197. Aeq. Vol. = 40,73. Mol. Vol. = 81,47.
2) 2,6376 Gr. bei 18,0° C. spec. Gew. = 2,197. Aeq. Vol. = 40,73. Mol. Vol. = 81,47.
3) 1,4916 Gr. bei 18,8° C. spec. Gew. = 2,198. Aeq. Vol. = 40,72. Mol. Vol. = 81,43.
Mittleres Aeqvivalentvolum = 40,72. *Molekularvolum* = 81,45.
Frühere Bestimmung: 2,162 — 2,192 (v. Topsoe).

Schwefelsaures Rubidion.

Aeq. Gew. = 133,40. Mol. Gew. = 266,80.

Die Darstellung des Salzes ist im Vorigen besprochen. Nur durch Glühen mit überschüssigem kohlensaurem Ammon gelang es, aus dem sauren schwefelsauren Rubidion neutrales Salz darzustellen.

1) 1,9929 Gr. bei 16,8 C spec. Gew. = 3,639. Aeq. Vol. = 36.65. Mol. Vol. = 73,30.
2) 1,1628 Gr. bei 16,8° C. spec. Gew. = 3,641. Aeq. Vol. = 36,62. Mol. Vol. = 73,24.
Mittleres Aeqvivalentvolum = 36,63. *Molekularvolum* = 73,27.

Selensaures Rubidion.

Aeq. Gew. = 156,90. Mol. Gew. = 313,80.

Eine Lösung von reinem Chlorrubidium, dessen Bereitung in dem Vorigen beschrieben ist, wurde durch Schütteln mit selensaurem Silberoxyd zersetzt und die abfiltrirte Lösung von selensaurem Rubidion zur Krystallisation eingedampft.

1) 1,4954 Gr. bei 19,8° C. spec. Gew. = 3,943. Aeq. Vol. = 39,78. Mol. Vol, = 79,56.
2) 1,4185 Gr. bei 18,6° C. spec. Gew. = 3,896. Aeq. Vol. = 40,26. Mol. Vol. = 80,53.
3) 1,6970 Gr. bei 18,0° C. spec. Gew. = 3,926. Aeq. Vol. = 39,96. Mol. Vol. = 79,93.
4) 1,5884 Gr. bei 18,0° C. spec. Gew. = 3.938. Aeq. Vol. = 39,84. Mol. Vol. = 79,68.
2) 1,7698 Gr. bei 19,0° C. spec. Gew. = 3,914. Aeq. Vol. = 40,09. Mol. Vol. = 80,18.
Mittleres Aeqvivalentvolum = 39,99. *Molekularvolum* = 79,97.

Schwefelsaures Cæsion.

Aeq. Gew. = 181,00 Mol. Gew. = 362,00.

Das schwefelsaure Cæsion war nicht vollkommen rein, denn bei der spektroskopischen Prüfung zeigte sich auch eine schwache Rubidium-Linie.

Die ausserordentlich geringe Menge des Materials machte es unmöglich, dasselbe vorher zu reinigen und gestattete nur eine Bestimmung des specifischen Gewichtes. Diese ist doch wichtig, weil sie annähernd den Platz erkennen lässt, den das schwefelsaure Cœsion in der Reihe der Molekülarvolumina der analogen Alkalisalze einnimmt.

1) 1,0925 Gr. bei 19,2° C. spec. Gew. = 4,105. Aeq. Vol. = 44,09. Mol. Vol. = 88,18.

Aeqvivalentvolum = 44,09. *Molekularvolum* = 88,18.

Schwefelsaures Lithion. (wasserhaltiges.)

Aeq. Gew. = 64,00 Mol. Gew. = 128,00.

Aus kohlensaurem Lithion, das spektralanalytisch auf seine Reinheit geprüft war, und Schwefelsäure.

1) 2,8031 Gr. bei 20,0° C. spec. Gew. = 2,056. Aeq. Vol. = 31,12. Mol. Vol. = 62,25.
2) 1,5869 Gr. bei 20,0° C. spec. Gew. = 2,066. Aeq. Vol. = 30,96. Mol. Vol. = 61,92.
3) 2,7274 Gr. bei 21,0° C. spec. Gew. = 2,052. Aeq. Vol. = 31,13. Mol. Vol. = 62,37.

Mittleres Aeqvivalentvolum = 31,07. *Molekularvolum* = 62,14.

Frühere Bestimmung: 2,020 (liegt dem von Topsoe berechneten Mol. Vol. zu Grunde.)

Beim Erhitzen zu 100° C. verlor das Salz genau 1 Aeq. Wasser [1]) entsprechend 14,07 %. Nach Rechnung sollte es 14,06 % verlieren. Das specifische Gewicht des wasserfreien schwefelsauren Lithions wurde gefunden:

1) 2,0217 Gr. bei 17,2° C. spec Gew. = 2,248. Aeq. Vol. = 24.46. Mol. Vol. = 48,92.
2) 1,5045 Gr. bei 17,2° C. spec. Gew. = 2,247. Aeq. Vol. = 24,47. Mol. Vol. = 48,95.

Mittleres Aeqvivalentvolum = 24,46. *Molekularvolum* = 48,93.

Selensaures Lithion (wasserhaltiges).

Aeq. Gew. = 87,50. Mol. Gew. = 175,00.

Aus kohlensaurem Lithion und Selensäure.

1) 2,3484 Gr. bei 19,5° C. spec. Gew. = 2,565. Aeq. Vol. = 34,13. Mol. Vol. = 68,26.
2) 2,3527 Gr. bei 18,0° C. spec. Gew. = 2,564. Aeq. Vol. = 34,14. Mol. Vol. = 68,28.

Mittleres Aeqvivalentvolum = 34,13 *Molekularvolum* = 68,27.

Frühere Bestimmung: 2,439 (v. Topsoe gebraucht).

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§. 6.

SCHWEFELSAURES EISENOXYD UND ALUMINIUMOXYD.

Schwefelsaures Eisenoxyd.

Aeq. Gew.: $Fe_2O_3 3SO_3 = 200$. Mol. Gew: $Fe_2 3SO_4 = 400$.

Dargestellt durch Erhitzen von wasserhaltigem schwefelsauren Eisenoxyd bis zum Abdampfen der überschüssigen Schwefelsäure. Es war ein

[1]) Herr Carleson hat gefälligst diese Wasserbestimmung ausgeführt.

gelblich weisses Pulver, das sich sehr langsam aber vollständig in Wasser löste. Eine Analyse an 1,0560 Gr. des Salzes ergab:

	gefunden	berechnet
Fe_2O_3	39,96 %	40,00 %
SO_3	60,24 %	60,00 %
	100,20.	100,00.

Es war also vollkommen neutral zusammengesetzt [1]).

1) 2,3560 Gr. bei 18,2° C. spec. Gew. = 3,103. Aeq. Vol. = 64,45. Mol. Vol. = 128,90.
2) 2,5283 Gr. bei 18,0° C. spec. Gew. = 3,097. Aeq. Vol. = 64,38. Mol. Vol. = 128,76.
3) 2,5800 Gr. bei 18,5° C. spec. Gew. = 3,098. Aeq. Vol. = 64,37. Mol. Vol. = 128,73.

Mittleres Aeqvivalentvolum = 64,4. *Molekularvolum* = 128,79.

Schwefelsaure Thonerde.

Aeq. Gew, $Al_2O_3\,3SO_3$ = 171,5. Mol. Gew: $Al_2\,3SO_4$ = 343,0.

Dargestellt durch vorsichtiges Erhitzen von Thonerdehydrat mit Schwefelsäure bis zum Vertreiben des Überschusses derselben. Löste sich vollständig in Wasser. Die Analyse zeigte, dass das Salz vollkommen neutral erhalten war.

1) 1,7725 Gr. bei 17,0° C. spec. Gew. = 2,710. Aeq. Vol. = 63,28. Mol. Vol. = 126,46.
2) 1,6763 Gr. bei 17,0° C. spec. Gew. = 2,716. Aeq. Vol. = 63,15. Mol. Vol. = 126,30.

Mittleres Aeqvivalentvolum = 63,21. *Molekularvolum* = 126,38.

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§ 7.

SCHWEFELSAURE UND SELENSAURE SALZE SCHWERER METALLE.

Schwefelsaures Silberoxyd.

Aeq. Gew. = 156. Mol. Gew. = 312.

Aus reinem salpetersaurem Silberoxyd und Schwefelsäure. Im Dunkeln längere Zeit bei 100° C. getrocknet.

1) 3,8650 Gr. bei 18,0° C. spec. Gew. = 5,49. Aeq. Vol. = 28,4. Mol. Vol. = 56,8.
2) 2,2975 Gr. bei 18,0° C. spec. Gew. = 5,54. Aeq. Vol. = 28,1. Mol. Vol. = 56,3.

Mittleres Aeqvivalentvolum = 28,3. *Molekularvolum* = 56,6.

Frühere Bestimmung: 5,34 (v. Karsten). 5,41 (v. Filhol).

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[1]) Die Ausführung dieser Analyse verdanken wir Herrn Kandidaten Santesson.

Selensaures Silberoxyd.

Aeq. Gew. = 179,5. Mol. Gew. = 359,0.

Aus salpetersaurem Silberoxyd und Selensäure. Der krystallinische Niederschlag wurde sorgfältig ausgewaschen und im Dunkeln lange bei 100° C. getrocknet.

1) 1,9087 Gr. bei 17,0° C. spec. Gew. = 5,93. Aeq. Vol. = 30,2. Mol. Vol. = 60,5.
2) 2,1362 Gr. bei $17{,}2_0$ C. spec. Gew. = 5,92. Aeq. Vol. = 30,3. Mol. Vol. = 60,4.

Mittleres Aeqvivalentvolum = 30,2. *Molekularvolum* = 60,5.

Schwefelsaures Thallion.

Aeq. Gew. = 252,0. Mol. Gew. = 504,0.

Die Darstellung desselben ist in dem Vorigen besprochen.

1) 3,3269 Gr. bei 16,8° C. spec. Gew. = 6,79. Aeq. Vol. = 37,0. Mol. Vol. = 74,1.
2) 3,4572 Gr. bei 17,2° C. spec. Gew. = 6,81. Aeq. Vol. = 37,0. Mol. Vol. = 74,0.
3) 3,4761 Gr. bei 17,0° C. spec. Gew. = 6,83. Aeq. Vol. = 36,8. Mol. Vol. = 73,7.

Mittleres Aeqvivalentvolum = 36,9 *Molekularvolum* = 73,8.

Selensaures Thallion..

Aeq. Gew. = 275,5. Mol. Gew. = 551,0.

Die Darstellung des Salzes ist bei den Alaunarten eingehend beschrieben.

1) 2,5450 Gr. bei 18,0° C. spec. Gew. = 7,019. Aeq. Vol. = 39,2. Mol Vol. = 78,4.
2) 2,7905 Gr. bei 18,2° C. spec. Gew. = 7,067. Aeq. Vol. = 38,9 Mol. Vol. = 77,9.

Mittleres Aeqvivalentvolum = 39,1 *Molekularvolum* = 78,2.

Schwefelsaures Bleioxyd.

Aeq. Gew. 151,5. Mol. Gew. = 303,0.

Bereitet durch Fällen von mehrmals umkristallisirtem salpetersauren Bleioxyd mit Schwefelsäure, Auswaschen und Trocknen des Niederschlages lange Zeit bei 100° C.

1) 4,1857 Gr. bei 16,8° C. spec. Gew. = 5,97. Aeq. Vol. = 25,3. Mol. Vol. = 50,7.
2) 2,0038 Gr. bei 17,1° C. spec. GeO. = 5,96. Aeq. Vol. = 25,3. Mol. Vol. = 50,7.

Mittleres Aeqvivalentvolum = 25,3. *Molekularvolum* = 50,7.

Frühere Bestimmungen: 6,30 (v. Filhol) 6,17 (v. Karsten.)

Selensaures Bleioxyd.

Aeq. Gew. = 175,0. Mol. Gew. = 350,0.

In vollkommen analoger Weise bereitet, wie das Schwefelsäuresalz.

1) 1,8729 Gr. bei 18,0° C. spec. Gew. = 6,22. Aeq. Vol. = 28,1. Mol. Vol. = 56,2.
2) 2,4174 Gr. bei 18,2° C. spec. Gew. = 6,23. Aeq. Vol. = 28,1. Mol. Vol. = 56,2.

Mittleres Aeqvivalentvolum = 28,1. *Molekularvolum* = 56,2.

Schwefelsaures Kupferoxyd (wasserhaltiges)

Aeq. Gew.: $CuOSO_3+5HO=127{,}70$. Mol. Gew.: $CuSO_4+5H_2O=249{,}4$.

Galvanisch ausgefälltes Kupfer wurde in Schwefelsäure gelöst und das Salz mehrmals umkrystallisirt.

1) 2,9688 Gr. bei 20,0° C. spec. Gew. = 2,292. Aeq. Vol. = 54,3. Mol. Vol. = 108,7.
2) 2,8570 Gr. bei 19,4° C spec. Gew. = 2,286. Aeq. Vol. = 54,5. Mol. Vol. = 109,0.
Grosse Krystalle.
3) 2,8813 Gr. bei 19,8° C. spec. Gew. = 2,286. Aeq. Vol. = 54,5. Mol. Vol = 109,0.
Mittleres Aeqvivalentvolum = 54,4. *Molekularvolum* = 108,9.
Frühere Bestimmungen: 2,286 (v. Filhol) 2,274 (v. Kopp.) u. s. w.

10,6300 Gr. von diesem Salz wurden bei 100° erhitzt, bis das Gewicht konstant blieb und verloren dabei 3,0544 Gr. Wasser = 28,92 %. Der berechnete Verlust (4 Aeq.) ist = 28.83 %. Von dem so getrockneten Salz gab:

1) 2,3937 Gr. bei 18,0° C. spec. Gew. = 3,246. Aeq. Vol. = 27,3. Mol. Vol. = 54,6.
2) 2,9753 Gr. bei 18,1° C. spec. Gew. = 3,239. Aeq. Vol. = 27,4. Mol. Vol. = 54,7.
3) 2,1910 Gr. bei 17,2° C. spec. Gew. = 3,235. Aeq. Vol. = 27,4. Mol. Vol. = 54,8.
Mittleres Aeqvivalentvolum = 27,4. *Molekularvolum* = 54,7.

Durch länger getriebenes Erhitzen verlor das schwefelsaure Kupferoxyd auch das letzte Aeq. Wasser. Völlig wasserfreies Salz gab:

1) 2,3100 Gr. bei 17,1° C. spec. Gew. = 3,82. Aeq. Vol. = 20,8. Mol. Vol. = 41,6.
2) 1,7432 Gr. bei 18,0° C. spec. Gew. = 3,83. Aeq. Vol. = 20,7. Mol. Vol. = 41,5.
Mittleres Aeqvivalentvolum = 20,7. *Molekularvolum* 41,5.
Frühere Bestimmungen: 3,57 (v. Karsten) u. s. w.

Selensaures Kupferoxyd (wasserhaltiges).

Aeq. Gew.: $CuOSeO_3+5HO$ = 148,4. Mol. Gew.: $CuSeO_4+5H_2O$ = 296,8.

Kohlensaures Kupferoxyd, dargestellt aus dem reinen schwefelsauren Salz wurde in Selensäure gelöst und das entstandene selensaure Kupferoxyd mehrmals umkrystallisirt.

1) 3,3737 Gr. bei 19 2° C. spec. Gew. = 2,561. Aeq Vol. = 57,9. Mol. Vol. = 115,8.
2) 3,0300 Gr. bei 17,8° C. spec. Gew. = 2,562. Aeq. Vol. = 57,9. Mol. Vol. = 115,8.
Aeqvivalentvolum = 57,9. *Molekularvolum* = 115,8.
Frühere Bestimmung: 2,559 (v. Topsoe).

§ 8.

PLATINDOPPELSALZE.

Wir haben nicht das Platin von Iridium u. s. w. zu reinigen versucht. Die gefundenen specifischen Gewichte der Doppelchloride können deshalb unrichtig sein, da aber dasselbe Platinchlorid zu der Darstellung von allen gedient hat, müssen wenigstens die Bestimmungen unter einander vergleichbar sein.

Chlorkalium Platinchlorid.

1) 1,4507 Gr. bei 17,2° C. spec. Gew. = 3,32. Aeq. Vol. = 73,6. Mol. Vol. = 147,2.
2) 1,5814 Gr. bei 17,0° C. spec. Gew. = 3,30. Aeq. Vol. = 74,0. Mol. Vol. = 148,0.

Aeqvivalentvolum $[KaCl + PtCl_2] = 73{,}8$. *Molekularvolum* $[Ka_2PtCl_6] = 147{,}6$.

Chlorammonium Platinchlorid.

2,5742 Gr. bei 17,2° C. spec. Gew. = 3,00. *Aeqvivalentvolum* $[AmCl + PtCl_2] = 74\,4$. *Molekularvolum* $[Am_2PtCl_6] = 148{,}8$.

Chlorrubidium Platinchlorid.

1) 2,3360 Gr. bei 17,5° C. spec. Gew. = 3,94. Aeq. Vol. = 73,7. Mol. Vol. = 147,4.
2) 1,3377 Gr. bei 17,4° C. spec. Gew. = 3,96. Aeq. Vol. = 73,4. Mol. Vol. = 146,8.

Aeqvivalentvolum $[RbCl + PtCl_2] = 73{,}5$. *Molekularvolum* $[Rb_2PtCl_6] = 147{,}0$.

Chlorthallium Platinchlorid.

1) 2,6814 Gr. bei 17,0° C. spec. Gew. = 5,76. *Aeqvivalentvolum* $[TlCl + PtCl_2] = 71{,}0$. *Molekularvolum* $[Tl_2PtCl_6] = 142{,}0$.

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§ 9.

ÜBERSICHT.

Man hat früher die Ansicht gehabt, dass anerkannt isomorphen Körpern auch annähernd dasselbe Atom- oder Molekülarvolum zukommen müsste. KOPP und DUMAS haben die Aufmerksamkeit darauf gelenkt, dass dies sehr oft bei den einfachen Körpern der Fall ist. SCHRÖDER hat diese Ansicht auch bei den zusammengesetzten Verbindungen durchzuführen versucht. Die Erfahrung hat aber gezeigt, dass gleiche Raumerfüllung keine unerlässliche

Bedingung für die Isomorphie der Körper ist. Innerhalb des Begriffes des Isomorphismus musste man den Begriff von Isostärismus aufstellen und bei isomorphen Körpern unterscheiden, ob sie gleiches Atom- oder Molekülarvolum haben oder nicht.

Topsoe hat gezeigt, dass die schwefelsauren und selensauren Salze, obgleich isomorph, doch nicht isostär sind, sondern dass die Salze der Selensäure durchgehend ein grösseres Molekülarvolum haben als die schwefelsauren.

Wir haben uns bei unsren Untersuchungen über die Raumerfüllung der Körper mit Vorliebe bei den Alaunen aufgehalten, weil diese eine grosse Reihe von Verbindungen darstellen, die alle vollkommen isomorph sind und dem regulären Krystallsystem angehören, worin keine Variationen der Axenverhältnisse vorkommen können (die bekanntlich bei isomorphen Salzen der übrigen Systeme etwas mit der chemischen Zusammensetzung variiren). Und da zugleich von einer jeden Theorie die Zusammensetzung aller Alaunarten durch eine und dieselbe allgemeine Formel ausgedrückt wird, weil bei dem jetzigen Zustand der Wissenschaft kein Grund vorhanden ist, die Lagerung der Atomen in einem von diesen Salzen in verschiedener Weise aufzufassen als in den übrigen, kann man vermuthen, dass die relative Raumerfüllung, die jedem Elemente zukommt, bei dem Aufbau der Moleküle dieser Salze, wo sonst alle Umstände gleich sind, ihren Einfluss mehr ungestört äussern muss, und dass Regelmässigkeiten zwischen den Molekülarvolumina der einzelnen Glieder der Reihe, wenn solche existiren, hier weniger verschleiert hervortreten werden, als sonst.

Es existiren in der That solche Regelmässigkeiten, und wir werden sie in der Weise darzulegen versuchen, dass wir einfach die Mittelwerthe der im Vorigen angeführten Bestimmungen zusammenstellen und die Aufmerksamkeit auf die Differenzen der Molekülarvolumina der verschiedenen Salze lenken.

Als Ausgangspunkt für die vergleichende Betrachtung dieser Differenzen wählen wir unter den Alaunarten das Aeqvivalent- oder Molekülarvolum des schwefelsauren Kali-Thonerdealauns, unter den schwefelsauren und selensauren Salzen das Aeqvivalentvolum des schwefelsauren Kalis. Wir wollen uns bei dieser Zusammenstellung der gefundenen Aeqvivalentvolumzahlen bedienen, weil das Ganze dadurch leichter übersichtlich wird. Bei den Alaunen sind die Zahlen der Aeqvivalent- und Molekülarvolumina gleich, bei den übrigen Salzen braucht man nur die ersteren zu verdoppeln, um die Molekülarvolumina zu erhalten.

Diff. von dem Volum des Kalialauns.	Schwefelsäurereihe.	Diff.	Selensäurereihe.	Diff. von dem Volum des Kalialauns.
	$KaOAl_2O_34SO_3+24HO$	13,2	$KaOAl_2O_34SeO_3+24HO$	
	270,8.		284,0.	
0,3	$KaOCr_2O_34SO_3+24HO$	14,4	$KaOCr_2O_34SeO_3+24HO$	1,5
	271,1.		285,5.	
— 1,0	$TlOAl_2O_34SO_3+24HO$			
	269,8.			
0,3	$NaOAl_2O_34SO_3+24HO$			
	271,1.			
4,9	$KaOFe_2O_34SO_3+24HO$			
	275,7.			
5,3	$AmOAl_2O_34SO_3+24HO$	13,2	$AmOAl_2O_34SeO_3+24HO$	5.3
	276,1.		289,3.	
6,0	$AmOCr_2O_34SO_3+24HO$	11,9	$AmOCr_2O_34SeO_3+24HO$	4,7
	276,8.		288,7.	
4,7	$RbOAl_2O_34SO_3+24HO$	12,6	$RbOAl_2O_34SeO_3+24HO$	4,1
	275,5.		288,1.	
6.5	$RbOCr_2O_34SO_3+24HO$	11,1	$RbOCr_2O_34SeO_3+24HO$	4,4
	277,3.		288,4.	
6,5	$TlOCr_2O_34SO_3+24HO$	11,2	$TlOCr_2O_34SeO_3+24HO$	4,3
	277,1.		288,3.	
8,9	$AmOFe_2O_34SO_3+24HO$			
	279,7.			
			$TlOAl_2O_34SeO_3+24HO$	9,0
			293,0.	
13,3	$TlOFe_2O_34SO_3+24HO$			
	284,1.			
13,8	$CsOAl_2O_34SO_3+24HO$	13,2	$CsOAl_2O_34SeO_3+24HO$	13.8
	284,6.		297,8.	

Der Natron- Thonerde-Selensäurealaun ist eine sehr bemerkenswerthe Ausnahme nicht nur von den Alaunarten sondern auch von der ganzen Reihe der selensauren Salze, indem das Aeqvivalentvolum desselben (267,2.) *kleiner* ist als das Volum der entsprechenden Schwefelsäureverbindung. Dass das Volum des selensauren Thallion-Thonerdealauns (dessen Bestimmnng jedoch nicht zuverlässig ist) auch ein anomales ist, geht aus der Tabelle deutlich hervor.

Die Aeqvivalentvolumina der Alaune können unmöglich genau bestimmt werden, weil das Aeqvivalentgewicht dieser Salze so gross ist: (Es variirt zwischen 458 und 758). Wir wollen jetzt nachsehen, welchen Einfluss die in dem Vorigen besprochenen unvermeidlichen Variationen in den Bestimmungen des specifischen Gewichtes der Alaunarten auf die Aeqvivalentvolumina derselben ausüben können. Es sei M das Aeqvivalentgew. s das specif. Gew. V das Aeqv. Volum.

$$V = \frac{M}{s}$$

$$dV = -\frac{M}{s_2} ds$$

Wir haben oben gesehen, dass die einzelnen Bestimmungen des specifischen Gewichtes bei einer und derselben Alaunart, auch wenn sie tadelfrei ausgeführt sind, gewöhnlich nicht mehr als in der zweiten Decimalstelle übereinstimmen, und also bis um 0,01 (gewöhnlich 0,6 bis 0,3 Procent) von einander abweichen, dass aber diese Grenze sehr selten überschritten wird. Angenommen, der Werth von s hätte um eben diese Grösse variirt, so würde die Variation in dem daraus berechneten Aeqvivalentvolum: dV bei dem Kali-Thonerde-Schwefelsäurealaun 1,54 betragen bei dem Rubidion-Chromoxyd-Selensäurealaun 1,29 u. s. w.

Es sind also bei den meisten Alaunarten die Aeqvivalentvolumina bis auf anderthalb Einheiten unsicher wegen der unvermeidlichen Messungsfehler. Bei den Differenzen zwischen den Aeqvivalentvolumzahlen *kann* der Fehler natürlich doppelt so viel, also ungefähr 3 Einheiten betragen. Mit Berücksichtigung dieses Verhältnisses wollen wir folgende Schlussfolgerungen aufstellen:

Die Alaunarten der Selensäure haben (Natronalaun ausgenommen) *durchgehend grössere Aeqvivalentvolumina als die schwefelsauren, und die Differenzen der analogen Glieder dieser Reihen variiren von 11,1 bis 14,4* (ausgenommen der Thallion-Thonerdealaun).

Es kann möglich sein, dass eine wirkliche Gesetzmässigkeit diesem Verhalten zu Grunde liegt (so, dass jeder Alaun der Selensäure um eine gleich grosse Quantität grösser ist als das entsprechende Schwefelsäuresalz), und dass sie durch die Variationen, die von den Messungsfehlern abhängen, verdeckt wird.

Dies wird wahrscheinlich dadurch, dass eben bei den am besten untersuchten Alaunen (wie der Kali-, Ammoniumoxyd-, Rubidion- und Cæsion-Thonerdealaun), wo die Resultate Mittelzahlen aus vielen Versuchen sind [1]), die Differenzen sehr scharf übereinstimmen und etwa = 13,0 sind.

Die Chromoxydalaune scheinen annähernd dasselbe Aeqvivalentvolum zu haben, wie die Thonerdealaune, wenigstens sind die Differenzen dieser Salze so klein, dass sie innerhalb der Fehlergrenzen fallen.

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[1]) Dagegen haben wir z. B. bei dem TlCrSelensäurealaun uns mit einer einzigen Bestimmung begnügen müssen u. s. w.

Dagegen haben die Eisenoxydalaune sicher ein grösseres Aeqvivalentvolum als die analogen Verbindungen von Thonerde und Chromoxyd, und zwar scheint dieser Unterschied ungefähr 4,3 zu sein bei den Kali- und Ammoniumoxydverbindungen und etwa 3 mal so gross 13,3 bei dem Thallionalaun.

Mit Ausnahme des Natron-Selensäurealauns *liegen die Aeqvivalentvolumina sämmtlicher von uns untersuchten Alaunarten zwischen den Zahlen 270 und 298. Innerhalb dieser Grenzen liegen die Aeqvivalentvolumina der schwefelsauren Alaunarten zwischen 270 und 285, die der selensauren zwischen 284 und 298. Die letzten Glieder der Schwefelsäurereihe* (mit dem grössten Aeqvivalentvolum) *und die ersten Glieder der Selensäurereihe haben also dieselbe Raumerfüllung.*

Innerhalb dieser Zahlengrenzen zeigt sich wieder eine Gesetzmässigkeit. Bei dem Betrachten der Reihe der Selensäurealaune findet man, dass die Volumina des Kali-Thonerde- und Kali-Chromoxydalauns nicht viel verschieden sind, dass ferner die Volumina der Ammoniumoxyd-Thonerde- und Chromoxyd-, der Rubidion-Thonerde- und Chromoxydalaune um einen Zahlenwerth schwanken, dessen Differenz von dem Aeqvivalentvolumzahl des Kalialauns im Mittel 4,6 ist. Dieselbe Differenz beträgt bei dem folgenden Glied der Reihe, dem Thallion-Thonerdealaun 9,0 also doppelt so viel, und ist bei dem letzten Glied, dem Cœsion-Thonerdealaun = 13,3 also fast dreimal so gross. Ganz dasselbe Verhältniss findet in der Reihe der schwefelsauren Alaune statt, nur ist hier die Gesetzmässigkeit der ersten Differenz durch die Versuchsfehler sehr undeutlich

Die Aeqvivalentvolumina der Alaunarten stimmen nicht mit irgend einer beliebigen Zahl zwischen den genannten Grenzen 270—298, sondern nur mit gewissen bestimmten Werthen, die im Durchsnitt 4 bis 5 Einheiten von einander entfernt sind.

Diff. von d. Kalisalz.	Schwefelsäure Reihe.	Diff.	Selensäure Reihe.	Diff. von d. Kalisalz.
	$KaSO_4$	3,3	$KaSeO_4$	
	32,6.		35,9.	
4.6	$AmSO_4$	3,5	$AmSeO_4$	4,8
	37,2.		40,7.	
4,0	$RbSO_4$	3,4	$RbSeO_4$	4,1
	36,6.		40,0.	
11,5	$CsSO_4$ [1])			
	44,1.			

[1]) Das Cæsiumsalz war nicht rein von Rubidium, und wir haben an demselben nur eine Dichtigkeitsbestimmung vornehmen können. Die Bestimmungen der übrigen Salze sind sehr zuverlässig.

Wir sehen, dass bei diesen Verbindungen die Differenz zwischen den analogen schwefelsauren und selensauren Salzen annähernd dieselbe und ungefähr = 3,4 ist.

Es ist auffallend, dass diese Differenz 3,4 bei den einfachen schwefelsauren und selensauren Salzen von Ka, Am, Rb fast genau gleich ist dem vierten Theil der Differenz 13, die wir für die Aeqvivalentvolumina der schwefelsauren und selensauren Thonerdealaune derselben Metalle (Ka, Am, Rb) festgestellt haben, und ferner, dass die Differenzen, die nach der letzten Tabelle zwischen dem schwefelsauren Kali einerseits und dem schwefelsauren Ammonium- Rubidium- und Cæsium-oxyd andererseits existiren, und die sich auch in der Selensäure-reihe wiederfinden, sehr annähernd mit den analogen Differenzen nach der vorigen Tabelle zwischen den Thonerdealaunarten von Ka, Am, Rb, Cs übereinstimmen.

Man kann zwei verschiedene Ansichten über die Konstitution der Alaunarten hegen. Entweder kann man sie als wirkliche Doppelsalze (molekuläre Verbindungen von zwei Salzen) betrachten, oder auch als isomorphe Atomkomplexe, worin eine Substitution der Bestandtheile durch verwandte Elemente statt finden kann.

Beide Anschauungen haben wir in dem Vorigen zu würdigen versucht. Als Ausdruck der ersteren dient die Aeqvivalentformel: $KaO\,SO_3 + Al_2O_3\,3SO_3 + 24HO$, die letztere wird durch die Molekülarformel $\left.\begin{matrix}Ka\\Al\end{matrix}\right\} 2SO_4 + 12H_2O$ repräsentirt. Welche von beiden die wahrscheinlichste ist, wollen wir nicht zu entscheiden versuchen; die Volumtheorie wird vielleicht einst solche Probleme lösen können, jetzt wäre es viel zu früh, darüber eine bestimmte Meinung zu äussern. Nur die folgende Andeutung mag genügen.

Es ist unzweifelhaft, dass zwischen vielen von den Alaunarten Differenzen der Aeqvivalentvolumina existiren, die sich auch bei den schwefelsauren und selensauren Salzen wiederfinden, und zwar werden diese sehr leicht begreiflich, wenn man die Alaune als Doppelsalze auffasst und die Formeln derselben in der ersten, oder analoger Weise schreibt. Geht man aber von der letzteren Ansicht aus, so drängt sich zuerst die Frage auf: wenn wir in irgend einer Verbindung ein Atom Kalium durch Am, Rb oder Cs ersetzen, wird dann das Volum der Verbindung in derselben Weise zuwachsen, wie es bei der Reihe der Alaunarten und der einfachen schwefelsauren und selensauren Salzen der Fall war? Wir nehmen zum Vergleich *die Volumina der Platindoppelsalze. Die in dem Vorigen angeführten Bestimmungen zeigen, dass die genannten Regelmässigkeiten sich hier nicht wiederfinden.*

Die Raumerfüllung des Wassers in den Salzen werden wir in dem zweiten Theil dieser Arbeit eingehend behandeln, hier wollen wir nur einige Resultate zusammenstellen, die ausschliesslich von der Reihe der Schwefelsäuresalze geholt sind.

$LiOSO_3 + HO$ Aeq. Vol. = 31,1		
$LiOSO_3$ Aeq. Vol. = 24,4	Diff. = 6,7.	
$CuOSO_3 + 5HO$ Aeq. Vol. = 54,4		
$CuOSO_3 + HO$ Aeq. Vol. = 27,4	Diff. = 27,0 = 6,7 × 4.	
$CuOSO_3$ Aeq. Vol. = 20,7	Diff. = 6,7.	
$AmOSO_3 + Fe_2O_3 3SO_3 + 24HO$ Aeq. Vol. = 279,7		Diff. = 175,0 = 24 × 7,3.
$AmOSO_3 + Fe_2O_3 3SO_3$ Aeq. Vol. = 104,7		
$TlOSO_3 + Al_2O_3 3SO_3 + 24HO$ Aeq. Vol. = 269,8		Diff. = 142,9 = 20 × 7,1.
$TlOSO_3 + Al_2O_3 3SO_3 + 4HO$ Aeq. Vol. = 126,9		

Es geht aus dieser Übersicht hervor, dass das Krystallwasser in mehreren schwefelsauren Salzen annähernd dasselbe Volum einnimmt. Es ist auch sehr bemerkenswerth, dass das Aeqvivalent-Volum des entwässerten Ammoniumoxyd-Eisenoxyd-Schwefelsäurealauns fast gleich ist der Summe der Aeqvivalentvolumina des schwefelsauren Ammoniumoxyds und des schwefelsauren Eisenoxyds.

$AmOSO_3 + Fe_2O_3 3SO_3$:	$= AmOSO_3 + Fe_2O_3 3SO_3$
Aeq. Vol. : 104,7	= Aeq. Vol. : 37,3 + Aeq. Vol. : 64,4.

Diess lässt sich zwar so erklären, dass durch den Wasserverlust das Molekül des Alauns zerstört, und einer Umsetzung in schwefelsaures Eisenoxyd und schwefelsaures Ammoniumoxyd unterworfen worden ist; aber auch bei dem Thallionalaun lässt sich das Volum der Verbindung $TlOSO_3 + Al_2O_3 3SO_3 + 4HO$ (= 126,9, annähernd (allerdings nicht genau) aus den Volumina des schwefelsauren Thallions (= 36,9), der schwefelsauren Thonerde (= 63,2) des Wassers (zu 7,1 angenommen) zusammensetzen. Hier kann dass Gebäude des Alaun-Moleküls noch nicht gesprengt und die Bestandtheile als frei existirend gedacht werden, denn das Wasser ist noch darin, nicht mit seinem ursprünglichen Volum (= 9), sondern in *annähernd demselben Zustand der Kondensation (Volum = 7,1) vorhanden, als in dem ursprünglichen Alaun.*

Wir theilen zuletzt hier eine Übersicht der Aeqvivalentvolumina der wichtigsten von den übrigen von uns untersuchten schwefelsauren und selensauren Salze mit.

Schwefelsäure reihe.	Diff.	Selensäure reihe.
$NaOSO_3$	3,0	$NaOSeO_3$
26,4.		29,4.
$LiOSO_3+HO$	3,0	$LiOSeO_3+HO$
31,1.		34,1.
$AgOSO_3$	2,0	$AgOSeO_3$
28,2.		30,2.
$TlOSO_3$	2,2	$TlOSeO_3$
36,9.		39,1.
$PbOSO_3$	2,8	$PbOSeO_3$
25,3.		28,1.
$CuOSO_3+5HO$	3,5	$CuOSeO_3+5HO$
54,4.		57,9.

D'UNE FONCTION TRANSCENDENTE.

PAR

C. F. LINDMAN.

(Présenté à la Société Royale des Sciences d'Upsal le 14 Février 1874).

UPSAL,
ED. BERLING, IMPRIMEUR DE L'UNIVERSITÉ
1874.

J'avais l'honneur, il y a près de vingt ans, de présenter à la Société royale un traité inscrit "de functione quadam transcendente", qui reçut place dans le premier volume des Nouv. Actes, imprimé en 1855 à Upsal. Depuis ce temps je me suis plusieurs fois occupé de cette même fonction et j'y ai trouvé de nouvelles propriétés, ainsi que plusieurs intégrales qui en dépendent. J'ai aussi eu l'occasion d'employer" Tables d'intégrales définies "par BIERENS DE HAAN (Amsterdam 1858) et" Exposé de la théorie, des propriétés etc. des intégrales définies" (Amsterdam 1862) du même auteur; deux ouvrages excellents qui m'ont fourni beaucoup d'éclaircissements et ainsi m'ont été d'une grande utilité. Du premier de ces ouvrages j'ai aussi appris que LOBATSCHEWSKY, géomètre Russe, a désigné par un signe particulier une fonction transcendente qui a beaucoup de rapport avec celle que j'ai proposé d'employer. De sa manière je ne connais rien; BIERENS DE HAAN ne communique (pag. 23) que le signe de la fonction et la définition, à laquelle je reviendrai plus tard. Enfin j'ai calculé de nouveau et un peu étendu la table*) des valeurs de la fonction qui se trouve dans le traité antérieur. M'étant trouvé obligé par les dites causes de refaire ce traité, je le présente ici respectueusement.

§ 1.

Quoique aucunes fonctions transcendentes en rapport pratique puissent se mésurer aux logarithmes et aux fonctions goniométriques, il y a bien des personnes qui ont entrepris de traiter séparément les fonctions qui se présentent souvent et dont plusieurs autres dépendent, et d'en faire des tables.

*) J'y ai découvert une faute grave. Pour l'argument $a = 1{,}85$, des nombres complétement faux s'y trouvent, sans que j'aie été capable de découvrir comment cela s'est fait.

C'est pourquoi $\int_0^{\frac{\pi}{2}} x \operatorname{Cot} ax\, dx$, désigné ci-dessous par $\mathrm{H}(a)$, m'a paru digne d'attention. Les Tables de Bierens de Haan indiquent que LEGENDRE dans ses "Exercices de calcul intégral" a déjà fixé la valeur de cette fonction pour $a = 1$ et $a = \frac{1}{2}$; même d'autres, p. e. CAUCHY, ont traité ces fonctions-ci ou d'autres pareilles. La recherche présente commence donc par l'équation

$$\mathrm{H}(a) = \int_0^{\frac{\pi}{2}} x \operatorname{Cot} ax\, dx, \quad \ldots\ldots\ldots \quad (1)$$

où a d'abord est supposé situé entre 0 et $\pm$ 2, ces deux exclus.

Évidemment est

$$\mathrm{H}(-a) = -\mathrm{H}(a), \quad \ldots\ldots\ldots \quad (2)$$

$$\int_0^{a} x \operatorname{Cot} \beta x\, dx = \left(\frac{2\alpha}{\pi}\right)^2 \mathrm{H}\left(\frac{2\alpha\beta}{\pi}\right), \quad \ldots\ldots\ldots \quad (3)$$

où $\alpha\beta$ est compris entre 0 et $\pm\, \pi$ (exclus). En intégrant par parties, nous aurons par (1)

$$\int_0^{\frac{\pi}{2}} \frac{x^2\, dx}{\operatorname{Sin}^2 ax} = \frac{2}{a}\mathrm{H}(a) - \frac{\pi^2}{4a} \operatorname{Cot} \frac{a\pi}{2} \text{ *)} \quad \ldots\ldots\ldots \quad (4)$$

$$\int_0^{\frac{\pi}{2}} l \operatorname{Sin} ax\, .\, dx = \frac{\pi}{2}\, l \operatorname{Sin} \frac{a\pi}{2} - a\,\mathrm{H}(a) \text{ **)} \quad \ldots\ldots \quad (5).$$

Si l'on substitue dans (4) $\frac{a}{2}$, $2y$ pour a, x respectivement, il vient

$$\int_0^{\frac{\pi}{4}} \frac{y^2\, dy}{\operatorname{Sin}^2 ay} = \frac{1}{2a}\mathrm{H}\left(\frac{a}{2}\right) - \frac{\pi^2}{16a} \operatorname{Cot} \frac{a\pi}{4} \quad \ldots\ldots \quad (6).$$

La même formule (4) donne aussi

$$\int_0^{\frac{\pi}{2}} x^2 \operatorname{Cot}^2 ax\, .\, dx = \frac{2}{a}\mathrm{H}(a) - \frac{\pi^2}{4a} \operatorname{Cot} \frac{a\pi}{2} - \frac{\pi^3}{24} \quad \ldots \quad (7),$$

*) Trouvée par CAUCHY pour $a = 1$. Voyez Bierens de Haan Tab. 239 N:o 8.

**) Bierens de Haan trouve dans son Exposé pag. 478 form. (1264)

$$\int_0^{\frac{a\pi}{2}} l \operatorname{Sin} x\, dx = -\frac{a\pi}{2}\, l2,$$

où il faut se souvenir que a doit être un nombre entier, puisque autrement $\int_0^{\frac{a\pi}{2}} \operatorname{Cos} 2nx\, dx$ n'est pas égale à zéro en général.

Par la notation l nous désignons toujours le logarithme naturel.

parce qu'on a

$$\frac{1}{\mathrm{Sin}^2 ax} = 1 + \mathrm{Cot}^2 ax.$$

§ 2.

La formule connue

$$\mathrm{Cot}\,\tfrac{1}{2}\psi - \mathrm{Cot}\,\psi = \frac{1}{\mathrm{Sin}\,\psi}$$

conduit à

$$\int_0^{\frac{\pi}{2}} \frac{x\,dx}{\mathrm{Sin}\,ax} = \mathrm{H}\left(\frac{a}{2}\right) - \mathrm{H}(a) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Dans ce qui va suivre nous désignons, pour abréger, cette intégrale par le signe L(a). Selon Bierens de Haan*), LEGENDRE a trouvé

$$\mathrm{H}(1) = \int_0^{\frac{\pi}{2}} x\,\mathrm{Cot}\,x\,.\,dx = \frac{\pi}{2}\,l\,2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

et EULER entre autres

$$\int_0^{\frac{\pi}{2}} l\,\mathrm{Sin}\,x\,.\,dx = \int_0^{\frac{\pi}{2}} l\,\mathrm{Cos}\,x\,.\,dx = -\frac{\pi}{2}\,l\,2 \quad . \quad . \quad . \quad . \quad (10)$$

qui, jointe à la formule (5), donne la même valeur à H(1) qu' auparavant.

En faisant $ax = \mathrm{Arc\ Cot}\ y$ dans la formule (5), on aura

$$\int_{\mathrm{Cot}\frac{a\pi}{2}}^{\infty} \frac{l(1+y^2)}{1+y^2}\,dy = 2a^2\,\mathrm{H}(a) - a\pi\ l\,\mathrm{Sin}\,\frac{a\pi}{2} \quad . \quad . \quad . \quad . \quad (11)$$

qui pour $a = 1$ et $a = \frac{1}{2}$ donne

$$\int_0^{\infty} \frac{l(1+y^2)}{1+y^2}\,dy = 2\,\mathrm{H}(1) = \pi\,l\,2 \quad . \quad . \quad . \quad . \quad . \quad (12),$$

$$\int_1^{\infty} \frac{l\,(1+y^2)}{1+y^2}\,dy = \tfrac{1}{2}\,\mathrm{H}(\tfrac{1}{2}) + \frac{\pi}{4}\,l\,2 \quad . \quad . \quad . \quad . \quad . \quad (13),$$

$$\int_0^{1} \frac{l\,(1+y^2)}{1+y^2}\,dy = \frac{\pi}{2}\,l\,2 - \tfrac{1}{2}\,\mathrm{L}(1) \quad . \quad . \quad . \quad . \quad . \quad (14);$$

mais LEGENDRE a donné**)

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*) Tab. 238 N:o 3.

**) B. d. H. Tab. 238 N:o 4 comparée avec Tab. 160 N:o 7.

$$\int_0^1 \frac{l(1+y^2)}{1+y^2}\,dy = \frac{\pi}{2}\,l\,2 - \sum_{p=0}^{p=\infty} \frac{(-1)^p}{(2p+1)^2},$$

donc

$$\sum_{p=0}^{p=\infty} \frac{(-1)^p}{(2p+1)^2} = \tfrac{1}{2}\,\mathrm{L}(1)^*) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

On peut aussi ramener $\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\mathrm{Cos}\,ax}$ à la fonction L(...) comme il suit. En premier lieu l'on a

$$\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\mathrm{Cos}\,ax} = \frac{1}{a^2} \int_{(1-\frac{a}{2})\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(\frac{\pi}{2}-x)\,dx}{\mathrm{Sin}\,x} = \frac{1}{a^2}\left[\frac{\pi}{2}\,l\,\mathrm{Cot}\,(1-\frac{a}{2})\,\frac{\pi}{4} - \int_{(1-\frac{a}{2})\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x\,dx}{\mathrm{Sin}\,x}\right];$$

mais on a

$$\int_{(1-\frac{a}{2})\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x\,dx}{\mathrm{Sin}\,x} = \mathrm{L}(1) - \int_0^{(1-\frac{a}{2})\frac{\pi}{2}} \frac{x\,dx}{\mathrm{Sin}\,x} = \mathrm{L}(1) - (1-\frac{a}{2})^2\,\mathrm{L}(1-\frac{a}{2}),$$

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*) B. d. H. Tab. 239 N:o 1. Cette série se présente plusieurs fois. Ainsi B. de Haan la trouve dans son Exposé (pag. 421), quand il calcule l'intégrale

$$\mathrm{J} = \int_0^1 l\,\frac{1+x}{\sqrt{x}}\cdot\frac{dx}{1+x^2}.$$ Il trouve $\mathrm{J} = \frac{\pi}{8}\,l\,2 + \frac{1}{2}\sum_{p=0}^{p=\infty}\frac{(-1)^p}{(2p+1)^2}$.

L'intégrale peut aussi s'obtenir de cette manière. En posant $x = \mathrm{tg}\varphi$ nous aurons

$$\mathrm{J} = \int_0^{\frac{\pi}{4}} l\,\frac{1+\mathrm{tg}\varphi}{\sqrt{\mathrm{tg}\varphi}}\,.\,d\varphi = \int_0^{\frac{\pi}{4}} l\,(1+\mathrm{tg}\varphi)\,d\varphi - \tfrac{1}{2}\int_0^{\frac{\pi}{4}} l\,\mathrm{tg}\varphi\,.\,d\varphi;$$

mais on a

$$l\,(1+\mathrm{tg}\varphi) = l\,(\mathrm{Cos}\,\varphi + \mathrm{Sin}\,\varphi) - l\,\mathrm{Cos}\,\varphi = \tfrac{1}{2}\,l2 + l\,\mathrm{Cos}\left(\frac{\pi}{4}-\varphi\right) - l\,\mathrm{Cos}\,\varphi,$$

donc

$$\mathrm{J} = \frac{\pi}{8}\,l\,2 + \int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\left(\frac{\pi}{4}-\varphi\right)d\varphi - \int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\,\varphi\,d\varphi - \tfrac{1}{2}\int_0^{\frac{\pi}{4}} l\,\mathrm{tg}\varphi\,d\varphi.$$

Par la substitution φ pour $\frac{\pi}{4}-\varphi$ on voit que

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\left(\frac{\pi}{4}-\varphi\right)d\varphi = \int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\,\varphi\,d\varphi$$

et qu'on ait $\mathrm{J} = \frac{\pi}{8}\,l\,2 - \frac{1}{2}\int_0^{\frac{\pi}{4}} l\,\mathrm{tg}\varphi\,d\varphi$; mais la formule (21) ci-après donne

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{tg}\varphi\,d\varphi = -\tfrac{1}{2}\,\mathrm{L}(1), \text{ donc } \mathrm{J} = \frac{\pi}{8}\,l2 + \tfrac{1}{4}\,\mathrm{L}(1).$$

donc

$$\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\operatorname{Cos} ax} = \frac{1}{a^2}\left[\frac{\pi}{2}\, l\operatorname{Cot}\left(1-\frac{a}{2}\right)\frac{\pi}{4} + \left(1-\frac{a}{2}\right)^2 \mathrm{L}\left(1-\frac{a}{2}\right) - \mathrm{L}(1)\right]. \quad (16).$$

En remplaçant a, x dans (8) par $\frac{2\alpha\beta}{\pi}$, $\frac{\pi x}{2\alpha}$ resp. et dans (16) par $\frac{4\alpha\beta}{\pi}$, $\frac{\pi x}{4\alpha}$ resp. proviennent les intégrales plus générales

$$\int_0^{\alpha} \frac{x\,dx}{\operatorname{Sin} \beta x} = \left(\frac{2\alpha}{\pi}\right)^2 \mathrm{L}\left(\frac{2\alpha\beta}{\pi}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

$$\int_0^{\alpha} \frac{x\,dx}{\operatorname{Cos} \beta x} = \frac{1}{\beta^2}\left[\frac{\pi}{2}\, l\operatorname{Cot}\left(\frac{\pi}{4}-\frac{\alpha\beta}{2}\right) + \left(1-\frac{2\alpha\beta}{\pi}\right)^2 \mathrm{L}\left(1-\frac{2\alpha\beta}{\pi}\right) - \mathrm{L}(1)\right] \quad (18)$$

($\alpha\beta$ compris entre 0 et $\pm\,\pi$, ces deux exclus).

§ 3.

En intégrant par parties dans les formules (8) et (16), on trouve

$$\int_0^{\frac{\pi}{2}} \frac{x^2 \operatorname{Cos} ax}{\operatorname{Sin}^2 ax}\, dx = \frac{2}{a}\,\mathrm{L}(a) - \frac{\pi^2}{4a \operatorname{Sin} \frac{a\pi}{2}} \quad . \quad . \quad . \quad . \quad (19),$$

$$\int_0^{\frac{\pi}{4}} \frac{x^2 \operatorname{Sin} ax}{\operatorname{Cos}^2 ax}\, dx = \frac{\pi^2}{16a \operatorname{Cos} \frac{a\pi}{4}} - \frac{2}{a^3}\left[\frac{\pi}{2}\, l\operatorname{Cot}\left(1-\frac{a}{2}\right)\frac{\pi}{4} - \mathrm{L}(1) + \left(1-\frac{a}{2}\right)^2 \mathrm{L}\left(1-\frac{a}{2}\right)\right] \quad (20).$$

Moyennant ces deux intégrales on peut avoir toutes les intégrales de la forme

$$\int_0^{\frac{\pi}{2}} \frac{x^2 \operatorname{Cos}^{2n+1} ax}{\operatorname{Sin}^2 ax}\, dx \quad \text{ou} \quad \int_0^{\frac{\pi}{4}} \frac{x^2 \operatorname{Sin}^{2n+1} ax}{\operatorname{Cos}^2 ax}\, dx.$$

($n =$ à un nombre entier).

En effet si l'on substitue $1 - \operatorname{Cos}^2 ax$ pour $\operatorname{Sin}^2 ax$ dans la première et $1 - \operatorname{Sin}^2 ax$ pour $\operatorname{Cos}^2 ax$ dans la dernière, on aura par la division

$$-x^2\,(\operatorname{Cos}^{2n-1} ax + \operatorname{Cos}^{2n-3} ax + \ldots + \operatorname{Cos} ax) - \frac{x^2 \operatorname{Cos} ax}{\operatorname{Sin}^2 ax}$$

$$-x^2\,(\operatorname{Sin}^{2n-1} ax + \operatorname{Sin}^{2n-3} ax + \ldots + \operatorname{Sin} ax) - \frac{x^2 \operatorname{Sin} ax}{\operatorname{Cos}^2 ax}.$$

Mais comme les intégrales

$$\int_0^{\frac{\pi}{2}} x^2 \operatorname{Cos}^{2p+1} ax\, dx, \quad \int_0^{\frac{\pi}{4}} x^2 \operatorname{Sin}^{2p+1} ax\, dx$$

peuvent être supposées connues*), il est évident que les intégrales ci-dessus peuvent être exprimées par la fonction L(...). De la même fonction dépendent aussi les intégrales de la forme

$$\int_0^{\frac{\pi}{2}} \frac{x\,dx}{\mathrm{Sin}^{2n+1}ax} \text{ et } \int_0^{\frac{\pi}{4}} \frac{x\,dx}{\mathrm{Cos}^{2n+1}ax}$$ **).

§ 4.

Si l'on remplace x par $2x$ dans la formule (8) et ensuite intègre par parties, on trouve

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{tg}\,ax\,dx = \frac{\pi}{4}\,l\,\mathrm{tg}\,\frac{a\pi}{4} - \frac{a}{2}\,\mathrm{L}(a) \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{Cot}\,ax\,dx = \frac{\pi}{4}\,l\,\mathrm{Cot}\,\frac{a\pi}{4} + \frac{a}{2}\,\mathrm{L}(a) \quad . \quad . \quad . \quad . \quad (22).$$

Par la même substitution ou tirera de la formule (5)

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{Sin}\,ax\,.\,dx + \int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\,ax\,.\,dx = \frac{\pi}{4}\,l\,\frac{\mathrm{Sin}\,\frac{a\pi}{2}}{2} - \frac{a}{2}\,\mathrm{H}(a)$$

qui, combinée avec (21) ou (22), donne

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{Sin}\,ax\,.\,dx = \frac{\pi}{4}\,l\,\mathrm{Sin}\,\frac{a\pi}{4} - \frac{a}{4}\,[\mathrm{L}(a) + \mathrm{H}(a)] \quad . \quad . \quad . \quad (23),$$

$$\int_0^{\frac{\pi}{4}} l\,\mathrm{Cos}\,ax\,.\,dx = \frac{\pi}{4}\,l\,\mathrm{Cos}\,\frac{a\pi}{4} + \frac{a}{4}\,[\mathrm{L}(a) - \mathrm{H}(a)] \quad . \quad . \quad . \quad (24).$$

En intégrant par parties ici, on trouve

$$\int_0^{\frac{\pi}{4}} x\,\mathrm{Cot}\,ax\,dx = \tfrac{1}{4}\,[\mathrm{L}(a) + \mathrm{H}(a)] = \tfrac{1}{4}\,\mathrm{H}\left(\frac{a}{2}\right) \quad . \quad . \quad . \quad . \quad (25),$$

$$\int_0^{\frac{\pi}{4}} x\,\mathrm{tg}\,ax\,dx = \tfrac{1}{4}\,[\mathrm{L}(a) - \mathrm{H}(a)] \quad . \quad . \quad . \quad . \quad . \quad . \quad (26).$$

§ 5.

En posant dans (1) $a = 1 + b$, $(b < 1)$ il vient

$$\mathrm{H}(1+b) = \int_0^{\frac{\pi}{2}} x\,\mathrm{Cot}\,(1+b)\,x\,.\,dx = \frac{1}{(1+b)^2}\int_0^{(1+b)\frac{\pi}{2}} x\,\mathrm{Cot}\,x\,dx \quad (27),$$

*) Minding, Integral-Tafeln pag. 115, 116. Berlin 1849.

**) Minding, l. c. pag. 139.

lorsq'on substitue x au lieu de $(1+b)\,x$. Il s'ensuit d'un théorème bien connu, que

$$\mathrm{H}(1+b) = \frac{1}{(1+b)^2}\left[\int_0^{\frac{\pi}{2}} x\,\mathrm{Cot}\,x\,.\,dx + \int_{\frac{\pi}{2}}^{(1+b)\frac{\pi}{2}} x\,\mathrm{Cot}\,x\,.\,dx\right].$$

En remplaçant x par $\frac{\pi}{2}+x$ dans le dernier terme, on trouve

$$\int_{\frac{\pi}{2}}^{(1+b)\frac{\pi}{2}} x\,\mathrm{Cot}\,x\,.\,dx = -\frac{\pi}{2}\int_0^{\frac{b\pi}{2}} \mathrm{tg}\,x\,dx - \int_0^{\frac{b\pi}{2}} x\,\mathrm{tg}\,x\,dx;$$

mais on sait que

$$\int_0^{\frac{b\pi}{2}} \mathrm{tg}\,x\,dx = -\,l\,\mathrm{Cos}\,\frac{b\pi}{2},$$

et on trouve par (26)

$$\int_0^{\frac{b\pi}{2}} x\,\mathrm{tg}\,x\,dx = 4b^2\int_0^{\frac{\pi}{4}} x\,\mathrm{tg}\,2b\,x\,dx = b^2\,[\mathrm{L}(2b) - \mathrm{H}(2b)],$$

donc

$$\mathrm{H}(1+b) = \frac{1}{(1+b)^2}\left[\frac{\pi}{2}\,l\left(2\,\mathrm{Cos}\,\frac{b\pi}{2}\right) - b^2\,\{\mathrm{L}(2b) - \mathrm{H}(2b)\}\right] \quad . \quad (28).$$

En substituant $-b$ pour b, on aura

$$\mathrm{H}(1-b) = \frac{1}{(1-b)^2}\left[\frac{\pi}{2}\,l\left(2\,\mathrm{Cos}\,\frac{b\pi}{2}\right) + b^2\,\{\mathrm{L}(2b) - \mathrm{H}(2b)\}\right] . \quad (29).$$

Si l'on multiplie la formule (28) par $(1+b)^2$ et (29) par $(1-b)^2$ et qu'on additionne les produits, il en résulte la relation fort simple

$$(1+b)^2\,\mathrm{H}(1+b) + (1-b)^2\,\mathrm{H}(1-b) = \pi\,l\left(2\,\mathrm{Cos}\,\frac{b\pi}{2}\right) . \quad . \quad (30).$$

Une pareille relation a lieu par rapport à la fonction L(...). La formule (16) donne

$$\left(1-\frac{a}{2}\right)^2 \mathrm{L}\left(1-\frac{a}{2}\right) = \mathrm{L}(1) - \frac{\pi}{2}\,l\,\mathrm{Cot}\left(1-\frac{a}{2}\right)\frac{\pi}{4} + a^2\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\mathrm{Cos}\,ax};$$

en changeant a en $-a$, nous aurons

$$\left(1+\frac{a}{2}\right)^2 \mathrm{L}\left(1+\frac{a}{2}\right) = \mathrm{L}(1) - \frac{\pi}{2}\,l\,\mathrm{Cot}\left(1+\frac{a}{2}\right)\frac{\pi}{4} + a^2\int_0^{\frac{\pi}{4}} \frac{x\,dx}{\mathrm{Cos}\,ax}$$

et en soustrayant

$$\left(1-\frac{a}{2}\right)^2 \mathrm{L}\left(1-\frac{a}{2}\right) - \left(1+\frac{a}{2}\right)^2 \mathrm{L}\left(1+\frac{a}{2}\right) = \frac{\pi}{2}\, l\, \frac{\mathrm{Cot}\,(1+\frac{a}{2})\frac{\pi}{4}}{\mathrm{Cot}\,(1-\frac{a}{2})\frac{\pi}{4}} = \pi\, l\, \mathrm{tg}\,(1-\frac{a}{2})\frac{\pi}{4}$$

ou en posant $a = 2b$ (b num. < 1)

$$(1-b)^2\,\mathrm{L}(1-b) - (1+b)^2\,\mathrm{L}(1+b) = \pi\, l\, \mathrm{tg}\,(1-b)\frac{\pi}{4} \quad . \quad (31).$$

Ces relations qui par leur simplicité semblent dignes d'attention sont aussi, comme la suite le démontrera, d'une grande utilité dans le calcul des valeurs numériques des fonctions H(...) et L(...). De ces relations on peut aussi déduire quelques autres formules que nous proposerons, lorsque nous aurons considéré quelques cas spéciaux qui peuvent être utiles dans diverses réductions. En faisant $b = \frac{1}{3}$, $b = \frac{2}{3}$ dans les formules (30) et (31), on trouve

$$4\,\mathrm{H}(\tfrac{4}{3}) + \mathrm{H}(\tfrac{2}{3}) = \frac{9\pi}{8}\, l\, 3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$25\,\mathrm{H}(\tfrac{5}{3}) + \mathrm{H}(\tfrac{1}{3}) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

$$\mathrm{L}(\tfrac{2}{3}) - 4\,\mathrm{L}(\tfrac{4}{3}) = -\frac{9\pi}{8}\, l\, 3 \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$\mathrm{L}(\tfrac{1}{3}) - 25\,\mathrm{L}(\tfrac{5}{3}) = 9\pi\, l\, \mathrm{tg}\,\frac{\pi}{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35).$$

Si l'on pose $b = \frac{1}{3}$ dans (29), on aura

$$6\,\mathrm{H}(\tfrac{2}{3}) = \frac{9\pi}{4}\, l\, 3 + \mathrm{H}(\tfrac{1}{3}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36).$$

Ces formules, liées avec (8), font voir que toutes ces fonctions peuvent être exprimées par $\mathrm{H}(\frac{1}{3})$ et $\mathrm{H}(\frac{1}{6})$ jointes à un logarithme.

§ 6.

En posant successivement dans la formule (29) $b = \frac{1}{n}, \frac{2}{n}, \ldots \frac{n-1}{n}$ et en faisant attention à l'équation $\mathrm{L}(2b) = \mathrm{H}(b) - \mathrm{H}(2b)$; on aura

$$(n-1)^2\,\mathrm{H}\left(1-\frac{1}{n}\right) = \frac{n^2\pi}{2}\, l\left(2\,\mathrm{Cos}\,\frac{\pi}{2n}\right) + \mathrm{H}\left(\frac{1}{n}\right) - 2\,\mathrm{H}\left(\frac{2}{n}\right)$$

$$(n-2)^2\,\mathrm{H}\left(1-\frac{2}{n}\right) = \frac{n^2\pi}{2}\, l\left(2\,\mathrm{Cos}\,\frac{2\pi}{2n}\right) + 4\,\mathrm{H}\left(\frac{2}{n}\right) - 8\,\mathrm{H}\left(\frac{4}{n}\right)$$

. . . . . .

$$4\,\mathrm{H}\left(\frac{2}{n}\right) = \frac{n^2\pi}{2}\, l\left(2\,\mathrm{Cos}\,\frac{(n-2)\pi}{2n}\right) + (n-2)^2\,\mathrm{H}\left(1-\frac{2}{n}\right) - 2\,(n-2)^2\,\mathrm{H}\left(2-\frac{4}{n}\right)$$

$$1\,\mathrm{H}\left(\frac{1}{n}\right) = \frac{n^2\pi}{2}\, l\left(2\,\mathrm{Cos}\,\frac{(n-1)\pi}{2n}\right) + (n-1)^2\,\mathrm{H}\left(1-\frac{1}{n}\right) - 2\,(n-1)^2\,\mathrm{H}\left(2-\frac{2}{n}\right).$$

Quand on prend la somme de toutes les équations, les termes du membre gauche détruiront les termes moyens du membre droit et l'on trouvera

$$\sum_{p=1}^{p=n-1} p^2 \,\mathrm{H}\left(\frac{2p}{n}\right) = \frac{n^2\pi}{4}\, l\left(2^{n-1}\,\mathrm{Cos}\,\frac{\pi}{2n}\,.\,\mathrm{Cos}\,\frac{2\pi}{2n}\,.\,.\,.\,\mathrm{Cos}\,\frac{(n-1)\pi}{2n}\right).$$

Supposons maintenant, que n soit un nombre pair $2m$, alors le nombre des facteurs entre les crochets, excepté 2^{n-1}, est impair et égal à $2m-1$. D'une formule bien connue il s'ensuit que

$$2\,\mathrm{Cos}\,\frac{\pi}{4m}\,.\,\mathrm{Cos}\,\frac{(2m-1)\pi}{4m} = \mathrm{Sin}\,\frac{\pi}{2m}$$

$$2\,\mathrm{Cos}\,\frac{2\pi}{4m}\,\mathrm{Cos}\,\frac{(2m-2)\pi}{4m} = \mathrm{Sin}\,\frac{2\pi}{2m}$$

. . . . . .

$$2\,\mathrm{Cos}\,\frac{(m-1)\pi}{4m}\,.\,\mathrm{Cos}\,\frac{(m+1)\pi}{4m} = \mathrm{Sin}\,\frac{(m-1)\pi}{2m}.$$

Puisque le facteur moyen est $\mathrm{Cos}\,\frac{\pi}{4} = 2^{-\frac{1}{2}}$, on peut écrire

$$\sum_{p=1}^{p=2m-1} p^2 \,\mathrm{H}\left(\frac{2p}{2m}\right) = \frac{(2m)^2\pi}{4}\, l\left(2^{m-\frac{1}{2}}\,\mathrm{Sin}\,\frac{\pi}{2m}\,.\,\mathrm{Sin}\,\frac{2\pi}{2m}\,.\,.\,.\,\mathrm{Sin}\,\frac{(m-1)\pi}{2m}\right);$$

mais GRUNERT a démontré*) la formule

$$2m = 2^{2m-1}\left(\mathrm{Sin}\,\frac{\pi}{2m}\,.\,\mathrm{Sin}\,\frac{2\pi}{2m}\,.\,.\,.\,\mathrm{Sin}\,\frac{(m-1)\pi}{2m}\right)^2,$$

dont il suit, qu'on a

$$l\left(2^{m-\frac{1}{2}}\,\mathrm{Sin}\,\frac{\pi}{2m}\,.\,\mathrm{Sin}\,\frac{2\pi}{2m}\,.\,.\,.\,\mathrm{Sin}\,\frac{(m-1)\pi}{2m}\right) = \tfrac{1}{2}\, l\, 2m$$

et

$$\sum_{p=1}^{p=2m-1} p^2 \,\mathrm{H}\left(\frac{2p}{2m}\right) = \frac{(2m)^2\pi}{8}\, l\, 2m.$$

Si n au contraire est un nombre impair $2m+1$, on a

$$\sum_{p=1}^{p=2m} p^2 \,\mathrm{H}\left(\frac{2p}{2m+1}\right) = \frac{(2m+1)^2\pi}{4}\, l\left(2^{2m}\,\mathrm{Cos}\,\frac{\pi}{2(2m+1)}\,.\,\mathrm{Cos}\,\frac{2\pi}{2(2m+1)}\,.\,.\,.\right.$$
$$\left.\mathrm{Cos}\,\frac{2m\pi}{2(2m+1)}\right).$$

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*) Voyez Supplemente zu Klügels Math. Wörterbuche, zweite Abtheilung pag. 643 form. (365).

De la même manière qu'auparavant, mais par une autre formule de GRUNERT*) savoir

$$2m+1 = 2^{2m}\left(\operatorname{Sin}\frac{\pi}{2m+1}\cdot\operatorname{Sin}\frac{2\pi}{2m+1}\cdots\operatorname{Sin}\frac{m\pi}{2m+1}\right)^2,$$

on trouve

$$\sum_{p=1}^{p=2m} p^2\,\mathrm{H}\left(\frac{2p}{2m+1}\right) = \frac{(2m+1)^2\pi}{8}\,l\,(2m+1).$$

Si l'on pose $2m = n$ dans la formule précédente et $2m+1 = n$ dans celle-ci, toutes les deux peuvent être réunies dans une seule, savoir

$$\sum_{p=1}^{p=n-1} p^2\,\mathrm{H}\left(\frac{2p}{n}\right) = \frac{n^2\pi}{8}\,l\,n \quad \ldots\ldots\ldots \quad (37).$$

Cette formule vaut pour les multiples paires de $\frac{1}{n}$. Une pareille qui vaut pour les multiples impaires peut être trouvée sans difficulté. Posons dans (29) $b = \frac{1}{2n}, \frac{3}{2n}\ldots\ldots\frac{2n-1}{2n}$. Alors on aura

$$(2n-1)^2\,\mathrm{H}\left(1-\frac{1}{2n}\right) = 2n^2\pi\,l\left(2\operatorname{Cos}\frac{\pi}{4n}\right) + 1\,.\,\mathrm{H}\left(\frac{1}{2n}\right) - 2\,\mathrm{H}\left(\frac{1}{n}\right)$$

$$(2n-3)^2\,\mathrm{H}\left(1-\frac{3}{2n}\right) = 2n^2\pi\,l\left(2\operatorname{Cos}\frac{3\pi}{4n}\right) + 9\,\mathrm{H}\left(\frac{3}{2n}\right) - 18\,.\,\mathrm{H}\left(\frac{3}{n}\right)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$9\,\mathrm{H}\left(\frac{3}{2n}\right) = 2n^2\pi\,l\left(2\operatorname{Cos}\frac{(2n-3)\pi}{4n}\right) + (2n-3)^2\,\mathrm{H}\left(1-\frac{3}{2n}\right) - 2\,(2n-3)^2\,\mathrm{H}\left(2-\frac{3}{n}\right)$$

$$1\,.\,\mathrm{H}\left(\frac{1}{2n}\right) = 2n^2\pi\,l\left(2\operatorname{Cos}\frac{(2n-1)\pi}{4n}\right) + (2n-1)^2\,\mathrm{H}\left(1-\frac{1}{2n}\right) - 2\,(2n-1)^2\,\mathrm{H}\left(2-\frac{1}{n}\right).$$

Par l'addition de ces équations il provient

$$\sum_{p=0}^{p=n-1} (2p+1)^2\,\mathrm{H}\left(\frac{2p+1}{n}\right) = n^2\pi\,l\left(2^n\operatorname{Cos}\frac{\pi}{4n}\,.\,\operatorname{Cos}\frac{3\pi}{4n}\ldots\operatorname{Cos}\frac{(2n-1)\pi}{4n}\right).$$

De la même manière qu'en cas précédent et par deux autres formules de GRUNERT**) on aura

$$\sum_{p=0}^{p=n-1} (2p+1)^2\,\mathrm{H}\left(\frac{2p+1}{n}\right) = \frac{n^2\pi}{2}\,l\,2 \quad \ldots\ldots \quad (38).$$

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*) Ibid. form. (366).

**) Ibid. pag. 641 form. (355) et (356).

§ 7.

Jusqu'ici nous avons supposé que la valeur numérique de l'argument a est comprise entre 0 et 2 (exclus.) ou b entre -1 et $+1$ (exclus.); et il est évident que l'une et l'autre des fonctions deviennent infinies pour chacune des dites valeurs. Mais si l'on donne à b, dans (30) et (31), une valeur entre 1 et 3, par ex. 2, et qu'en même temps au lieu de $l\left(2\,\mathrm{Cos}\,\frac{b\pi}{2}\right)$, $l\,\mathrm{tg}\,(1-b)\,\frac{\pi}{4}$ on écrive $\frac{1}{2}\,l\left(2\,\mathrm{Cos}\,\frac{b\pi}{2}\right)^2$, $\frac{1}{2}\,l\,\mathrm{tg}^2(1-b)\,\frac{\pi}{4}$, nulle des fonctions ne devient infinie. Cette circonstance donne lieu à croire que les formules (28)...(31) valent plus généralement, qu'il ne semblait être le cas auparavant. Pour éclaircir ce point, considérons d'abord l'intégrale

$$\mathrm{J} = \int_0^{\frac{\pi}{2}} x\,\mathrm{Cot}\,(2n+1+\alpha)\,x\,.\,dx,$$

où n est un nombre entier et α num. < 1. Comme la fonction sous le signe de l'intégration devient discontinue pour $x = \frac{m\pi}{2n+1+\alpha}$, m étant un des nombres 1, 2, ... n, il faut écrire selon CAUCHY*)

$$\mathrm{J} = \int_0^{\frac{\pi}{2\nu}} x\,\mathrm{Cot}\,\nu\,x\,dx + \int_{\frac{\pi}{2\nu}}^{\frac{\pi}{\nu}-\delta} x\,\mathrm{Cot}\,\nu\,x\,dx + \int_{\frac{\pi}{\nu}+\delta}^{\frac{3\pi}{2\nu}} x\,\mathrm{Cot}\,\nu\,x\,dx + \ldots$$

$$\int_{\frac{(2n-1)\pi}{2\nu}}^{\frac{n\pi}{\nu}-\delta} x\,\mathrm{Cot}\,\nu\,x\,dx + \int_{\frac{n\pi}{\nu}+\delta}^{\frac{\pi}{2}} x\,\mathrm{Cot}\,\nu\,x\,dx,$$

$$(\lim \delta = 0)$$

où pour abréger nous avons posé $2n+1+\alpha = \nu$. Désignons aussi les termes du membre droit par $\mathrm{J}_1, \mathrm{J}_2, \ldots \mathrm{J}_{2n}, \mathrm{J}_{2n+1}$ et faisons $x = \frac{(m-1)\pi + y}{\nu}$ dans J_{2m-1} et $x = \frac{m\pi - y}{\nu}$ dans J_{2m}. De cette manière et en posant $\nu\,\delta = \delta_1$, nous aurons

$$\mathrm{J}_{2m-1} = \frac{1}{\nu^2}\left[(m-1)\pi \int_{\delta_1}^{\frac{\pi}{2}} \mathrm{Cot}\,y\,dy + \int_{\delta_1}^{\frac{\pi}{2}} y\,\mathrm{Cot}\,y\,dy\right]$$

*) Voyez Bierens de Haan, Exposé pag. 7 et Serret, Calc. int. pag. 103 Paris 1868.

$$J_{2m} = \frac{1}{\nu^2}\left[-m\pi \int_{\delta_1}^{\frac{\pi}{2}} \operatorname{Cot} y\, dy + \int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right]$$

$$J_{2n+1} = \frac{1}{\nu^2}\left[n\pi \int_{\delta_1}^{\frac{\pi}{2}} \operatorname{Cot} y\, dy + n\pi\, l \operatorname{Cos}\frac{\alpha\pi}{2} + \int_{\delta_1}^{(1+\alpha)\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right].$$

En faisant $m = 1, 2, \ldots n$ et en prenant la somme des intégrales deux à deux on trouve

$$J_1 + J_2 = \frac{1}{\nu^2}\left[-\pi \int_{\delta_1}^{\frac{\pi}{2}} \operatorname{Cot} y\, dy + \int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy + \int_{0}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right]$$

$$J_3 + J_4 = \frac{1}{\nu^2}\left[-\pi \int_{\delta_1}^{\frac{\pi}{2}} \operatorname{Cot} y\, dy + 2 \int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right]$$

. . . . . .

$$J_{2n-1} + J_{2n} = \frac{1}{\nu^2}\left[-\pi \int_{\delta_1}^{\frac{\pi}{2}} \operatorname{Cot} y\, dy + 2 \int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right].$$

Si l'on ajoute J_{2n+1} à la somme de ces équations, il en viendra

$$J = \frac{1}{\nu_2}\left[n\pi\, l \operatorname{Cos}\frac{\alpha\pi}{2} + (2n-1)\int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy + \int_{0}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy + \int_{\delta_1}^{(1+\alpha)\frac{\pi}{2}} y \operatorname{Cot} y\, dy\right],$$

parceque les premières intégrales entre les crochets se détruisent mutuellement. En passant à la limite on trouve

$$\int_{\delta_1}^{\frac{\pi}{2}} y \operatorname{Cot} y\, dy = \mathrm{H}(1) = \frac{\pi}{2} l 2,$$

$$\int_{\delta_1}^{(1+\alpha)\frac{\pi}{2}} y \operatorname{Cot} y\, dy = (1+\alpha)^2\, \mathrm{H}(1+\alpha),$$

comme en (27). Si l'on introduit $2n + 1 + \alpha$ au lieu de ν, on a enfin la valeur principale

$$J = \mathrm{H}(2n+1+\alpha) = \frac{1}{(2n+1+\alpha)^2}\left[n\pi\, l\left(2 \operatorname{Cos}\frac{\alpha\pi}{2}\right) + (1+\alpha)^2\, \mathrm{H}(1+\alpha)\right] \quad (39).$$

Voyons maintenant, si la formule (30), supposée valable, donne le même résultat. Si l'on y pose $b = 2n + \alpha$ (α num. < 1), il vient

$$(2n+1+\alpha)^2\, \mathrm{H}(2n+1+\alpha) = \frac{\pi}{2}\, l\left(2 \operatorname{Cos}\overline{2n+\alpha}\,.\,\frac{\pi}{2}\right)^2 + (2n-1+\alpha)^2\, \mathrm{H}(2n-1+\alpha)$$

$$(2n-1+\alpha)^2\,\mathrm{H}(2n-1+\alpha) = \frac{\pi}{2}\,l\left(2\,\mathrm{Cos}\,\overline{2(n-1)+\alpha}\,.\,\frac{\pi}{2}\right)^2 + (2n-3+\alpha)^2\,\mathrm{H}(2n-3+\alpha)$$

. . . . . .

$$(3+\alpha)^2\,\mathrm{H}(3+\alpha) = \frac{\pi}{2}\,l\left(2\,\mathrm{Cos}\,\overline{2+\alpha}\,.\,\frac{\pi}{2}\right)^2 + (1+\alpha)^2\,\mathrm{H}(1+\alpha).$$

En ajoutant ces équations on trouve

$$\mathrm{H}(2n+1+\alpha) = \frac{1}{(2n+1+\alpha)^2}\left[\frac{\pi}{2}\sum_{p=1}^{p=n} l\left(2\,\mathrm{Cos}\,\overline{2p+\alpha}\,.\,\frac{\pi}{2}\right)^2 + (1+\alpha)^2\mathrm{H}(1+\alpha)\right];$$

mais on a

$$\mathrm{Cos}\,(2p+\alpha)\,\frac{\pi}{2} = (-1)^p\,\mathrm{Cos}\,\frac{\alpha\pi}{2},$$

donc

$$\sum_{p=1}^{p=n} l\left(2\,\mathrm{Cos}\,\overline{2p+\alpha}\,.\,\frac{\pi}{2}\right)^2 = 2n\,l\left(2\,\mathrm{Cos}\,\frac{\alpha\pi}{2}\right)$$

et

$$\mathrm{H}(2n+1+\alpha) = \frac{1}{(2n+1+\alpha)^2}\left[n\pi\,l\left(2\,\mathrm{Cos}\,\frac{\alpha\pi}{2}\right) + (1+\alpha)^2\,\mathrm{H}(1+\alpha)\right]$$

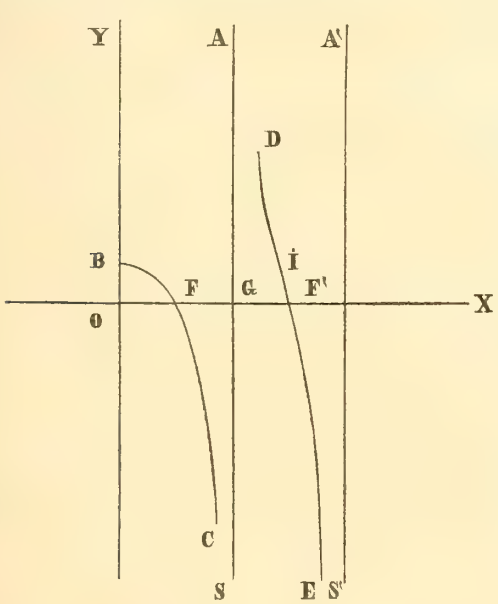
ou la formule (39).

La formule (30) vaut ainsi pour toute valeur de b, excepté $b =$ un nombre impair.

§ 8.

Peut-être ne semblera-t-il pas inutile, si nous tâchons d'éclaircir cette matière par une considération géométrique. A cet effet, construisons la courbe, dont l'équation en coordonnées rectangulaires est

$$y = x\,\mathrm{Cot}\,3x.$$



Dans la figure, BC est l'une de ses branches, DE une autre. Du côté gauche de l'axe des ordonnées se trouvent des branches semblables. La droite AS est l'asymptote de BC et de ED, $A'S'$ est l'asymptote de DE. Si l'on pose $n = 1$, $\alpha = 0$ dans le paragraphe précédent, on aura

$$\mathrm{J} = \int_0^{\frac{\pi}{2}} x\,\mathrm{Cot}\,3x\,dx.$$

Parceque la quantité contenue sous le signe de l'intégration devient infinie pour $x = \frac{\pi}{3}$, il faut mettre

$$J = \int_0^{\frac{\pi}{6}} x \operatorname{Cot} 3x\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}-\delta} x \operatorname{Cot} 3x\, dx + \int_{\frac{\pi}{3}+\delta}^{\frac{\pi}{2}} x \operatorname{Cot} 3x\, dx.$$

Alors on a

$$\int_0^{\frac{\pi}{6}} x \operatorname{Cot} 3x\, dx = \text{l'aire } BFO.$$

La somme des deux autres termes n'est autre chose que la différence des aires infiniment grandes $DFGA$ et $CFGS$, qui ont des signes contrairs. Ainsi cette considération géométrique constate la conclusion à laquelle nous sommes parvenu auparavant.

§ 9.

Par un procédé semblable à celui que nous avons suivi dans le paragraphe 7ième, on peut examiner, si la formule (31) vaut aussi pour chaque valeur de b qui n'est pas un nombre impair. Nous considérerons donc l'intégrale

$$J' = \int_0^{\frac{\pi}{2}} \frac{x\, dx}{\operatorname{Sin}(2n + 1 + \alpha)x},$$

où n et α ont la même signification que dans § 7. La fonction sous le signe $\int$ devient discontinue pour $x = \frac{m\pi}{2n+1+\alpha}$, m étant un des nombres entiers 1, 2, ... n. Écrivons donc

$$J' = \int_0^{\frac{\pi}{2\nu}} \frac{x\, dx}{\operatorname{Sin} \nu x} + \int_{\frac{\pi}{2\nu}}^{\frac{\pi}{\nu}-\varepsilon} \frac{x\, dx}{\operatorname{Sin} \nu x} + \int_{\frac{\pi}{\nu}+\varepsilon}^{\frac{3\pi}{2}} \frac{x\, dx}{\operatorname{Sin} \nu x} \cdots + \int_{\frac{(2n-1)\pi}{2\nu}}^{\frac{n\pi}{\nu}-\varepsilon} \frac{x\, dx}{\operatorname{Sin} \nu x} + \int_{\frac{n\pi}{\nu}+\varepsilon}^{\frac{\pi}{2}} \frac{x\, dx}{\operatorname{Sin} \nu x},$$

$$(\lim \varepsilon = 0)$$

où nous avons posé, pour abréger, $2n + 1 + \alpha = \nu$. Désignons les termes de J' par J'_1, J'_2, ... J'_{2n}, J'_{2n+1} et posons $x = \frac{(m-1)\pi + y}{\nu}$ dans J'_{2m-1} et $x = \frac{m\pi - y}{\nu}$ dans J'_{2m}. Alors nous aurons, en posant $\nu\varepsilon = \varepsilon_1$,

$$J'_{2m-1} = \frac{(-1)^{m-1}}{\nu^2} \left[(m-1)\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y} + \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{y\, dy}{\operatorname{Sin} y} \right],$$

$$J'_{2m} = \frac{(-1)^{m-1}}{\nu^2}\left[m\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y} - \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y}\right],$$

$$J'_{2n+1} = \frac{(-1)^n}{\nu^2}\left[n\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y} + n\pi\, l\operatorname{tg}(1+\alpha)\frac{\pi}{4} + \int_{\varepsilon_1}^{(1+\alpha)\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y}\right].$$

En prenant la somme de ces intégrales deux à deux, on trouve

$$J'_1 + J'_2 = \frac{1}{\nu^2}(0+1)\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y} + \frac{1}{\nu^2}\left(\int_0^{\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y} - \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y}\right)$$

$$J'_3 + J'_4 = -\frac{1}{\nu^2}(1+2)\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y}$$

. . . . . .

$$J'_{2n-1} + J'_{2n} = \frac{(-1)^{n-1}}{\nu^2}(n-1+n)\,\pi \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y}.$$

La somme de celles-ci est évidemment

$$= \frac{(-1)^{n-1} n\pi}{\nu^2} \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{dy}{\operatorname{Sin} y} + \frac{1}{\nu^2}\left(\int_0^{\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y} - \int_{\varepsilon_1}^{\frac{\pi}{2}} \frac{y\,dy}{\operatorname{Sin} y}\right).$$

En ajoutant J'_{2n+1}, les premières intégrales se détruisent, et en passant à la limite, on trouve la valeur principale

$$J' = L(2n+1+\alpha) = \frac{(-1)^n}{(2n+1+\alpha)^2}\left[n\pi\, l\operatorname{tg}^2(1+\alpha)\frac{\pi}{4} + (1+\alpha)^2\, L(1+\alpha)\right] \quad (40).$$

Cette formule peut être retrouvée par la formule (31). Faisons là $b = 2n + \alpha$ (α num. < 1) et distinguons d'abord entre n pair et impair. Soit donc $n = 2m$; nous aurons

$$(4m+1+\alpha)^2\, L(4m+1+\alpha) = -(4m-1+\alpha)^2\, L(4m-1+\alpha) - \frac{\pi}{2}\, l\operatorname{tg}^2(4m-1+\alpha).\frac{\pi}{4}$$

$$(4m-1+\alpha)^2\, L(4m-1+\alpha) = -(4m-3+\alpha)^2\, L(4m-3+\alpha) - \frac{\pi}{2}\, l\operatorname{tg}^2(4m-3+\alpha).\frac{\pi}{4}$$

. . . . . .

$$(3+\alpha)^2\, L(3+\alpha) = -(1+\alpha)^2\, L(1+\alpha) - \frac{\pi}{2}\, l\operatorname{tg}^2(1+\alpha)\frac{\pi}{4}.$$

Le nombre de ces termes-ci est $= 2m$. En changeant le signe de chaque terme d'ordre pair et en ajoutant depuis les équations, les termes du côté gauche hors le premier détruiront les termes antérieurs du côté droit hors le dernier qui a le signe $+$. Quant aux termes postérieurs du côté droit on a

$$\operatorname{tg}(4p-1+\alpha)\frac{\pi}{4} = -\operatorname{tg}(1-\alpha)\frac{\pi}{4},$$

$$\operatorname{tg}(4p-3+\alpha)\frac{\pi}{4} = \operatorname{tg}(1+\alpha)\frac{\pi}{4},$$

où le nombre p a toutes les valeurs depuis m jusqu' à 1. Puisque tous les termes d'ordre pair ont + (après le changement des signes), mais les autres —, la somme des termes postérieurs du côté droit devient

$$= \frac{\pi}{2} l \frac{\operatorname{tg}^{2m}(1+\alpha)\frac{\pi}{4}}{\operatorname{tg}^{2m}(1-\alpha)\frac{\pi}{4}} = 2m\pi l \operatorname{tg}(1+\alpha)\frac{\pi}{4},$$

donc

$$(4m+1+\alpha)^2 \mathrm{L}(4m+1+\alpha) = (1+\alpha)^2 \mathrm{L}(1+\alpha) + 2m\pi l \operatorname{tg}(1+\alpha)\frac{\pi}{4}.$$

Si l'on ôte la première équation ci-dessus, le nombre des termes devient impair $= 2m-1$. En traitant les équations qui restent de la même manière, on aura

$$(4m-1+\alpha)^2 \mathrm{L}(4m-1+\alpha) = -(1+\alpha)^2 \mathrm{L}(1+\alpha) - (2m-1)\pi l \operatorname{tg}(1+\alpha)\frac{\pi}{4}.$$

En faisant $2m = n+1$ dans celle-ci et $2m = n$ dans celle-là, on retrouve la formule (40).

§ 10.

Comme on le sait, il peut arriver que la différence de deux fonctions est finie pour une valeur de l'argument qui rend les fonctions mêmes infinies. Les formules (39) et (40) doivent prouver, comment il est des fonctions $\mathrm{H}(a)$ et $\mathrm{L}(a)$ à cet égard. Mais avant de les consulter considérons un cas special. Par ce qui précède nous avons

$$\mathrm{H}(a) - \mathrm{H}(2a) = \mathrm{L}(2a) = \int_0^{\frac{\pi}{2}} \frac{x\,dx}{\operatorname{Sin} 2ax}$$

$$\mathrm{H}(2a) - \mathrm{H}(4a) = \mathrm{L}(4a) = \int_0^{\frac{\pi}{2}} \frac{x\,dx}{\operatorname{Sin} 4ax} \quad \ldots\ldots \quad (e)$$

où nous supposons d'abord $a < \frac{1}{2}$.

En soustrayant on aura

$$\mathrm{H}(a) - 2\,\mathrm{H}(2a) + \mathrm{H}(4a) = \int_0^{\frac{\pi}{2}} \left(\frac{1}{\operatorname{Sin} 2ax} - \frac{1}{\operatorname{Sin} 4ax}\right) x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{x \operatorname{Cos} 3ax}{\operatorname{Cos} ax \operatorname{Sin} 4ax}\,.\,dx.$$

Parce que on a

$$\frac{\operatorname{Cos} 3ax}{\operatorname{Cos}\ ax \operatorname{Sin} 4ax} = \frac{\operatorname{Cot} ax}{\operatorname{Cos} 2ax} - \frac{3}{\operatorname{Sin} 4ax},$$

il viendra

$$\mathrm{H}(a) - 2\,\mathrm{H}(2a) + \mathrm{H}(4a) = \int_0^{\frac{\pi}{2}} \frac{x \operatorname{Cot} ax}{\operatorname{Cos} 2ax}\, dx - 3\int_0^{\frac{\pi}{2}} \frac{x\, dx}{\operatorname{Sin} 4ax}.$$

Ainsi l'on trouve en vertu de l'équation (e) ci-dessus

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \frac{x \operatorname{Cot} ax}{\operatorname{Cos} 2ax}\, dx &= \mathrm{H}(a) + \mathrm{H}(2a) - 2\,\mathrm{H}(4a) \\ &= \mathrm{H}(a) + \mathrm{L}(4a) - \mathrm{H}(4a) \ldots\ldots\ldots \quad (41).\end{aligned}$$

Cette formule vaut sans doute, tant que a est $< \frac{1}{2}$; vaut-elle aussi, lorsque $a = 1$? Pour y répondre nous chercherons l'intégrale

$$\mathrm{J} = \int_0^{\frac{\pi}{2}} \frac{x \operatorname{Cot} x\, dx}{\operatorname{Cos} 2x}$$

au lieu de laquelle on peut écrire

$$\mathrm{J} = \int_0^{\frac{\pi}{2}} x \operatorname{Cot} x\, dx + \int_0^{\frac{\pi}{2}} x \operatorname{tg} 2x\, dx$$

à cause de la formule connue

$$\frac{\operatorname{Cot} x}{\operatorname{Cos}\ 2x} = \operatorname{Cot} x + \operatorname{tg} 2x.$$

Parce que la fonction $x \operatorname{tg} 2x$ devient discontinue pour $x = \frac{\pi}{4}$, il faut mettre

$$\int_0^{\frac{\pi}{2}} x \operatorname{tg} 2x\, dx = \int_0^{\frac{\pi}{4}-\delta} x \operatorname{tg}\ 2x\, dx + \int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} x \operatorname{tg} 2x\, dx,$$

$$(\lim \delta = 0).$$

Très facilement nous aurons

$$\int_0^{\frac{\pi}{4}-\delta} x \operatorname{tg} 2x\, dx = -\tfrac{1}{2}\left(\left(\frac{\pi}{4} - \delta\right)\right) l \operatorname{Sin} 2\delta + \tfrac{1}{4}\int_0^{\frac{\pi}{4}-\delta} l \operatorname{Cos}^2 2x\, dx,$$

$$\int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} x \operatorname{tg}\ 2x\, dx = \tfrac{1}{2}\left(\frac{\pi}{4} + \delta\right) l \operatorname{Sin} 2\delta + \tfrac{1}{4}\int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} l \operatorname{Cos}^2 2x\, dx$$

donc

$$\int_0^{\frac{\pi}{4}-\delta} x \operatorname{tg} 2x\, dx + \int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} x \operatorname{tg} 2x\, dx =$$

$$= \delta\, l \operatorname{Sin} 2\delta + \tfrac{1}{4} \int_0^{\frac{\pi}{4}-\delta} l \operatorname{Cos}^2 2x\, dx + \tfrac{1}{4} \int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} l \operatorname{Cos}^2 2x\, dx.$$

Si l'on pose $\frac{x}{2}$ pour x dans la première intégrale du membre droit et $\frac{\pi}{2} - \frac{x}{2}$ pour x dans la seconde, on trouve

$$\int_0^{\frac{\pi}{4}-\delta} x \operatorname{tg} 2x\, dx + \int_{\frac{\pi}{4}+\delta}^{\frac{\pi}{2}} x \operatorname{tg} 2x\, dx = \delta\, l \operatorname{Sin} 2\delta + \tfrac{1}{2} \int_0^{\frac{\pi}{2}-2\delta} l \operatorname{Cos} x\, dx.$$

En passant à la limite on aura par (10) $-\frac{\pi}{4} l 2$ comme valeur principale et on trouvera enfin en vertu de (9)

$$J = \int_0^{\frac{\pi}{2}} \frac{x \operatorname{Cot} x}{\operatorname{Cos} 2x}\, dx = \frac{\pi}{4} l 2, \quad \ldots\ldots\ldots\ldots \quad (42)$$

comme CAUCHY l'a trouvé *) d'une autre manière. Posons maintenant $a = 1$ dans (41); il viendra

$$L(4) - H(4) = -\frac{\pi}{4} l 2, \quad \ldots\ldots\ldots\ldots \quad (43)$$

c'est à dire, que la différence est finie. Passons aux formules (39) et (40) qui donnent

$$H(1+b) = \frac{1}{(1+b)^2}\left[\frac{\pi}{2} l\left(2 \operatorname{Cos} \frac{b\pi}{2}\right)^2 - (1-b)^2 H(1-b)\right]$$

$$L(1+b) = \frac{1}{(1+b)^2}\left[(1-b)^2 L(1-b) - \frac{\pi}{2} l \operatorname{tg}^2 (1-b)\frac{\pi}{4}\right].$$

En soustrayant nous aurons

$$L(1+b) - H(1+b) =$$

$$= \frac{1}{(1+b)^2}\left[(1-b)^2 \{L(1-b) + H(1-b)\} - \frac{\pi}{2} l \left(2 \operatorname{Cos} \frac{b\pi}{2} \cdot \operatorname{tg} (1-b)\frac{\pi}{4}\right)^2\right]$$

*) Voyez Bierens de Haan, Tab. 239 N:o 21 et Tab. 265 N:o 13.

$$= \frac{1}{(1+b)^2}\left[(1-b)^2 \mathrm{H}\left(\frac{1-b}{2}\right) - \frac{\pi}{2} l\left(2 \operatorname{Cos}\frac{b\pi}{2} \operatorname{tg}(1-b)\frac{\pi}{4}\right)^2\right].$$

Soit $b = 4n+1$; alors on a $\mathrm{H}\left(\frac{1-b}{2}\right) = -\mathrm{H}(2n) = -\infty$. Si l'on pose $b = 4n+3$, on trouve $\mathrm{H}\left(\frac{1-b}{2}\right) = -\mathrm{H}(2n+1)$ or finie. Quant au second terme on a

$$\operatorname{Cos}\frac{b\pi}{2} \operatorname{tg}(1-b)\frac{\pi}{4} = \frac{\operatorname{Cos}\frac{b\pi}{2}}{\operatorname{Cot}(1-b)\frac{\pi}{4}}$$

ce qui devient $= \frac{0}{0}$ pour $b = 4n+3$; mais par la méthode ordinaire on trouve

$$\operatorname{Cos}\frac{b\pi}{2} \operatorname{tg}(1-b)\frac{\pi}{4} = 2$$

pour $b = 4n+3$. Dans ce cas on a donc

$$\mathrm{L}[4(n+1)] - \mathrm{H}[4(n+1)] = -\frac{1}{16(n+1)^2}[4(2n+1)^2\mathrm{H}(2n+1) + 2\pi l 2]. \quad (44).$$

En posant $n = 0$, on retrouve la formule (43). La différence dont il s'agit est donc finie, quand l'argument est une multiple de 4, pas en cas contrair.

§ 11.

Dans les paragraphes 1—4 nous avons rassemblé un nombre d'intégrales qui ont été presque immédiatement trouvées par la formule (1) ou par des procédés les plus usuelles faites par rapport à celle-là. Cela s'est fait non-seulement pour faire voir, qu'un grand nombre d'intégrales dépendent de la fonction $\mathrm{H}(a)$, mais aussi pour avoir des formules qui seraient utiles en déduisant certaines propriétés de $\mathrm{H}(a)$ et de $\mathrm{L}(a)$ desquelles nous avons fait mention dans les paragraphes 5—10. Passons maintenant à quelques intégrales qu'on ne peut trouver aussi facilement que les précédentes et commençons par l'intégrale

$${}^{m}\mathrm{J}_{\nu} = \int_0^1 \frac{x^{m-1} l x}{1+x^{\nu}}\, dx,$$

où m et ν sont des nombres entiers et m supposé $< \nu$ *). Mais comme il

*) Etant $m > \nu$, la fonction est décomposable en une partie entière et en une fraction; étant $m = \nu$ l'intégrale peut être simplifiée en posant $x^{\nu} = y$.

devient nécessaire de distinguer ν pair d'avec ν impair, il semble mieux de traiter les intégrales ${}^{m}J_{2n}$, ${}^{m}J_{2n+1}$ séparément déjà en commençant.
Si l'on pose pour abréger

$$(2p+1)\frac{\pi}{\nu} = \varphi_p,\ l(1-2x\,\mathrm{Cos}\,\varphi_p+x^2) = \mathrm{L}_p,\ \mathrm{Arc\,tg}\frac{x\,\mathrm{Sin}\,\varphi_p}{1-x\,\mathrm{Cos}\,\varphi_p} = \mathrm{A}_p,$$

on trouve par intégration indéfinie*)

$$\int \frac{x^{m-1}lx}{1+x^{2n}}\,dx = -\frac{lx}{2n}\left\{\sum_{p=0}^{p=n-1} \mathrm{L}_p\,\mathrm{Cos}\,m\varphi_p - 2\sum_{p=0}^{p=n-1} \mathrm{A}_p\,\mathrm{Sin}\,m\varphi_p\right\}$$

$$+\frac{1}{2n}\int\frac{dx}{x}\left[\sum_{p=0}^{p=n-1} \mathrm{L}_p\,\mathrm{Cos}\,m\varphi_p - 2\sum_{p=0}^{p=n-1} \mathrm{A}_p\,\mathrm{Sin}\,m\varphi_p\right],$$

$$\int\frac{x^{m-1}lx}{1+x^{2n+1}}dx = \frac{lx}{2n+1}\left\{(-1)^{m-1}l(1+x) - \sum_{p=0}^{p=n-1} \mathrm{L}_p\,\mathrm{Cos}\,m\varphi_p + 2\sum_{p=0}^{p=n-1} \mathrm{A}_p\,\mathrm{Sin}\,m\varphi_p\right\}$$

$$+\frac{(-1)^m}{2n+1}\int\frac{l(1+x)}{x}\,dx + \frac{1}{2n+1}\int\frac{dx}{x}\left[\sum_{p=0}^{p=n-1} \mathrm{L}_p\,\mathrm{Cos}\,m\varphi_p - 2\sum_{p=0}^{p=n-1} \mathrm{A}_p\,\mathrm{Sin}\,m\varphi_p\right].$$

Pour avoir les intégrales définies ${}^{m}J_{2n}$, ${}^{m}J_{2n+1}$ il faut prendre les intégrales indéfinies entre les limites 0 et 1; mais alors il arrive que les termes intégrés prennent la forme indéterminée $0\times\infty$ pour $x=0$. Afin de trouver leurs valeurs véritables en ce cas, on peut procéder de la manière suivante. Posons pour abréger

$$lx\,.\,l(1-2x\,\mathrm{Cos}\,\varphi_p+x^2) = u,\ lx\,.\,\mathrm{Arc\,tg}\frac{x\,\mathrm{Sin}\,\varphi_p}{1-x\,\mathrm{Cos}\,\varphi_p} = v.$$

D'abord il est évident qu'il faut que u soit compris entre les expressions $2\,lx\,.\,l(1-x)$ et $2\,lx\,.\,l(1+x)$. Les fonctions $l(1-x)$ et $l(1+x)$ peuvent à présent être développées en séries convergentes dont tous les termes ont la forme $\frac{x^r}{r}$ ou, multipliées par lx, la forme $\frac{x^r}{r}\,lx$. Parce que cette fonction est $=0$ pour $x=0$, il en est de même des fonctions $lx\,.\,l(1-x)$ et $lx\,.\,l(1+x)$ et de la fonction u qui est comprise entre celles-ci.

Si l'on pose dans la fonction v

$$\frac{x\,\mathrm{Sin}\,\varphi_p}{1-x\,\mathrm{Cos}\,\varphi_p} = \mathrm{tg}\,\omega,$$

*) Voyez Minding, Integral-Tafeln pag. 57.

on aura

$$x = \frac{\operatorname{tg}\omega}{\operatorname{Sin}\varphi_p + \operatorname{Cos}\varphi_p \operatorname{tg}\omega} = \frac{\operatorname{Sin}\omega}{\operatorname{Sin}(\varphi_p + \omega)}$$

et

$$v = \omega\, l \operatorname{Sin}\omega - \omega\, l \operatorname{Sin}(\varphi_p + \omega).$$

En supposant $x = 0$, on a $\omega = 0$ et l'on trouve facilement, que la fonction v est aussi $= 0$ en ce cas. Tous les termes intégrés sont donc $= 0$ pour $x = 0$. Ils sont encore $= 0$ pour $x = 1$. Donc nous aurons

$${}^{m}\mathrm{J}_{2n} = \frac{1}{2n}\int_0^1 \frac{dx}{x} \sum_{p=0}^{p=n-1} \operatorname{Cos} m\varphi_p\, l(1 - 2x\operatorname{Cos}\varphi_p + x^2)$$

$$- \frac{1}{n}\int_0^1 \frac{dx}{x} \sum_{p=0}^{p=n-1} \operatorname{Sin} m\varphi_p . \operatorname{Arc\,tg} \frac{x \operatorname{Sin}\varphi_p}{1 - x\operatorname{Cos}\varphi_p}, \quad (45)$$

$${}^{m}\mathrm{J}_{2n+1} = \frac{(-1)^m}{2n+1}\int_0^1 \frac{l(1+x)}{x}\, dx + \frac{1}{2n+1}\int_0^1 \frac{dx}{x} \sum_{p=0}^{p=n-1} \operatorname{Cos} m\varphi_p\, l(1 - 2x\operatorname{Cos}\varphi_p + x^2)$$

$$- \frac{2}{2n+1}\int_0^1 \frac{dx}{x} \sum_{p=0}^{p=n-1} \operatorname{Sin} m\varphi_p . \operatorname{Arc\,tg} \frac{x \operatorname{Sin}\varphi_p}{1 - x\operatorname{Cos}\varphi_p} \ldots . \quad (46).$$

§ 12.

Les intégrales (45) et (46) dépendent donc de trois autres savoir

$$\int_0^1 \frac{l(1+x)}{x}\, dx,$$

$$\int_0^1 \frac{dx}{x}\, l(1 - 2x\operatorname{Cos}\varphi_p + x^2)$$

$$\int_0^1 \frac{dx}{x} \operatorname{Arc\,tg} \frac{x \operatorname{Sin}\varphi_p}{1 - x\operatorname{Cos}\varphi_p}.$$

D'abord on a

$$\int_0^1 \frac{l(1+x)}{x}\, dx = \frac{\pi^2}{12};\ ^{*})$$

puis

$$\int_0^1 \frac{dx}{x}\, l(1 - 2x\operatorname{Cos}\varphi_p + x^2) = -\frac{\pi^2}{3} + \pi\varphi_p - \tfrac{1}{2}\varphi_p{}^2\ ^{**})$$

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*) Bierens de Haan, Tab. 160 N:o 1.

**) B. d. H. Tab. 160 N:o 14, où les signes dans le membre droit sont fautifs.

Il reste encore à trouver la troisième intégrale. A cet effet posons

$$\text{Arc tg}\,\frac{x\,\text{Sin}\,\varphi_p}{1-x\,\text{Cos}\,\varphi_p} = \omega;$$

nous aurons comme dans le paragraphe précédent

$$x = \frac{\text{Sin}\,\omega}{\text{Sin}\,(\varphi_p+\omega)}, \quad \frac{dx}{x} = \text{Cot}\,\omega\,d\omega - \text{Cot}\,(\varphi_p+\omega)\,d\omega.$$

Aux limites $x = 0$ et $x = 1$ répondent $\omega = 0$ et $\omega = \frac{1}{2}(\pi - \varphi_p)$ resp.. On aura donc

$$\int_0^1 \frac{dx}{x}\,\text{Arc tg}\,\frac{x\,\text{Sin}\,\varphi_p}{1-x\,\text{Cos}\,\varphi_p} = \int_0^{\frac{1}{2}(\pi-\varphi_p)} \omega\,\text{Cot}\,\omega\,d\omega - \int_0^{\frac{1}{2}(\pi-\varphi_p)} \omega\,\text{Cot}\,(\varphi_p+\omega)\,d\omega.$$

La formule (3) donne

$$\int_0^{\frac{1}{2}(\pi-\varphi_p)} \omega\,\text{Cot}\,\omega\,d\omega = \left(1-\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1-\frac{\varphi_p}{\pi}\right).$$

En posant dans l'intégrale postérieure $\varphi_p + \omega = y$, on trouve

$$\int_0^{\frac{1}{2}(\pi-\varphi_p)} \omega\,\text{Cot}\,(\varphi_p+\omega)\,d\omega = \int_{\varphi_p}^{\frac{1}{2}(\pi+\varphi_p)} y\,\text{Cot}\,y\,dy - \varphi_p \int_{\varphi_p}^{\frac{1}{2}(\pi+\varphi_p)} \text{Cot}\,y\,dy$$

$$= \int_0^{\frac{1}{2}(\pi+\varphi_p)} y\,\text{Cot}\,y\,dy - \int_0^{\varphi_p} y\,\text{Cot}\,y\,dy - \varphi_p \int_{\varphi_p}^{\frac{1}{2}(\pi+\varphi_p)} \text{Cot}\,y\,dy$$

$$= \left(1+\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1+\frac{\varphi_p}{\pi}\right) - \frac{4\varphi_p^2}{\pi^2}\,\text{H}\left(\frac{2\varphi_p}{\pi}\right) + \varphi_p\, l(2\,\text{Sin}\,\tfrac{1}{2}\varphi_p).$$

Nous aurons donc

$$\int_0^1 \frac{dx}{x}\,\text{Arc tg}\,\frac{x\,\text{Sin}\,\varphi_p}{1-x\,\text{Cos}\,\varphi_p} = \left(1-\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1-\frac{\varphi_p}{\pi}\right) - \left(1+\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1+\frac{\varphi_p}{\pi}\right)$$

$$+ \frac{4\varphi_p^2}{\pi^2}\,\text{H}\left(\frac{2\varphi_p}{\pi}\right) - \varphi_p\, l(2\,\text{Sin}\,\tfrac{1}{2}\varphi_p);$$

mais en vertu des formules (28) et (29) on a

$$\left(1-\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1-\frac{\varphi_p}{\pi}\right) - \left(1+\frac{\varphi_p}{\pi}\right)^2 \text{H}\left(1+\frac{\varphi_p}{\pi}\right) = \frac{2\varphi_p^2}{\pi^2}\left[\text{L}\left(\frac{2\varphi_p}{\pi}\right) - \text{H}\left(\frac{2\varphi_p}{\pi}\right)\right]$$

$$= \frac{2\varphi_p^2}{\pi^2}\left[\text{H}\left(\frac{\varphi_p}{\pi}\right) - 2\,\text{H}\left(\frac{2\varphi_p}{\pi}\right)\right].$$

Nous obtenons donc en dernier lieu

$$\int_0^1 \frac{dx}{x} \operatorname{Arc\,tg} \frac{x \operatorname{Sin} \varphi_p}{1 - x \operatorname{Cos} \varphi_p} = \frac{2\varphi_p^2}{\pi^2} \mathrm{H}\left(\frac{\varphi_p}{\pi}\right) - \varphi_p \, l(2 \operatorname{Sin} \tfrac{1}{2} \varphi_p) \ldots \quad (47).$$

Introduisons les intégrales trouvées dans les formules (45) et (46) et en même temps la valeur de ϕ_p; il viendra

$${}^m\mathrm{J}_{2n} = \frac{\pi^2}{2n} \sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1) \frac{m\pi}{2n} \left[-\frac{1}{3} + \frac{2p+1}{2n} - \frac{(2p+1)^2}{8n^2} \right]$$

$$- \frac{1}{2n^2} \sum_{p=0}^{p=n-1} (2p+1) \operatorname{Sin}(2p+1) \frac{m\pi}{2n} \left[\frac{2p+1}{n} \mathrm{H}\left(\frac{2p+1}{2n}\right) - \pi \, l\left(2 \operatorname{Sin} \left(\frac{(2p+1)\pi}{4n}\right)\right) \right], \quad (48)$$

$${}^m\mathrm{J}_{2n+1} = \frac{(-1)^m \pi^2}{12(2n+1)} + \frac{\pi^2}{2n+1} \sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1) \frac{m\pi}{2n+1} \left[-\frac{1}{3} + \frac{2p+1}{2n+1} - \frac{(2p+1)^2}{2(2n+1)^2} \right]$$

$$- \frac{2}{(2n+1)^2} \sum_{p=0}^{p=n-1} (2p+1) \operatorname{Sin} \frac{(2p+1)m\pi}{2n+1} \left[\frac{2(2p+1)}{2n+1} \mathrm{H}\left(\frac{2p+1}{2n+1}\right) - \pi \, l\left(2 \operatorname{Sin} \frac{(2p+1)\pi}{2(2n+1)}\right) \right]. \quad (49).$$

§ 13.

Dans ces intégrales une reduction se peut faire par laquelle les formules (48) et (49) deviennent bien plus simples. Cela se fait en vertu des formules connues

$$\sum_{p=0}^{p=n-1} \operatorname{Sin}(2p+1)\alpha = \frac{\operatorname{Sin}^2 n\alpha}{\operatorname{Sin} \alpha}, \quad \sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\alpha = \frac{\operatorname{Sin} 2n\alpha}{2 \operatorname{Sin} \alpha}.$$

En différentiant la première une fois et la seconde deux fois par rapport à α, on trouvera

$$\sum_{p=0}^{p=n-1} (2p+1) \operatorname{Cos}(2p+1)\alpha = \frac{n \operatorname{Sin} 2n\alpha}{\operatorname{Sin} \alpha} - \frac{\operatorname{Sin}^2 n\alpha \operatorname{Cos} \alpha}{\operatorname{Sin}^2 \alpha},$$

$$\sum_{p=0}^{p=n-1} (2p+1)^2 \operatorname{Cos}(2p+1)\alpha = \frac{2n^2 \operatorname{Sin} 2n\alpha}{\operatorname{Sin} \alpha} + \frac{2n \operatorname{Cos} 2n\alpha \operatorname{Cos} \alpha}{\operatorname{Sin}^2 \alpha}$$

$$- \frac{\operatorname{Sin} 2n\alpha \, (1 + \operatorname{Cos}^2 \alpha)}{2 \operatorname{Sin}^3 \alpha}.$$

En employant ces formules dans (48), où $\alpha = \frac{m\pi}{2n}$, on aura

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\frac{m\pi}{2n} = 0;$$

$$\sum_{p=0}^{p=n-1} (2p+1)\operatorname{Cos}(2p+1)\frac{m\pi}{2n} = 0 \quad \text{pour } m \text{ pair},$$

$$= -\frac{\operatorname{Cos}\frac{m\pi}{2n}}{\operatorname{Sin}^2\frac{m\pi}{2n}} \quad \text{pour } m \text{ impair};$$

$$\sum_{p=0}^{p=n-1} (2p+1)^2 \operatorname{Cos}(2p+1)\frac{m\pi}{2n} = (-1)^m \cdot 2n \cdot \frac{\operatorname{Cos}\frac{m\pi}{2n}}{\operatorname{Sin}^2\frac{m\pi}{2n}}.$$

Il faut multiplier la première somme par $-\frac{1}{3}$, la deuxième par $\frac{1}{2n}$ et la troisième par $-\frac{1}{8n^2}$. Cela fait on aura toujours, m soit pair ou impair,

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\frac{m\pi}{2n}\left[-\frac{1}{3}+\frac{2p+1}{2n}-\frac{(2p+1)^2}{8n^2}\right] = -\frac{\operatorname{Cos}\frac{m\pi}{2n}}{4n\operatorname{Sin}^2\frac{m\pi}{2n}} \quad (50).$$

Dans la formule (49) on a $\alpha = \frac{m\pi}{2n+1}$. En posant $m = 2q$, on trouve

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\cdot\frac{2q\pi}{2n+1} = -\frac{1}{2}$$

$$\sum_{p=0}^{p=n-1} (2p+1)\operatorname{Cos}(2p+1)\frac{2q\pi}{2n+1} = -n - \frac{\operatorname{Sin}^2\frac{q\pi}{2n+1}\cdot\operatorname{Cos}\frac{2q\pi}{2n+1}}{\operatorname{Sin}^2\frac{2q\pi}{2n+1}},$$

$$\sum_{p=0}^{p=n-1} (2p+1)^2 \operatorname{Cos}(2p+1)\frac{2q\pi}{2n+1} = -2n^2+2n\cdot\frac{\operatorname{Cos}^2\frac{2q\pi}{2n+1}}{\operatorname{Sin}^2\frac{2q\pi}{2n+1}}$$

$$+\frac{1+\operatorname{Cos}^2\frac{2q\pi}{2n+1}}{2\operatorname{Sin}^2\frac{2q\pi}{2n+1}}.$$

Comme la quantité entre les crochets en ce cas est $= -\frac{1}{3}+\frac{2p+1}{2n+1}-\frac{(2p+1)^2}{2(2n+1)^2}$, il faut qu'on multiplie la première des sommes ci-dessus par $-\frac{1}{3}$, la deuxième par $\frac{1}{2n+1}$ et la troisième par $-\frac{1}{2(2n+1)^2}$.

Cela fait, on trouve après quelques reductions

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\frac{2q\pi}{2n+1}\left[-\frac{1}{3}+\frac{2p+1}{2n+1}-\frac{(2p+1)^2}{2(2n+1)^2}\right] =$$

$$= -\frac{1}{12} - \frac{\operatorname{Cos}\dfrac{2q\pi}{2n+1}}{2(2n+1)\operatorname{Sin}^2\dfrac{2q\pi}{2n+1}}.$$

Par un procédé tout-à-fait semblable on aura pour $m = 2q+1$

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1).\frac{(2q+1)\pi}{2n+1}\left[-\frac{1}{3}+\frac{2p+1}{2n+1}-\frac{(2p+1)^2}{2(2n+1)^2}\right] =$$

$$= \frac{1}{12} - \frac{\operatorname{Cos}\dfrac{(2q+1)\pi}{2n+1}}{2(2n+1)\operatorname{Sin}^2\dfrac{(2q+1)\pi}{2n+1}}.$$

Ces deux formules peuvent être remplacées par la seule formule

$$\sum_{p=0}^{p=n-1} \operatorname{Cos}(2p+1)\frac{m\pi}{2n+1}\left[-\frac{1}{3}+\frac{2p+1}{2n+1}-\frac{(2p+1)^2}{2(2n+1)^2}\right] =$$

$$= \frac{(-1)^{m-1}}{12} - \frac{\operatorname{Cos}\dfrac{m\pi}{2n+1}}{2(2n+1)\operatorname{Sin}^2\dfrac{m\pi}{2n+1}} \quad \ldots\ldots\ldots \quad (51).$$

Si l'on introduit les formules (50) et (51) dans (48) et (49) respectivement, on aura

$${}^{m}\mathrm{J}_{2n} = -\frac{\pi^2}{8n^2}\frac{\operatorname{Cos}\dfrac{m\pi}{2n}}{\operatorname{Sin}^2\dfrac{m\pi}{2n}}$$

$$-\frac{1}{2n^2}\sum_{p=0}^{p=n-1}(2p+1)\operatorname{Sin}(2p+1)\frac{m\pi}{2n}\left[\frac{2p+1}{n}\,\mathrm{H}\left(\frac{2p+1}{2n}\right)-\pi\, l\left(2\operatorname{Sin}\frac{(2p+1)\pi}{4n}\right)\right],$$

$${}^{m}\mathrm{J}_{2n+1} = -\frac{\pi^2}{2(2n+1)^2}\cdot\frac{\operatorname{Cos}\dfrac{m\pi}{2n+1}}{\operatorname{Sin}^2\dfrac{m\pi}{2n+1}}$$

$$-\frac{2}{(2n+1)^2}\sum_{p=0}^{p=n-1}(2p+1)\operatorname{Sin}(2p+1)\frac{m\pi}{2n+1}\left[\frac{2(2p+1)}{2n+1}\mathrm{H}\left(\frac{2p+1}{2n+1}\right)-\pi\, l\left(2\operatorname{Sin}\frac{(2p+1)\pi}{2(2n+1)}\right)\right].$$

Ces deux formules peuvent aussi être remplacées par une seule, savoir

$$ {}^{m}\mathrm{J}_{n} = \int_0^1 \frac{x^{m-1} l x}{1+x^n}\, dx = -\frac{\pi^2}{2n^2} \cdot \frac{\mathrm{Cos}\,\frac{m\pi}{n}}{\mathrm{Sin}^2\frac{m\pi}{n}} $$

$$ -\frac{2}{n^2}\sum_{p=0}^{p=r} (2p+1)\,\mathrm{Sin}\,(2p+1)\,\frac{m\pi}{n}\left[\frac{2(2p+1)}{n}\,\mathrm{H}\left(\frac{2p+1}{n}\right) - \pi\, l\left(2\,\mathrm{Sin}\,\frac{(2p+1)\pi}{2n}\right)\right], \quad (52) $$

où r est égal à $\frac{n}{2} - 1$, quand n est un nombre pair, mais $r = \frac{n-3}{2}$, quand n est un nombre impair.

§ 14.

Pour les valeurs spéciales de n et m l'intégrale ${}^{m}\mathrm{J}_{n}$ prend souvent des valeurs fort simples. Supposons par exemple $n=3$, $m=1$ et $m=2$; nous aurons

$$ \int_0^1 \frac{l x}{1+x^3}\, dx = -\frac{\pi^2}{27} - \frac{2}{9\sqrt{3}}\,\mathrm{H}\left(\tfrac{1}{3}\right) \ldots\ldots\ldots (53), $$

$$ \int_0^1 \frac{x\, l x}{1+x^3}\, dx = \frac{\pi^2}{27} - \frac{2}{9\sqrt{3}}\,\mathrm{H}\left(\tfrac{1}{3}\right) \ldots\ldots\ldots\ldots (54). $$

En soustrayant on trouve

$$ \int_0^1 \frac{(1-x)\, l x}{1+x^3}\, dx = -\frac{2\pi^2}{27}, \ldots\ldots\ldots\ldots (55) $$

comme EULER l'a trouvé *).

Ajoutons (54) à (53); il viendra

$$ \int_0^1 \frac{(1+x)\, l x}{1+x^3}\, dx = -\frac{4}{9\sqrt{3}}\,\mathrm{H}\left(\tfrac{1}{3}\right) $$

ou en vertu de

$$ \frac{1+x}{1+x^3} = \frac{1}{1-x+x^2} $$

*) Bierens de Haan Tab. 152 N:o 15.

$$\int_0^1 \frac{l\,x}{1-x+x^2}\,dx = -\frac{4}{9\sqrt{3}}\,\mathrm{H}\left(\tfrac{1}{3}\right)^{*}). \quad \ldots\ldots\ldots \quad (56).$$

Si l'on pose $n=4$, $m=1$ et $m=3$, on trouve en employant les formules (28) et (8)

$$\int_0^1 \frac{l\,x\,dx}{1+x^4} = -\frac{1}{8\sqrt{2}}\left[\frac{\pi^2}{2} + \pi\,l(\sqrt{2}+1) + \mathrm{L}\left(\tfrac{1}{2}\right)\right], \quad \ldots\ldots \quad (57)$$

$$\int_0^1 \frac{x^2\,l\,x}{1+x^4}\,dx = \frac{1}{8\sqrt{2}}\left[\frac{\pi^2}{2} - \pi\,l(\sqrt{2}+1) - \mathrm{L}\left(\tfrac{1}{2}\right)\right]. \quad \ldots\ldots \quad (58).$$

En soustrayant celles-ci, on retrouve une formule donnée par EULER**).

Lorsqu'on cherche l'intégrale (57) par la méthode ordinaire, on rencontre

$$\int_0^1 \frac{dx}{x}\,\mathrm{Arc\,tg}\,\frac{x\sqrt{2}}{1-x^2}.$$

On peut l'avoir par (57), mais aussi par ce que

$$\mathrm{Arc\,tg}\,\frac{x\sqrt{2}}{1-x^2} = \mathrm{Arc\,tg}\,\frac{x}{\sqrt{2}-x} + \mathrm{Arc\,tg}\,\frac{x}{\sqrt{2}+x}.$$

Après quelques réductions il vient

$$\int_0^1 \frac{dx}{x}\,\mathrm{Arc\,tg}\,\frac{x\sqrt{2}}{1-x^2} = \tfrac{1}{4}\left\{\pi\,l(\sqrt{2}+1) + \mathrm{L}\left(\tfrac{1}{2}\right)\right\}. \quad \ldots\ldots \quad (59).$$

§ 15.

La formule (52) fait voir que le nombre des termes croît avec n; il faut donc profiter de chaque occasion de diminuer n par des substitutions convenables. Une telle occasion se présente par exemple, lorsqu'on cherche les intégrales

$$\int_0^1 \frac{x\,l\,x\,dx}{1+x^6}, \quad \int_0^1 \frac{x^2\,l\,x}{1+x^6}\,dx,$$

*) Ibid., Tab. 153 N:o 3, se trouve cette intégrale, et il est dit qu'elle a été trouvée par EULER et par LEGENDRE qui pourtant lui ont assigné la valeur $-\frac{4\pi^2}{27}$, N'ayant pas eu l'occasion de consulter les ouvrages cités par B. d. H., il m'est impossible d'expliquer cette chose. J'ai abordé l'intégrale de plusieurs manières, mais j'ai toujours trouvé le même résultat qu'en (56).

**) B. d. H., Tab. 152 N:o 19 pour $p=2$.

si l'on fait $x^2 = y$ dans la première et $x^3 = y$ dans la deuxième. Ainsi on trouve

$$\int_0^1 \frac{x\,lx}{1+x^6}\,dx = -\frac{\pi^2}{108} - \frac{1}{18\sqrt{3}}\,\mathrm{H}\left(\tfrac{1}{3}\right) \quad \ldots\ldots\ldots \quad (60),$$

$$\int_0^1 \frac{x^2\,lx}{1+x^6}\,dx = -\frac{1}{18}\,\mathrm{L}(1) \quad \ldots\ldots\ldots\ldots \quad (61).$$

Si l'on cherche l'intégrale (60) sans substitution, directement par (52), on trouve le même résultat; quant à l'intégrale (61), il n'en est pas ainsi, mais en égalant les résultats, on aura la relation

$$6\,\mathrm{H}\left(\frac{1}{2}\right) - \mathrm{L}\left(\frac{1}{3}\right) = \frac{3\pi}{2}\,l\left(4\,\mathrm{Cot}\,\frac{\pi}{12}\right) \quad \ldots\ldots\ldots \quad (62).$$

D'autres formules pareilles peuvent être trouvées de la même manière.

§ 16.

De la fonction H(....) dépend aussi

$${}^{m}\mathrm{J}'_{2n} = \int_0^1 \frac{x^{m-1}\,lx}{1-x^{2n}}\,dx\,.$$

Posons pour abréger

$$l\left(1-2x\,\mathrm{Cos}\,\frac{p\pi}{n}+x^2\right) = \mathrm{L}'_p,\ \mathrm{Arc\,tg}\,\frac{x\,\mathrm{Sin}\,\frac{p\pi}{n}}{1-x\,\mathrm{Cos}\,\frac{p\pi}{n}} = \mathrm{A}'_p;$$

par intégration indéfinie il vient *)

$$\int\frac{x^{m-1}lx}{1-x^{2n}}dx = \frac{lx}{2n}\left[-l(1-x)-(-1)^m l(1+x) - \sum_{p=1}^{p=n-1}\mathrm{L}'_p\,\mathrm{Cos}\,\frac{mp\pi}{n} + 2\sum_{p=1}^{p=n-1}\mathrm{A}'_p\,\mathrm{Sin}\,\frac{mp\pi}{n}\right]$$

$$+\frac{1}{2n}\int\frac{l(1-x)}{x}\,dx + \frac{(-1)^m}{2n}\int\frac{l(1+x)}{x}\,dx$$

$$+\frac{1}{2n}\int\frac{dx}{x}\left[\sum_{p=1}^{p=n-1}\mathrm{L}'_p\,\mathrm{Cos}\,\frac{mp\pi}{n} - 2\sum_{p=1}^{p=n-1}\mathrm{A}'_p\,\mathrm{Sin}\,\frac{mp\pi}{n}\right].$$

De la même manière que dans le paragraphe (11) on trouve que les termes intégrés disparaissent pour $x = 0$ et pour $x = 1$. On a donc

*) Voyez Minding, Integral-Tafeln pag. 58.

$$^{m}J'_{2n} = \frac{1}{2n}\int_0^1 \frac{l(1-x)}{x}\,dx + \frac{(-1)^m}{2n}\int_0^1 \frac{l(1+x)}{x}\,dx$$

$$+ \frac{1}{2n}\int_0^1 \frac{dx}{x}\sum_{p=1}^{p=n-1} L'_p \operatorname{Cos}\frac{mp\pi}{n} - \frac{1}{n}\int_0^1 \frac{dx}{x}\sum_{p=1}^{p=n-1} A'_p \operatorname{Sin}\frac{mp\pi}{n}.$$

Cette intégrale dépend donc des mêmes intégrales que $^{m}J_{\nu}$ et en outre de

$$\int_0^1 \frac{l(1-x)}{x}\,dx = -\frac{\pi^2}{6}. \quad *)$$

Introduisons ces intégrales en $^{m}J'_{2n}$; attendu que maintenant $\varphi_p = \frac{p\pi}{n}$ on aura

$$\int_0^1 \frac{x^{m-1}lx}{1-x^{2n}}\,dx = -\frac{\pi^2}{12n} + \frac{(-1)^m\pi^2}{24n} + \frac{\pi^2}{2n}\sum_{p=1}^{p=n-1} \operatorname{Cos}\frac{mp\pi}{n}\left(-\frac{1}{3} + \frac{p}{n} - \frac{p^2}{2n^2}\right)$$

$$- \frac{1}{n^2}\sum_{p=1}^{p=n-1} p \operatorname{Sin}\frac{mp\pi}{n}\left[\frac{2p}{n}\operatorname{H}\left(\frac{p}{n}\right) - \pi\, l\left(2\operatorname{Sin}\frac{p\pi}{2n}\right)\right] \cdots\cdots \quad (63).$$

Dans cette intégrale on peut aussi faire une réduction telle qu'en § 13.

D'abord on a

$$\sum_{p=1}^{p=n-1} \operatorname{Sin} p\alpha = \frac{\operatorname{Sin}\frac{n\alpha}{2}\cdot\operatorname{Sin}\frac{(n-1)\alpha}{2}}{\operatorname{Sin}\frac{\alpha}{2}}, \quad \sum_{p=1}^{p=n-1} \operatorname{Cos} p\alpha = \frac{\operatorname{Cos}\frac{n\alpha}{2}\cdot\operatorname{Sin}\frac{(n-1)\alpha}{2}}{\operatorname{Sin}\frac{\alpha}{2}}.$$

En différentiant celle-là une fois et celle-ci deux fois, on trouvera

$$\sum_{p=1}^{p=n-1} p \operatorname{Cos} p\alpha = \frac{n}{2}\cdot\frac{\operatorname{Sin}(2n-1)\frac{\alpha}{2}}{\operatorname{Sin}\frac{\alpha}{2}} - \frac{\operatorname{Sin}^2\frac{n\alpha}{2}}{2\operatorname{Sin}^2\frac{\alpha}{2}},$$

$$\sum_{p=1}^{p=n-1} p^2 \operatorname{Cos} p\alpha = \frac{n(2n-1)}{4}\cdot\frac{\operatorname{Sin}(2n-1)\frac{\alpha}{2}}{\operatorname{Sin}\frac{\alpha}{2}} + \frac{n}{4}\cdot\frac{\operatorname{Cos}(2n-1)\frac{\alpha}{2}\cdot\operatorname{Cos}\frac{\alpha}{2}}{\operatorname{Sin}^2\frac{\alpha}{2}}$$

$$+ \frac{n}{4}\cdot\frac{\operatorname{Cos} n\alpha}{\operatorname{Sin}^2\frac{\alpha}{2}} - \frac{\operatorname{Sin} n\alpha \operatorname{Cos}\frac{\alpha}{2}}{4\operatorname{Sin}^3\frac{\alpha}{2}}.$$

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*) Voyez Bierens de Haan, Tab. 160 N:o 5.

En posant ici $\alpha = \frac{m\pi}{n}$, il viendra

$$\sum_{p=1}^{p=n-1} \operatorname{Cos} \frac{mp\pi}{n} = -1 \quad \text{pour } m \text{ pair},$$
$$= 0 \quad \text{pour } m \text{ impair};$$

$$\sum_{p=1}^{p=n-1} p \operatorname{Cos} \frac{mp\pi}{n} = -(-1)^m \cdot \frac{n}{2} - \frac{\operatorname{Sin}^2 \frac{m\pi}{2}}{2 \operatorname{Sin}^2 \frac{m\pi}{2n}};$$

$$\sum_{p=1}^{p=n-1} p^2 \operatorname{Cos} \frac{mp\pi}{n} = -(-1)^m \cdot \frac{n(2n-1)}{4} + (-1)^m \cdot \frac{n}{4} \frac{1+\operatorname{Cos}^2 \frac{m\pi}{2n}}{\operatorname{Sin}^2 \frac{m\pi}{2n}}.$$

En multipliant ces sommes par $-\frac{1}{3}$, $\frac{1}{n}$, $-\frac{1}{2n^2}$ respectivement, on aura

$$\sum_{p=1}^{p=n-1} \operatorname{Cos} \frac{mp\pi}{n} \left(-\frac{1}{3} + \frac{p}{n} - \frac{p^2}{2n^2}\right) = \frac{1}{12} - \frac{1}{4n \operatorname{Sin}^2 \frac{m\pi}{2n}},$$

si m est un nombre pair, mais

$$\sum_{p=1}^{p=n-1} \operatorname{Cos} \frac{mp\pi}{n} \left(-\frac{1}{3} + \frac{p}{n} - \frac{p^2}{2n^2}\right) = \frac{1}{4} - \frac{1}{4n \operatorname{Sin}^2 \frac{m\pi}{2n}},$$

si m est un nombre impair.

En mettant ces expressions dans (63), on trouvera, que m soit pair ou impair,

$$\int_0^1 \frac{x^{m-1} l x}{1-x^{2n}} dx = -\frac{\pi^2}{8n^2 \operatorname{Sin}^2 \frac{m\pi}{2n}}$$
$$- \frac{1}{n^2} \sum_{p=1}^{p=n-1} p \operatorname{Sin} \frac{mp\pi}{n} \left[\frac{2p}{n} \mathrm{H}\left(\frac{p}{n}\right) - \pi l \left(2 \operatorname{Sin} \frac{p\pi}{2n}\right)\right] \ldots\ldots \quad (64).$$

Cette formule présente aussi des cas spéciaux auxquels cependant nous ne nous arrêterons pas ici. Quelques-uns d'eux se trouvent du reste dans les tables de BIERENS DE HAAN Tab. 152.

§ 17.

Par l'intégration indéfinie on trouve

$$\int \frac{lx}{1+2x\,\mathrm{Cos}\,\alpha+x^2}\,dx = \frac{lx}{\mathrm{Sin}\,\alpha}\cdot \mathrm{Arc\ tg}\,\frac{x\,\mathrm{Sin}\,\alpha}{1+x\,\mathrm{Cos}\,\alpha} - \frac{1}{\mathrm{Sin}\,\alpha}\int \frac{dx}{x}\,\mathrm{Arc\ tg}\,\frac{x\,\mathrm{Sin}\,\alpha}{1+x\,\mathrm{Cos}\,\alpha}.$$

Lorsque l'intégrale est prise entre les limites 0 et 1, le terme intégré s'évanouit et l'on a

$$\int_0^1 \frac{lx}{1+2x\,\mathrm{Cos}\,\alpha+x^2}\,dx = -\frac{1}{\mathrm{Sin}\,\alpha}\int_0^1 \frac{dx}{x}\cdot \mathrm{Arc\ tg}\,\frac{x\,\mathrm{Sin}\,\alpha}{1+x\,\mathrm{Cos}\,\alpha}.$$

Posons dans (47) $\varphi_p = \pi - \alpha$; il viendra

$$\int_0^1 \frac{lx}{1+2x\,\mathrm{Cos}\,\alpha+x^2}\,dx = \frac{1}{\mathrm{Sin}\,\alpha}\left[(\pi-\alpha)\,l\left(2\,\mathrm{Cos}\,\frac{\alpha}{2}\right) - 2\left(1-\frac{\alpha}{\pi}\right)^2 \mathrm{H}\left(1-\frac{\alpha}{\pi}\right)\right]. \quad (65).$$

En donnant à α des valeurs spéciales, on retrouvera quelques formules connues. Ainsi en faisant $\alpha = \frac{\pi}{2}$, on aura la formule N:o 11 dans la Table 152, formule que BIDONE a donnée; pour $\alpha = \frac{2\pi}{3}$ on retrouve la formule (56). Si l'on pose $x = e^{-y}$ dans (65), il vient

$$\int_0^\infty \frac{y\,dy}{e^y+e^{-y}+2\,\mathrm{Cos}\,\alpha} = \frac{1}{\mathrm{Sin}\,\alpha}\left[2\left(1-\frac{\alpha}{\pi}\right)^2 \mathrm{H}\left(1-\frac{\alpha}{\pi}\right) - (\pi-\alpha)\,l\left(2\,\mathrm{Cos}\,\frac{\alpha}{2}\right)\right]. \quad (66);$$

En faisant $\alpha = \pi - 2\lambda$, on aura

$$\int_0^\infty \frac{y\,dy}{e^y+e^{-y}-2\,\mathrm{Cos}\,2\lambda} = \frac{2}{\mathrm{Sin}\,2\lambda}\left[\left(\frac{2\lambda}{\pi}\right)^2 \mathrm{H}\left(\frac{2\lambda}{\pi}\right) - \lambda\,l\,(2\,\mathrm{Sin}\,\lambda)\right]. \quad . \quad . \quad . \quad (67).$$

Il faudrait que cette formule s'accordât avec celle qui se trouve dans les tables de BIERENS DE HAAN Tab. 124 N:o 3; mais ce n'est pas ainsi, si l'on ne pose dans celle-ci $\frac{\pi}{2} - \lambda$ au lieu de λ. Cela convient aussi à la formule N:o 4 de la même table, où l'intégrale est un quart de la précédente.

§ 18.

Si dans l'intégrale

$$\mathrm{J} = \int_0^b \frac{l\,(1+ax)}{1+x^2}\,dx$$

on fait $x = \mathrm{tg}\,\varphi$, $a = \mathrm{tg}\,\alpha$, $\mathrm{Arc\ tg}\,b = \beta$, on aura

$$J = \int_0^\beta l\,(1+\operatorname{tg}\alpha\operatorname{tg}\varphi)\,d\varphi;$$

mais parce que

$$l\,(1+\operatorname{tg}\alpha\operatorname{tg}\varphi) = l\operatorname{Cos}(\alpha-\varphi) - l\operatorname{Cos}\varphi - l\operatorname{Cos}\alpha,$$

on a

$$J = \int_0^\beta l\operatorname{Cos}(\alpha-\varphi)\,d\varphi - \int_0^\beta l\operatorname{Cos}\varphi\,d\varphi - \beta\, l\operatorname{Cos}\alpha.$$

En vertu de la formule (24) on trouve

$$\int_0^\beta l\operatorname{Cos}(\alpha-\varphi)\,d\varphi = \alpha\, l\operatorname{Cos}\alpha - (\alpha-\beta)\, l\operatorname{Cos}(\alpha-\beta) + \frac{4\alpha^2}{\pi^2}\left[\mathrm{L}\left(\frac{4\alpha}{\pi}\right) - \mathrm{H}\left(\frac{4\alpha}{\pi}\right)\right] - \frac{4(\alpha-\beta)^2}{\pi^2}\left[\mathrm{L}\left(\frac{4(\alpha-\beta)}{\pi}\right) - \mathrm{H}\left(\frac{4(\alpha-\beta)}{\pi}\right)\right],$$

$$\int_0^\beta l\operatorname{Cos}\varphi\,d\varphi = \beta\, l\operatorname{Cos}\beta + \frac{4\beta^2}{\pi^2}\left[\mathrm{L}\left(\frac{4\beta}{\pi}\right) - \mathrm{H}\left(\frac{4\beta}{\pi}\right)\right].$$

Si l'on met ces valeurs en J, on aura, en employant la formule (29) et en faisant quelques réductions

$$J = \left(\frac{\pi}{2} - \overline{\alpha-\beta}\right) l\frac{\operatorname{Cos}(\alpha-\beta)}{\operatorname{Cos}\alpha} + \left(\frac{\pi}{2}-\beta\right) l\operatorname{Cos}\beta + \frac{\pi}{2}\, l\,2$$

$$+\left(1-\frac{2\alpha}{\pi}\right)^2 \mathrm{H}\left(1-\frac{2\alpha}{\pi}\right) - \left(1-\frac{2(\alpha-\beta)}{\pi}\right)^2 \mathrm{H}\left(1-\frac{2(\alpha-\beta)}{\pi}\right) - \left(1-\frac{2\beta}{\pi}\right)^2 \mathrm{H}\left(1-\frac{2\beta}{\pi}\right). \quad (68).$$

Quand on donne des valeurs spéciales à α et β, quelques formules connues proviennent. Ainsi, α étant $= \beta$, il vient

$$\int_0^a \frac{l\,(1+ax)}{1+x^2}\,dx = -\alpha\, l\operatorname{Cos}\alpha = \frac{1}{2}\, l\,(1+a^2)\,.\,\operatorname{Arc\,tg}\,a,$$

qui pour $a = 1$ ou $\alpha = \frac{\pi}{4}$ se change en

$$\int_0^1 \frac{l\,(1+x)}{1+x^2}\,dx = \frac{\pi}{8}\, l\,2\,.$$

M:r BERTRAND a le premier proposé ces deux intégrales *).

*) Bierens de Haan, Tab. 188 N:o 5 et Tab. 160 N:o 2.

En faisant $\beta = \frac{\pi}{2}$, $\alpha = \frac{\pi}{4}$ ou $b = \infty$, $a = 1$ on aura

$$\int_0^\infty \frac{l(1+x)}{1+x^2}\,dx = \frac{1}{2}\mathrm{H}\left(\frac{1}{2}\right) *) \ldots\ldots\ldots\ldots \quad (69)$$

qui peut aussi être trouvée autrement.

De la même manière on peut avoir

$$\int_0^b \frac{l(1-ax)}{1+x^2}\,dx$$

ou, pour mieux dire, si l'on donne à α et β la même signification qu'auparavant, on n'a qu'à substituer $-\alpha$ au lieu de α en (68) pour avoir la dernière intégrale. On trouve donc

$$\int_0^b \frac{l(1-ax)}{1+x^2}\,dx = \left(\frac{\pi}{2}+\alpha+\beta\right) l\frac{\mathrm{Cos}\,(\alpha+\beta)}{\mathrm{Cos}\,\alpha} + \left(\frac{\pi}{2}-\beta\right) l\,\mathrm{Cos}\,\beta + \frac{\pi}{2}\,l\,2$$
$$+\left(1+\frac{2\alpha}{\pi}\right)^2 \mathrm{H}\left(1+\frac{2\alpha}{\pi}\right) - \left(1+\frac{2(\alpha+\beta)}{\pi}\right)^2 \mathrm{H}\left(1+\frac{2(\alpha+\beta)}{\pi}\right)$$
$$-\left(1-\frac{2\beta}{\pi}\right)^2 \mathrm{H}\left(1-\frac{2\beta}{\pi}\right), \ldots\ldots\ldots \quad (70),$$

où pourtant $\mathrm{tg}\,\alpha\ \mathrm{tg}\,\beta$ doit être < 1.

En posant $a = b = \frac{1}{\sqrt{3}}$; nous aurons

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{l\left(1-\frac{x}{\sqrt{3}}\right)}{1+x^2}\,dx = \frac{\pi}{2}\,l\,2 - \frac{2}{9}\,\mathrm{H}\left(\tfrac{2}{3}\right) \ldots\ldots\ldots \quad (71).$$

§ 19.

En faisant $x = \mathrm{tg}\,\varphi$, on trouve

$$\int_0^\infty \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{u^2+\mathrm{tg}^2\varphi}}$$
$$= \int_0^{\frac{\pi}{2}} \frac{\mathrm{Cos}\,\varphi\,d\varphi}{\sqrt{u^2\,\mathrm{Cos}^2\varphi+\mathrm{Sin}^2\varphi}},$$

*) En déterminant cette intégrale par (68) il faut connaître la valeur de $a^2\,\mathrm{H}(a)$ pour $a = 0$. Très facilement on s'en aperçoit, que $a^2\,\mathrm{H}(a)$ en ce cas s'annule.

d'où pour $u \geqq 1$

$$\int_0^\infty \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \frac{\operatorname{Arc\,Cos}\frac{1}{u}}{\sqrt{u^2-1}} \quad \ldots\ldots\ldots (72)$$

et pour $1 > u > 0$

$$\int_0^\infty \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \frac{1}{\sqrt{1-u^2}}\, l\,\frac{1+\sqrt{1-u^2}}{u} \quad \ldots\ldots (73).$$

Si en (72) nous multiplions par du et intégrons entre les limites $u = 1$ et $u = \alpha$ $(\alpha > 1)$, nous aurons

$$\int_1^\alpha du \int_0^\infty \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \int_1^\alpha \frac{\operatorname{Arc\,Cos}\frac{1}{u}}{\sqrt{u^2-1}}\, du\,.$$

Si l'on intervertit l'ordre de l'intégration ce qui est permis ici, il vient

$$\int_1^\alpha du \int_0^\infty \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \int_0^\infty \frac{dx}{1+x^2} \int_1^\alpha \frac{du}{\sqrt{u^2+x^2}}$$

$$= \int_0^\infty \frac{dx}{1+x^2}\, l\,(\alpha+\sqrt{\alpha^2+x^2}) - \int_0^\infty \frac{dx}{1+x^2}\, l\,(1+\sqrt{1+x^2})\,.$$

En posant $x = \operatorname{tg}\downarrow$, on a

$$\int_0^\infty \frac{dx}{1+x^2}\, l\,(1+\sqrt{1+x^2}) = \int_0^{\frac{\pi}{2}} l\left(1+\frac{1}{\operatorname{Cos}\downarrow}\right) d\downarrow$$

et en vertu des formules déjà proposées

$$\int_0^\infty \frac{dx}{1+x^2}\, l\,(1+\sqrt{1+x^2}) = \mathrm{L}(1) \quad \ldots\ldots\ldots (74).$$

Soit encore $\operatorname{Arc\,Cos}\frac{1}{u} = y$; il viendra

$$\int_1^\alpha \frac{\operatorname{Arc\,Cos}\frac{1}{u}}{\sqrt{u^2-1}}\, du = \int_0^{\operatorname{Arc\,Cos}\frac{1}{\alpha}} \frac{y\,dy}{\operatorname{Cos} y}\,.$$

Moyennant la formule (18) et parce que $\operatorname{Arc\,Cos}\frac{1}{\alpha}$ est $= \frac{\pi}{2} - \operatorname{Arc\,Sin}\frac{1}{\alpha}$, nous trouvons

$$\int_1^{\alpha} \frac{\text{Arc Cos}\,\frac{1}{u}}{\sqrt{u^2-1}}\,du = \frac{\pi}{2}\,l\,(\alpha+\sqrt{\alpha^2-1}) - \mathrm{L}(1)$$
$$+ \left(\frac{2\,\text{Arc Sin}\,\frac{1}{\alpha}}{\pi}\right)^2 \mathrm{L}\left(\frac{2\,\text{Arc Sin}\,\frac{1}{\alpha}}{\pi}\right) \ldots\ldots\ldots \quad (75)$$

et enfin

$$\int_0^{\infty} \frac{dx}{1+x^2}\,l\,(\alpha+\sqrt{\alpha^2+x^2}) = \frac{\pi}{2}\,l\,(\alpha+\sqrt{\alpha^2-1})$$
$$+ \left(\frac{2\,\text{Arc Sin}\,\frac{1}{\alpha}}{\pi}\right)^2 \mathrm{L}\left(\frac{2\,\text{Arc Sin}\,\frac{1}{\alpha}}{\pi}\right) \ldots\ldots \quad (76).$$
$$(\alpha \geq 1)$$

En traitant la formule (73) de la même manière, il vient

$$\int_{\alpha}^{1} du \int_0^{\infty} \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \int_{\alpha}^{1} \frac{du}{\sqrt{1-u^2}}\,l\,\frac{1+\sqrt{1-u^2}}{u}.$$
$$(1 > \alpha > 0)$$

On trouve en permutant l'ordre de l'intégration

$$\int_{\alpha}^{1} du \int_0^{\infty} \frac{dx}{(1+x^2)\sqrt{u^2+x^2}} = \mathrm{L}(1) - \int_0^{\infty} \frac{dx}{1+x^2}\,l\,(\alpha+\sqrt{\alpha^2+x^2}).$$

Puis on aura

$$\int_{\alpha}^{1} \frac{du}{\sqrt{1-u^2}}\,l\left(\frac{1+\sqrt{1-u^2}}{u}\right) = \int_{\text{Arc Sin}\,\alpha}^{\frac{\pi}{2}} l\,\text{Cot}\,\tfrac{1}{2}\varphi\,d\varphi,$$

en faisant $u = \text{Sin}\,\varphi$. Par des réductions faciles et au moyen de (22) on trouve

$$\int_{\alpha}^{1} \frac{du}{\sqrt{1-u^2}}\,l\,\frac{1+\sqrt{1-u^2}}{u} = \mathrm{L}(1) - \text{Arc Sin}\,\alpha\,.\,l\,(1+\sqrt{1-\alpha^2})$$
$$- \left(\frac{2\,\text{Arc Sin}\,\alpha}{\pi}\right)^2 \mathrm{L}\left(\frac{2\,\text{Arc Sin}\,\alpha}{\pi}\right) \ldots\ldots \quad (77).$$

Dernièrement on obtiendra

$$\int_0^{\infty} \frac{dx}{1+x^2}\,l\,(\alpha+\sqrt{\alpha^2+x^2}) = \text{Arc Sin}\,\alpha\,.\,l\,(1+\sqrt{1-\alpha^2})$$
$$+ \left(\frac{2\,\text{Arc Sin}\,\alpha}{\pi}\right)^2 \mathrm{L}\left(\frac{2\,\text{Arc Sin}\,\alpha}{\pi}\right) \ldots\ldots\ldots \quad (78)$$
$$(1 > \alpha > 0)$$

Cette formule vaut aussi pour $a = 0$, car les termes du membre droit s'evanouissent en ce cas, parce que $a^2 \mathrm{L}(a)$ est $= 0$ pour $a = 0$, et le membre gauche se change en

$$\int_0^\infty \frac{lx}{1+x^2}\, dx$$

laquelle est $= 0$, ce qui en outre peut être sans difficulté démontré directement.

§ 20.

La dernière intégrale rappelle une autre que j'ai autrefois proposée, savoir

$$\int_0^\infty \frac{lx}{a^2+b^2x^2}\, dx = \frac{\pi}{2ab} l\frac{a}{b} \cdot \cdots \cdots \cdots \quad (79).$$

Tâchons maintenant de trouver l'intégrale

$$\mathrm{J} = \int_0^\alpha \frac{lx}{a^2+b^2x^2}\, dx.$$

En faisant $x = \frac{a}{b} \operatorname{tg} \varphi$ et $\operatorname{Arc\,tg} \frac{b\alpha}{a} = \beta$, on aura

$$\int_0^\alpha \frac{lx}{a^2+b^2x^2}\, dx = \frac{\beta}{ab} l\frac{a}{b} + \frac{1}{ab} \int_0^\beta l \operatorname{tg} \varphi\, d\varphi,$$

où le dernier terme est $= 0$ pour $\beta = \frac{\pi}{2}$, comme on voit par la formule (10).

Posons $\varphi = \frac{4\beta}{\pi} \psi$; il viendra

$$\int_0^\beta l \operatorname{tg} \varphi\, d\varphi = \frac{4\beta}{\pi}\left[\frac{\pi}{4} l \operatorname{tg} \beta - \frac{2\beta}{\pi} \mathrm{L}\left(\frac{4\beta}{\pi}\right)\right]$$

$$= \beta l \frac{b\alpha}{a} - \frac{1}{2}\left(\frac{4\beta}{\pi}\right)^2 \mathrm{L}\left(\frac{4\beta}{\pi}\right).$$

En mettant cette intégrale dans la précédente, on trouve

$$\int_0^\alpha \frac{lx}{a^2+b^2x^2}\, dx = \frac{\beta}{ab} l\,\alpha - \frac{1}{2ab}\left(\frac{4\beta}{\pi}\right)^2 \mathrm{L}\left(\frac{4\beta}{\pi}\right) \cdots \cdots \quad (80)$$

$$\left(\beta = \operatorname{Arc\,tg} \frac{b\alpha}{a}\right).$$

§ 21.

Très aisément on trouve

$$\int_0^{\frac{\pi}{2}} \frac{\operatorname{Cos} x\, dx}{1+\operatorname{Sin}\alpha \operatorname{Cos} x} = \frac{\pi}{2\operatorname{Sin}\alpha} - \frac{\pi-2\alpha}{2\operatorname{Sin}\alpha\operatorname{Cos}\alpha}$$

$$= -\frac{\pi \operatorname{tg}\frac{1}{2}\alpha}{2\operatorname{Cos}\alpha} + \frac{\alpha}{\operatorname{Sin}\alpha\operatorname{Cos}\alpha} \quad \ldots\ldots\ldots\ldots \quad (81)$$

et

$$\int_0^{\frac{\pi}{2}} \frac{\operatorname{Cos} x\, dx}{1-\operatorname{Sin}\alpha \operatorname{Cos} x} = \frac{\pi}{2}\cdot\frac{\operatorname{tg}\frac{1}{2}\alpha}{\operatorname{Cos}\alpha} + \frac{\alpha}{\operatorname{Sin}\alpha\operatorname{Cos}\alpha} \quad \ldots\ldots \quad (82).$$

En multipliant celles-ci par $\operatorname{Cos}\alpha\,.\,d\alpha$ et en intégrant entre les limites 0 et $\beta\left(\beta < \frac{\pi}{2}\right)$ et en permutant l'ordre de l'intégration, on aura

$$\int_0^{\frac{\pi}{2}} l\,(1+\operatorname{Sin}\beta \operatorname{Cos} x)\, dx = \pi\, l \operatorname{Cos}\tfrac{1}{2}\beta + \left(\frac{2\beta}{\pi}\right)^2 \mathrm{L}\left(\frac{2\beta}{\pi}\right), \quad \ldots \quad (83),$$

$$\int_0^{\frac{\pi}{2}} l\,(1-\operatorname{Sin}\beta \operatorname{Cos} x)\, dx = \pi\, l \operatorname{Cos}\tfrac{1}{2}\beta - \left(\frac{2\beta}{\pi}\right)^2 \mathrm{L}\left(\frac{2\beta}{\pi}\right) \quad \ldots \quad (84).$$

Par addition et soustraction ces intégrales donnent

$$\int_0^{\frac{\pi}{2}} l\,(1-\operatorname{Sin}^2\beta \operatorname{Cos}^2 x)\, dx = 2\pi\, l \operatorname{Cos}\tfrac{1}{2}\beta\ ^*), \quad \ldots\ldots \quad (85),$$

$$\int_0^{\frac{\pi}{2}} l\,\frac{1+\operatorname{Sin}\beta \operatorname{Cos} x}{1-\operatorname{Sin}\beta \operatorname{Cos} x}\, .\, dx = 2\left(\frac{2\beta}{\pi}\right)^2 \mathrm{L}\left(\frac{2\beta}{\pi}\right)\ ^{**}) \quad \ldots\ldots \quad (86).$$

L'intégrale (85) n'appartient pas au sujet, mais on en peut déduire une autre dans laquelle la fonction sous le signe d'intégration a quelque ressem-

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*) Bierens de Haan, Tab. 334 N:o 13.

**) Commençant par $\int_0^{\alpha} \frac{\operatorname{Cos} x\, dx}{1-y^2\operatorname{Cos}^2 x}$ M:r UNFERDINGER a trouvé

$$\int_0^{\frac{\pi}{2}} l\,\frac{a+b\operatorname{Cos} x}{a-b\operatorname{Cos} x}\, dx = 2\int_0^{\operatorname{Arc Sin}\frac{b}{a}} \frac{x\, dx}{\operatorname{Sin} x}, \text{ où } b < a.$$

Voyez Archiv der Math. édit par GRUNERT Tom. LIII. 27.

blance à la même fonction dans (86). Multiplions donc (85) par $\operatorname{Cos}\beta\, d\beta$ et intégrons entre les limites 0 et a $\left(a < \frac{\pi}{2}\right)$; il viendra

$$\int_0^a \operatorname{Cos}\beta\, d\beta \int_0^{\frac{\pi}{2}} l\,(1-\operatorname{Sin}^2\beta\operatorname{Cos}^2 x)\, dx = 2\pi \int_0^a \operatorname{Cos}\beta\, l \operatorname{Cos}\tfrac{1}{2}\beta\, d\beta\,.$$

Lorsqu'on intervertit l'ordre de l'intégration, on trouve

$$\int_0^a \operatorname{Cos}\beta\, l\,(1-\operatorname{Sin}^2\beta\operatorname{Cos}^2 x)d\beta = \operatorname{Sin} a\, l\,(1-\operatorname{Sin}^2 a\operatorname{Cos}^2 x) - 2\operatorname{Sin} a + \frac{1}{\operatorname{Cos} x}\, l\,\frac{1+\operatorname{Sin} a\operatorname{Cos} x}{1-\operatorname{Sin} a\operatorname{Cos} x}\,.$$

En multipliant par dx et en intégrant depuis $x = 0$ jusqu'à $x = \frac{\pi}{2}$, on aura

$$\operatorname{Sin} a \int_0^{\frac{\pi}{2}} l\,(1-\operatorname{Sin}^2 a\operatorname{Cos}^2 x)\, dx - \pi \operatorname{Sin} a + \int_0^{\frac{\pi}{2}} \frac{dx}{\operatorname{Cos} x}\, l\,\frac{1+\operatorname{Sin} a\operatorname{Cos} x}{1-\operatorname{Sin} a\operatorname{Cos} x} = 2\pi \int_0^a \operatorname{Cos}\beta\, l \operatorname{Cos}\tfrac{1}{2}\beta\, d\beta;$$

mais en vertu de la formule (85) on a

$$\operatorname{Sin} a \int_0^{\frac{\pi}{2}} l\,(1-\operatorname{Sin}^2 a\operatorname{Cos}^2 x)\, dx = 2\pi \operatorname{Sin} a\, l \operatorname{Cos}\tfrac{1}{2} a$$

et on trouve aisément

$$\int_0^a \operatorname{Cos}\beta\, l \operatorname{Cos}\tfrac{1}{2}\beta\, d\beta = \operatorname{Sin} a\, l \operatorname{Cos}\tfrac{1}{2} a + \frac{a}{2} - \frac{1}{2}\operatorname{Sin} a\,.$$

Ainsi on a enfin

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\operatorname{Cos} x}\, l\,\frac{1+\operatorname{Sin} a\operatorname{Cos} x}{1-\operatorname{Sin} a\operatorname{Cos} x} = a\pi\,, \quad \ldots\ldots\ldots \quad (87),$$

comme LOBATSCHEWSKY l'a trouvé auparavant *).

*) Bierens de Haan, Tab. 340 N:o 10, jointe à la Tab. 116 N:o 7.

§ 22.

Quoique un nombre encore plus grand d'intégrales qui dépendent des fonctions H(...) et L(...) puissent être indiquées, les intégrales déjà trouvées suffisent pour prouver que ces fonctions se présentent souvent. Les tables de BIERENS DE HAAN contiennent d'ailleurs quantité d'intégrales qui puissent être exprimées par la fonction transcendente de LOBATSCHEWSKY et qui aussi dépendent de H(...) ou de L(...) à cause de la liaison que les fonctions ont entre elles. On peut parvenir à expliquer cela de la manière suivante. Par la formule (15) nous avons déjà vu que la somme d'une série peut s'exprimer par la fonction L(...). Nous verrons maintenant qu'on peut atteindre le même but par la fonction H(...). En plusieurs endroits *) on démontre que

$$\sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p} \operatorname{Cos} p\varphi = l\,2 + l \operatorname{Cos} \tfrac{1}{2}\varphi,$$

autant que $\frac{\pi}{2} > \varphi > 0$.

En multipliant par $d\varphi$ et en intégrant, on aura

$$\sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p^2} \operatorname{Sin} p\varphi = \varphi\, l\,2 + \int l \operatorname{Cos} \tfrac{1}{2}\varphi\, d\varphi.$$

Nonobstant la condition que nous venons de faire, l'intégrale peut être prise entre les limites 0 et α $\left(\alpha \leqq \frac{\pi}{2}\right)$, parce que la seconde série neanmoins converge. On trouve donc

$$\sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p^2} \operatorname{Sin} p\alpha = \alpha\, l\,2 + \int_0^{\alpha} l \operatorname{Cos} \tfrac{1}{2}\varphi\, d\varphi \quad \ldots\ldots\ldots\ldots \text{(A)}$$

$$= \alpha\, l\,2 + \frac{4\alpha}{\pi} \int_0^{\frac{\pi}{4}} l \operatorname{Cos} \frac{2\alpha x}{\pi}\, dx,$$

en faisant $\varphi = \frac{4\alpha x}{\pi}$. En vertu de (24) on a

$$\frac{4\alpha}{\pi} \int_0^{\frac{\pi}{4}} l \operatorname{Cos} \frac{2\alpha x}{\pi}\, dx = \alpha\, l \operatorname{Cos} \frac{\alpha}{2} + \frac{2\alpha^2}{\pi} \left[\mathrm{L}\left(\frac{2\alpha}{\pi}\right) - \mathrm{H}\left(\frac{2\alpha}{\pi}\right) \right].$$

*) Voyez par ex. CATALAN, Traité élém. des séries pag. 106. Paris 1860.

En introduisant cela on trouve par (29)

$$\sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p^2} \operatorname{Sin} p\alpha = 2\left(1-\frac{\alpha}{\pi}\right)^2 \mathrm{H}\left(1-\frac{\alpha}{\pi}\right) - (\pi-\alpha)\, l\left(2 \operatorname{Cos}\frac{\alpha}{2}\right). \quad (88).$$

Si l'on traite la série connue

$$\sum_{p=1}^{p=\infty} \frac{\operatorname{Cos} p\varphi}{p^2} = -l\,2 - l \operatorname{Sin}\tfrac{1}{2}\varphi$$

de la même manière, on obtient

$$\sum_{p=1}^{p=\infty} \frac{\operatorname{Sin} p\alpha}{p^2} = -\alpha\, l\,2 - \int_0^{\alpha} l \operatorname{Sin}\tfrac{1}{2}\varphi\, d\varphi$$

$$= -\alpha\, l\,(2 \operatorname{Sin}\tfrac{1}{2}\alpha) + \frac{2\alpha^2}{\pi^2} \mathrm{H}\left(\frac{\alpha}{\pi}\right) \ldots\ldots\ldots \quad (89).$$

En posant dans l'une ou l'autre de ces formules $\alpha = \frac{\pi}{2}$, on aura

$$\frac{1}{1} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \text{etc.} = -\frac{\pi}{4} l\,2 + \frac{1}{2} \mathrm{H}\left(\frac{1}{2}\right) \ldots\ldots \quad (90).$$

En remplaçant dans la formule (A) α par 2α et φ par $2x$, on trouvera

$$\sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p^2} \operatorname{Sin} 2p\alpha = 2\alpha\, l\,2 + 2\int_0^{\alpha} l \operatorname{Cos} x\, dx$$

et après transposition et après division par 2

$$\int_{\alpha}^{0} l \operatorname{Cos} x\, dx = \alpha\, l\,2 - \tfrac{1}{2} \sum_{p=1}^{p=\infty} \frac{(-1)^{p-1}}{p^2} . \operatorname{Sin} 2p\alpha .$$

C'est cette fonction à laquelle LOBATSCHEWSKY a donné le signe L (α), hors que $(-1)^p$ se trouve sous le signe S au lieu de $(-1)^{p-1}$ chez BIERENS DE HAAN (Tables pag. 23), ce qui est probablement une faute d'impression. En vertu de ce qui précède on a

$$\int_{\alpha}^{0} l \operatorname{Cos} x\, dx = \frac{\pi}{2} l\,2 + \left(\frac{\pi}{2} - \alpha\right) l \operatorname{Cos}\alpha - \left(1 - \frac{2\alpha}{\pi}\right)^2 \mathrm{H}\left(1 - \frac{2\alpha}{\pi}\right); \quad (91)$$

formule qui exprime la relation entre la fonction H (...) et la fonction de LOBATSCHEWSKY.

§ 23.

Passons maintenant au calcul des valeurs numériques des fonctions $H(a)$, $L(a)$. Ce calcul peut se faire de plus d'une manière. Ainsi on peut mettre dans les formules fondamentales (1) et (8) au lieu de Cot ax et Cosec ax leurs valeurs en séries. Cependant celles-ci ne convergent pas bien, que pour des valeurs très petites de l'argument; c'est aussi pourquoi cette manière n'est par généralement convenable. On peut aussi commencer par les formules (23) et (24) dans lesquelles on met

$$l \operatorname{Sin} ax = l\, ax - \frac{2^2 B_1}{1.2} \cdot \frac{a^2 x^2}{2} - \frac{2^4 B_3}{1.2.3.4} \cdot \frac{a^4 x^4}{4} - \text{etc.} \;^*)$$

$$l \operatorname{Cos} ax = - \frac{2^2(2^2-1) B_1}{1.2} \cdot \frac{a^2 x^2}{2} - \frac{2^4(2^4-1).B_3}{1.2.3.4} \cdot \frac{a^4 x^4}{4} - \text{etc.}$$

Si l'on multiplie par dx et intègre entre $x = 0$ et $x = \frac{\pi}{4}$, il vient

$$\int_0^{\frac{\pi}{4}} l \operatorname{Sin} ax\, dx = \frac{\pi}{4}\left\{ l\frac{a\pi}{4} - 1 - \frac{B_1}{1.2} \cdot \frac{\left(\frac{\pi}{2}\right)^2}{2.3} a^2 - \frac{B_3}{1.2.3.4} \cdot \frac{\left(\frac{\pi}{2}\right)^4}{4.5} a^4 - \text{etc.} \right\}$$

$$\int_0^{\frac{\pi}{4}} l \operatorname{Cos} ax\, dx = - \frac{\pi}{4}\left\{ \frac{(2^2-1) B_1}{1.2} \cdot \frac{\left(\frac{\pi}{2}\right)^2}{2.3} a^2 + \frac{(2^4-1) B_3}{1.2.3.4} \cdot \frac{\left(\frac{\pi}{2}\right)^4}{4.5} a^4 + \text{etc.} \right\}.$$

En désignant les coëfficients des puissances de a dans celle-là par C_1, C_3 et en général

$$\frac{B_{2n-1}}{1.2....2n} \cdot \frac{\left(\frac{\pi}{2}\right)^{2n}}{2n(2n+1)} \text{ par } C_{2n-1}$$

on aura

$$L(a) + H(a) = \frac{\pi}{a}\left[l \operatorname{Sin} \frac{a\pi}{4} - l\frac{a\pi}{4} + 1 + C_1 a^2 + C_3 a^4 + \text{etc.} \right] \ldots (92)$$

$$L(a) - H(a) = - \frac{\pi}{a}\left[l \operatorname{Cos} \frac{a\pi}{4} + (2^2-1) C_1 a^2 + (2^4-1) C_3 a^4 + \text{etc.} \right] \ldots (93)$$

qui par addition et soustraction donnent $L(a)$ et $H(a)$. Ces formules furent employées dans le calcul de la table primitive, mais celle-ci a l'inconvénient de converger lentement, aussitôt que l'argument devient $> \frac{1}{2}$. Elles exigent aussi qu'on connaît $l \operatorname{Sin} \frac{a\pi}{4}$ et $l \operatorname{Cos} \frac{a\pi}{4}$. Pourtant cela n'est pas un inconvénient véritable, car si pour éviter ces logarithmes on les développait en

*) Voyez SCHLÖMILCH, Differenzial-Rechnung, pag. 235, 233. Greifswald 1847.

séries, on serait ramené aux séries dont nous avons parlé en commençant et qui convergent trop peu. On les obtient d'ailleurs, si l'on prend les logarithmes vulgaires dans les tables de CALLET et qu'on les transforme en logarithmes naturels. Comme ces logarithmes vulgaires ont 14 décimales, ont peut avoir les logarithmes naturels avec 12 et puis $L(a)$ et $H(a)$ avec 10 décimales sûres. Si ce nombre des décimales ne suffit pas, il faut calculer les dites logarithmes avec plusieurs décimales. C'est aussi ce que j'ai fait. Le calcul s'est fait de la maniére qu' EULER *) a montrée, mais je n'ai pas employé ses chiffres; au contraire j'ai conservé un logarithme de plus dans le membre droit pour augmenter la convergence. C'est ainsi que j'ai obtenu les formules suivantes:

$$l \operatorname{Sin} \frac{m\pi}{2n} = l\,m + l\,(2n-m) + l\,(2n+m) + l\,(4n-m) + l\,(4n+m)$$
$$- 5\,ln - 3{,}707300378070216991 78$$
$$- \frac{m^2}{n^2} \cdot 0{,}098733516712056609 12$$
$$- \frac{m^4}{n^4} \cdot 0{,}000619476053473068 48$$
$$- \frac{m^6}{n^6} \cdot 0{,}000008948239502339 27$$
$$- \frac{m^8}{n^8} \cdot 0{,}000000167095896430 02$$
$$- \frac{m^{10}}{n^{10}} \cdot 0{,}000000003518091370 72$$
$$- \frac{m^{12}}{n^{12}} \cdot 0{,}000000000079180025 56$$
$$- \frac{m^{14}}{n^{14}} \cdot 0{,}000000000001857027 84$$
$$- \frac{m^{16}}{n^{16}} \cdot 0{,}000000000000044766 13$$
$$- \frac{m^{18}}{n^{18}} \cdot 0{,}000000000000001100 33$$
$$- \frac{m^{20}}{n^{20}} \cdot 0{,}000000000000000027 44$$
$$- - - - - - - - - -$$

$$l \operatorname{Cos} \frac{m\pi}{2n} = l\,(n-m) + l\,(n+m) + l\,(3n-m) + l\,(3n+m)$$
$$- 4\,ln - 2{,}197224577336219382 79$$
$$- \frac{m^2}{n^2} \cdot 0{,}122589439025058716 24$$
$$- \frac{m^4}{n^4} \cdot 0{,}001166176295923187 77$$
$$- \frac{m^6}{n^6} \cdot 0{,}000025111509486422 89$$
$$- \frac{m^8}{n^8} \cdot 0{,}000000690808755061 68$$
$$- \frac{m^{10}}{n^{10}} \cdot 0{,}000000021255047279 04$$
$$- \frac{m^{12}}{n^{12}} \cdot 0{,}000000000695359993 44$$
$$- \frac{m^{14}}{n^{14}} \cdot 0{,}000000000023623011 82$$
$$- \frac{m^{16}}{n^{16}} \cdot 0{,}000000000000823031 72$$
$$- \frac{m^{18}}{n^{18}} \cdot 0{,}000000000000029196 11$$
$$- \frac{m^{20}}{n^{20}} \cdot 0{,}000000000000001049 84$$
$$- - - - - - - - - -$$

*) Introductio in analysin infinitorum Tom. I pag. 151 et suiv. Lausannæ 1748.

En calculant j'ai employé 20 décimales dont 18 ont été conservées comme sûres dans la table I.

Depuis nous avons entamé le calcul de $L(a)$ et de $H(a)$ d'après les formules (23) et (24), mais puisque la série pour $L(a) - H(a)$ surtout converge mal, nous avons abandonné les dites séries et employé les formules qu' EULER a donné au lieu cité. En posant dans celles-ci $m = x$, $n = \frac{\pi}{2a}$, on aura

$$l \operatorname{Sin} ax = lx + l\left(\frac{\pi}{a} - x\right) + l\left(\frac{\pi}{a} + x\right) - 2l\pi + 3la$$

$$- \frac{s_2 a^2}{\pi^2} x^2 - \frac{1}{2} \cdot \frac{s_4 a^4}{\pi^4} \cdot x^4 - \frac{1}{3} \cdot \frac{s_6 a^6}{\pi^6} x^6 - \text{etc.}$$

$$l \operatorname{Cos} ax = l\left(\frac{\pi}{2a} - x\right) + l\left(\frac{\pi}{2a} + x\right) - 2l\pi + 2l2a$$

$$- \frac{s'_2 (2a)^2}{\pi^2} \cdot x^2 - \frac{1}{2} \cdot \frac{s'_4 (2a)^4}{\pi^4} x^4 - \frac{1}{3} \cdot \frac{s'_6 (2a)^6}{\pi^6} \cdot x^6 - \text{etc.},$$

où on a

$$s_r = \frac{1}{2^r} + \frac{1}{3^r} + \frac{1}{4^r} + \text{etc.},$$

$$s'_r = \frac{1}{3^r} + \frac{1}{5^r} + \frac{1}{7^r} + \text{etc.}$$

Les valeurs de ces quantités se trouvent au lieu cité. En multipliant par dx et en intégrant entre les limites 0 et $\frac{\pi}{4}$, on obtiendra

$$\int_0^{\frac{\pi}{4}} l \operatorname{Sin} ax\, dx = \frac{\pi}{4}\left[l\pi - 6\,l2 - 3 + \frac{4}{a}\, l\frac{4+a}{4-a} + l(4+a) + l(4-a) + la\right.$$

$$\left. - \frac{s_2 a^2}{1.3.4^2} - \frac{s_4 a^4}{2.5.4^4} - \frac{s_6 a^6}{3.7.4^6} - \text{etc.}\right],$$

$$\int_0^{\frac{\pi}{4}} l \operatorname{Cos} ax\, dx = \frac{\pi}{4}\left[\frac{2}{a}\, l\frac{2+a}{2-a} + l(2+a) + l(2-a) - 2(l2+1)\right.$$

$$\left. - \frac{s'_2 a^2}{1.3.2^2} - \frac{s'_4 a^4}{2.5.2^4} - \frac{s'_6 a^6}{3.7.2^6} - \text{etc.}\right].$$

En introduisant ces valeurs dans (23) et (24), on trouve enfin

$$L(a) + H(a) = \frac{\pi}{a}\Big[l\,\mathrm{Sin}\,\frac{a\pi}{4} - \frac{4}{a}\,l\,\frac{4+a}{4-a} - l\,(4+a) - l\,(4-a) - l\,a - l\,\pi + 6\,l\,2 + 3$$
$$+ \frac{s_2\,a^2}{1\,.\,3\,.\,4^2} + \frac{s_4\,a^4}{2\,.\,5\,.\,4^4} + \frac{s_6\,a^6}{3\,.\,7\,.\,4^6} + \text{etc.}\Big],$$

$$L(a) - H(a) = \frac{\pi}{a}\Big[- l\,\mathrm{Cos}\,\frac{a\pi}{4} + \frac{2}{a}\,l\,\frac{2+a}{2-a} + l\,(2+a) + l\,(2-a)$$
$$- 2\,(l\,2 + 1) - \frac{s'_2\,a^2}{1\,.\,3\,.\,2^2} - \frac{s'_4\,a^4}{2\,.\,5\,.\,2^4} - \frac{s'_6\,a^6}{3\,.\,7\,.\,2^6} - \text{etc.}\Big].$$

Afin que ces formules deviennent propres au calcul numérique, reste à introduire les valeurs des coëfficients constants. Cela fait on aura

$$L(a) + H(a) = \frac{\pi}{a}\Big[l\,\mathrm{Sin}\,\frac{a\pi}{4} - \frac{4}{a}\,l\,\frac{4+a}{4-a} - l\,(4+a) - l\,(4-a)$$
$$- l\,a + 6{,}014153197510271682\,36\ (= k)$$
$$+ a^2\ .\ 0{,}013436126392671384\,09$$
$$+ a^4\ .\ 0{,}000321575131684133\,6$$
$$+ a^6\ .\ 0{,}000002016259996332\,0$$
$$+ a^8\ .\ 0{,}000000017282088377\,0$$
$$+ a^{10}\ .\ 0{,}000000000172454682\,8$$
$$+ a^{12}\ .\ 0{,}000000000001880500\,2$$
$$+ a^{14}\ .\ 0{,}000000000000021730\,2$$
$$+ a^{16}\ .\ 0{,}000000000000000261\,6$$
$$- - - - - - - - - -\Big] \quad \ldots\ldots\ldots \quad (94)$$
$$k + 3\,l\,10 = 12{,}921908476492408734\,41$$
$$k + 6\,l\,10 = 19{,}829663755474545786\,46$$

$$L(a) - H(a) = \frac{\pi}{a}\Big[- l\,\mathrm{Cos}\,\frac{a\pi}{4} + \frac{2}{a}\,l\,\frac{2+a}{2-a} + l\,(2+a) + l\,(2-a)$$
$$- \quad 3{,}386294361119890618\,83\ (= k')$$
$$- a^2\ .\ 0{,}019475045844680818\,95$$
$$- a^4\ .\ 0{,}000091737697526200\,34$$
$$- a^6\ .\ 0{,}000001076693929272\,41$$
$$- a^8\ .\ 0{,}000000016838001876\,75$$
$$- a^{10}\ .\ 0{,}000000000302581019\,97$$
$$- a^{12}\ .\ 0{,}000000000005902721\,18$$
$$- a^{14}\ .\ 0{,}000000000000121628\,84$$
$$- a^{16}\ .\ 0{,}000000000000002607\,14$$
$$- a^{18}\ .\ 0{,}000000000000000057\,59$$
$$- - - - - - - - - -\Big] \quad \ldots\ldots\ldots \quad (95)$$
$$k' + 2\,l\,10 = 7{,}991464547107981\,86\,87$$
$$k' + 4\,l\,10 = 12{,}596347330960733\,54\,91.$$

Le calcul des fonctions $L(a)$ et $H(a)$ d'après ces formules est moins pénible que par les formules (92) et (93). Il s'entend d'ailleurs, qu'il a été facilité par des tables qui donnent les multiples des coëfficients et par l'usage des logarithmes. Une table des multiples de π jusqu' à la $99^{\text{ième}}$ a aussi été dressée. Les fonctions étant calculées pour $a < 1$ par les formules (94) et (95), leurs valeurs pour $a > 1$ ont été trouvées par les formules (30) et (31).

Le nombre des décimales employées n'a jamais été moindre que seize, mais bien plus grand, lorsque l'exactitude semblait l'exiger. Dans la table II que nous avons ainsi obtenu quatorze décimales seulement sont admises comme sûres; quand la quinzième fut 5 ou au delà, la quatorzième a été augmentée par une unité, ce qui à la manière usuelle a été marqué par un — sous la dernière décimale.

La table II fait voir que la fonction $L(a)$ a un minimum pour une valeur de a entre $1{,}40$ et $1{,}45$. Pour avoir cette valeur plus exactement j'ai calculé directement les valeurs de $L(a)$ pour $a = 1{,}41$; $1{,}42$; ... $1{,}46$. Ainsi j'ai trouvé que la fonction $L(a)$ a sa valeur minimum $= 1{,}5854879756$ pour $a = 1.432346\ldots$

Tab I *a*.

a	$l \operatorname{Sin} \frac{a\pi}{4}$ *)		a	$l \operatorname{Sin} \frac{a\pi}{4}$ *)	
0,05	3,237553782985673778	1,95	1,00	0,346573590279972655	1,00
0,10	2,545177863530402766	1,90	1,05	0,308806960717290019	0,95
0,15	2,140998719800144235	1,85	1,10	0,273902823054041327	0,90
0,20	1,855118110440303548	1,80	1,15	0,241667909072166788	0,85
0,25	1,634292637831385545	1,75	1,20	0,211935355500341862	0,80
0,30	1,454807217021735857	1,70	1,25	0,184560526654310642	0,75
0,35	1,304012501179675952	1,65	1,30	0,159417667674614937	0,70
0,40	1,174359005619548757	1,60	1,35	0,136397201336889576	0,65
0,45	1,060978245887243818	1,55	1,40	0,115403529216735288	0,60
0,50	0,960547178929730495	1,50	1,45	0,096353232386996829	0,55
0,55	0,870696771226989808	1,45	1,50	0,079173591910187469	0,50
0,60	0,789679006843551883	1,40	1,55	0,063801367917795992	0,45
0,65	0,716167646897670671	1,35	1,60	0,050181789921618492	0,40
0,70	0,649133042102065660	1,30	1,65	0,038267721482335018	0,35
0,75	0,587760245815822157	1,25	1,70	0,028018970381761336	0,30
0,80	0,531393614981221939	1,20	1,75	0,019401721658290259	0,25
0,85	0,479498241869539857	1,15	1,80	0,012388075739190518	0,20
0,90	0,431632433245094501	1,10	1,85	0,006955677781536932	0,15
0,95	0,387427647312501382	1,05	1,90	0,003087427469846091	0,10
1,00	0,346573590279972655	1,00	1,95	0,000771261104674297	0,05
	$l \operatorname{Cos} \frac{a\pi}{4}$ *)	a		$l \operatorname{Cos} \frac{a\pi}{4}$ *)	a

*) Tous ces logarithmes sont négatifs.

Tab. I *b*.

b	$\pi l\left(2\,\mathrm{Cos}\frac{b\pi}{2}\right)$	b	$\pi\, l\,\mathrm{tg}\frac{b\pi}{4}$
0,05	+ 2,167886650845842327	0,05	− 10,168652192009192155
0,10	+ 2,138667802569247251	0,10	− 7,986212638688718866
0,15	+ 2,089561898791110714	0,15	− 6,704293943250071150
0,20	+ 2,019935347741859150	0,20	− 5,789107139566281461
0,25	+ 1,928854915600240891	0,25	− 5,073329438598286770
0,30	+ 1,815035210717971484	0,30	− 4,482387473872405954
0,35	+ 1,676760716684612787	0,35	− 3,976454501216797554
0,40	+ 1,511771534427786958	0,40	− 3,531706882169646108
0,45	+ 1,317094993599520843	0,45	− 3,132723554158423028
0,50	+ 1,088793045151801065	0,50	− 2,768916786048680717
0,55	+ 0,821572808969726407	0,55	− 2,432671972974444670
0,60	+ 0,508163813314072191	0,60	− 2,118298887008156009
0,65	+ 0,138274494033358696	0,65	− 1,821402572542231043
0,70	— 0,303263676290184526	0,70	− 1,538486222651254091
0,75	— 0,840061870448439826	0,75	− 1,266689275647272336
0,80	— 1,511771534427786959	0,80	− 1,003607721113714767
0,85	— 2,392825575081295240	0,85	− 0,747166026317042894
0,90	— 3,650439336997034210	0,90	− 0,495522184629794436
0,95	— 5,818325987842876537	0,95	− 0,246994171427701551

Tab. II.

a	L(a)	a	H(a)
0,05	31,42669725508703	0,05	31,39438908424216
0,10	15,72953266367248	0,10	15,66485642056968
0,15	10,50439975987563	0,15	10,40723511139226
0,20	7,89734565538246	0,20	7,76751076518722
0,25	6,33760357387281	0,25	6,17485346750475
0,30	5,30160538961874	0,30	5,10562972177352
0,35	4,56498369504588	0,35	4,33540387122700
0,40	4,01557258792459	0,40	3,75193817726264
0,45	3,59107487412786	0,45	3,29285923225159
0,50	3,25412923399851	0,50	2,92072423350624
0,55	2,98097742087087	0,55	2,61168716111544
0,60	2,75579819280904	0,60	2,34983152896448
0,65	2,56765672237042	0,65	2,12411841423696
0,70	2,40876181999231	0,70	1,92664205123469
0,75	2,27342074766783	0,75	1,75158270281047
0,80	2,15738653617294	0,80	1,59455164108970
0,85	2,05743632081250	0,85	1,45216664625892
0,90	1,97109102214471	0,90	1,32176821010688
0,95	1,89642450801690	0,95	1,20122439449383
1,00	1,83193118835444	1,00	1,08879304515180
1,05	1,77643291602082	1,05	0,98302189098881
1,10	1,72901315088182	1,10	0,88267401023361
1,15	1,68897071312217	1,15	0,78667031899360
1,20	1,65578826685027	1,20	0,69404326211420
1,25	1,62911260557467	1,25	0,60389737297239
1,30	1,60874527482100	1,30	0,51537313941596
1,35	1,59464336776062	1,35	0,42761080201344
1,40	1,58693175327521	1,40	0,33971029795948
1,45	1,58592991333550	1,45	0,25068234357294
1,50	1,59219959757703	1,50	0,15938310523344
1,55	1,60662485584571	1,55	0,06441990195163
1,60	1,63054628759280	1,60	− 0,03599464650311
1,65	1,66599265522862	1,65	− 0,14428373928079
1,70	1,71610102385401	1,70	− 0,26393437759509
1,75	1,78593621615195	1,75	− 0,40032333458530
1,80	1,88425955733999	1,80	− 0,56249134723311
1,85	2,02794534341776	1,85	− 0,76756416803145
1,90	2,25581938097658	1,90	− 1,05459498648275
1,95	2,69486362528518	1,95	− 1,55077237621393
$\frac{1}{3}$	4,78556924902753	$\frac{1}{3}$	+ 4,56723722884344
$\frac{2}{3}$	2.51175891332750	$\frac{2}{3}$	+ 2,05547831551594
$\frac{4}{3}$	1,59864381136340	$\frac{4}{3}$	+ 0,45683450415254
$\frac{5}{3}$	1,68086706142489	$\frac{5}{3}$	− 0,18268948915374

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VOL. IX.

FASC. II.

1875.

SMITHSONIAN INSTITUTION
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NOVA ACTA

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SCIENTIARUM

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SERIEI TERTIÆ VOL. IX.

FASCICULUS POSTERIOR.

UPSALIÆ,
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INTRODUCTIO.

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Proximo biennio, quod post Acta Regiæ Societatis Scientiarum Upsaliensis (Ser. III. Vol. VIII) anno 1873 mense Novembri edita præteriit,

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MICROMETRICAL OBSERVATIONS

OF

500 NEBULÆ

BY

D:r HERMAN SCHULTZ.

(Presented to the Royal Society of Upsala, the 18:th April 1874).

UPSALA,
PRINTED BY ED. BERLING.
1874.

INTRODUCTION.

The earliest systematical study of the nebulæ is contemporaneous with the rise of stellar astronomy under the auspicies of WILLIAM HERSCHEL, and may half a century be said to have been a kind of Herschelian science, with which scarcely any others than the HERSCHEL family occupied themselves. It is only within the last few decennia that some few other men of science, e. g. Lord ROSSE, LASSEL, D'ARREST and some others have begun to turn their attention in this direction and with more or less magnificient instruments have prosecuted the researches in this quarter commenced by Sir WILLIAM and Sir JOHN HERSCHEL.

The first question as regards these objects, that observation had to decide, was that of their absolute and relative frequency in the heavens, and the first problem was accordingly to determine on a large scale with sufficient accuracy for the purpose approximate positions, so as to obtain a catalogue of nebulæ as comprehensive as possible. But as these objects differ widely in their external appearance, and thus offerd the greatest variety in singular forms, size, brilliancy, etc., it further became a matter of great importance, by means of accurate descriptions and figures of the occurring nebula-types, to endeavour to gain some knowledge of their apparent nature in general, so as to provide material for the possible study of their physical construction. In this respect the resolubility for example of the nebulæ and their variability as regards form and brilliancy became questions of the first importance. Hence soon followed the classification of the nebulæ in more or less natural groups, the chief value of which lay in the facilities they offered for prosecuting the study of the objects' peculiar properties. The fundamental works on this subject, not only those of a general character but monographies referring to particular remarkable nebulæ, accordingly treat almost exclusively the above mentioned chief points of nebular study, even when we include the spectro-analytical researches of late years instituted by HUGGINS, D'ARREST and others.

An important point in regard to nebular science, but which, among the many other interesting questions that here present themselves, has not hitherto received sufficient attention, is the question of the proper motion of the nebulæ, and the principal object of the present work is to offer a contribution to that particular branch of research in the nebular-world. The question of the proper motion of nebulæ and telescopic clusters, which must necessarily be preparatory to that of the dynamical relations within the clusters and between the nebulæ and the starry universe in a restricted sense, is without doubt one of the most important in the present state of astronomical science; and the demand for a body of observations, which it is our duty to leave to posterity, appears in this case to be inignorable, although the study in question is as yet so little advanced that we can hardly hope in our own time to obtain any important results from our observations.

The question of the nebulæ's proper motion had indeed been early raised, and the determinations made of their positions were, some at least of them, intended for the future solution of the questions relative to this subject. It seems however to be beyond a doubt, that, with the exception of the positions of some few more brilliant nebulae, those, which were determined before LAUGIER'S observation at Paris and those af D'ARREST at Leipzig, are too deficient in accuracy for it to be probable, that they can be of any real use for that purpose. It is also a well known fact, that the above mentioned series of observations were the first that were undertaken exclusively for the purpose here referred to, whereas the chief object as well of Sir W. and Sir J. HERSCHEL'S, Lord ROSSE'S and LASSEL'S work, as of D'ARREST'S later observations, was, as has been already intimated, such as did not require the greatest possible precision in the positions. The uncertainty in W. HERSCHEL'S positions is never less than 1′ or 2′. Already those of J. HERSCHEL are far more accurate, inasmuch as that irrespective of confusions and incidental errors, the probable errors in a position do not, according to D'ARREST, exceed 1ˢ and 20″. D'ARREST'S Copenhagen-observations, approaching in form nearly those of Sir J. HERSCHEL, yet are superior to them as regards the absence of incident errors and confusions; but even here, according to the author's own statement, the probable errors in a position amount to about 0,ˢ8 and 18″. But although the right ascensions and declinations given in all these series of observations do not possess sufficient accuracy for the purpose that we have before us, yet we must not overlook the fact, that in many instances these catalogues contain differential determinations of the positions of nebulæ referred to neighbouring

small stars, which will without doubt be found of the greatest importance in the future study of their proper motions. D'ARREST'S work "Sid. Neb. Observationes Havnienses" in particular is extremely rich in determinations of this kind.

The first series of determinations of the positions of nebulæ, made on the other hand exclusively for the purpose of forming a basis to the study of their proper motion, dates from a period no longer back than about 20 years, and consists, as has been already intimated, of LAUGIER'S observations of 53 brighter nebulæ made at Paris and D'ARREST'S Leipzig-observations of more than 200 nebulæ. These two series of observations, which were both executed with small instruments, appear to be of about equal accuracy and the probable error in the definitive positions to amount to about 6″. These works have the high merit of having originated a new and important branch in the study of the nebulæ; and D'ARREST has done especial service to this study, by showing, that when what is required is simply good determinations of positions, a much greater number of nebulæ than has been usually supposed may be advantageously observed with instruments of but very moderate dimensions. But his series of observations is chiefly and especially important as proving beyond the possibility of a doubt, that the positions of nebulæ in general are determinable with far greater accuracy than it had been previously usual to suppose; and D'ARREST'S work thus made an epoch in the study of nebulæ by freeing it from deterring prestige which had before that period been attached to it. D'ARREST soon found followers, who prosecuted his so happily commenced work. We have accordingly latterly been provided with accurate positions of nebulæ by AUVERS, SCHMIDT, SCHÖNFELD, VOGEL, RÜMKER, STEPHAN and others; there are moreover admirable monographies in this way, as for example BOND'S and LIAPOUNOV'S excellent measures of the stars in the great nebula of Orion. Among the general series of observations of this kind SCHÖNFELD'S famous Mannheim-observations of 235 nebulæ are the most comprehensive; and, as the probable error in one of SCHÖNFELD'S positions appears not to exceed 2″ or 3″, and in for exemple those of AUVERS and VOGEL to be still less, it is evident that by a sufficient multiplication of observations the determination of positions may be easily carried to a very high degree of accuracy. as indeed my own experience has shown me.

My own observations, which were commenced in 1863, and have ever since been continued with more or less regularity, consist of determinations by means of a wire-micrometer of differences of right ascension and declination between about 500 nebulæ and neighbouring stars of sufficient bright-

ness to admit of being conveniently observed in the meridian and thus to give the right ascensions and declinations of the observed nebulæ. I have more over in many instances also compared the nebulæ with fainter stars in their immediate vicinity, or, where it was found more convenient to do so, compared such neighbouring stars of greater interest with the star of comparison employed in observing the nebula. The whole number of single measures, on which the results here registered are founded, amounts to about 12000. I have in all cases appended to the position as careful a description of the nebula as circumstances permitted me to give. I ought at once to mention that this series of observations is in many cases deficient in that completeness of detail, which is desirable, as well, not to mention other points, with respect to the observation of the fainter neighbouring stars, of which but few have as yet been observed, as also with respect to satisfactoriness in the number of positions of the nebulæ as refferred to the stars of comparison. As however my observations have already been protracted through so long a period, I have, in order not unnecessarily to isolate my work, considered it wisest no longer to defer the publication of the materials here before us, especially as I anticipate from such a communication some advantage towards the final completion of the work. I confine myself for the present to communicating the evening-means of the differential-coordinates observed, and accordingly defer the calculation of definitive means untill I have fully completed my own observations and succeeded in obtaining reliable positions for all the stars of comparison employed, so as to be able at the same time to give definitively the right ascensions and declinations of the observed objects.

It may seem astonishing that I have not during so many years collected a richer harvest of observations. But those, who are experienced in the observation of such objects, will not need to be reminded how much time is required for the determination by the parallel-line-micrometer of nebulæ's positions, and that under a northern sky such a work must necessarily proceed with distressing slowness. The heavens at Upsala so rarely for any length of time free from clouds, the almost always disturbed atmosphere, the long twilight at the seasons of the year most favourable for observation, and the remarkably brilliant aurora borealis during a considerable part of the last few years, are all local circumstances that have left but very few nights here disposable for the observation of nebulæ, and even among these only an extremely small number on which the observations could be made under circumstances favourable for the purpose. Add to this, that so many clear nights were, in consequence of bright moonlight

entirely unfit for these observations, which can not be advantageously made except in a dark and clear sky. It should be further remarked that the aurora is in many instances as formidable an enemy to these observations as moderate moonlight, since on such occasions, not to mention the illumination of the sky, the air, even when the aurora is but faint, often becomes very opake, just as if the heavens were curtained with a fine veil of cirrhi. All these circumstances combined render it impossible to follow out in detail any determinate plan, so that long periods often have intervened before a control-observation could be obtained, and in some cases indeed this confirmation is still wanting. In many cases for example I have had to look up the same nebula, not belonging to the brighter class, as often as five or six times before I was fortunate enough to get an evening on which I could see and observe it satisfactorily.

Besides all this it must, as I have already said, be added, that, even under tolerably favourable circumstances, the work must proceed very slowly, whenever greater accuracy is aimed at and fainter nebulæ are observed. I can not on an average assume, that a satisfactory determination of position can be obtained in less time than an hour, even when a nebula is pretty bright and the star of comparison not too distant. And though in some cases this time may be somewhat curtailed, yet the same operation will require under less favourable circumstances frequently double that time. In some cases in order to get through an observation and thus obtain something like a position, the determinations have therefore not so unfrequently — especially in the earlier years of the series — been based upon a decidedly insufficient number of individual observations. Moreover if our business be in certain cases to determine a nebula's position with respect to the fainter neighbouring stars, and to study it under favourable atmospheric circumstances in a dark field, it is evident, that a long time must in most instances elapse, before one could be ready with a single nebula. Here is also the place to remark, that the determination of accurate positions of nebulæ is a work, that soon tires the eye, so that even during the most favourable evenings it can not be arbitrarily continued for any length of time. On the most favourable occasions I have somtimes been able to continue working uninterruptedly for 6 hours, or even somewhat longer, but in most cases, when the state of the atmosphere was not so excellent, half that time has often been found sufficient to tire the eye, and render further observation impossible.

What has here been said could seem to lead to the conclusion, that the results obtained can hardly be considered as in any reasonable degree pro-

portional to the time and labour actually employed in observation, or in other words, that the observation of nebulæ with the parallel-line-micrometer is too tedious a work and, especially in so high a latitud, oltogether inappropriate. On the other hand I can only express my opinion that the parallel-line-micrometer is perhaps the only one of the instruments at present employed, with which a really sharp determination of a nebula's position is possible, and moreover remind the reader, that most observatories are, as regards climatic circumstances, curiously enough, by no means advantageously situated, whereas in localities, which would in those respects be best fitted for observation, there are either very few observatories or none at all. It evidently follows from this, that an astronomer can not afford to be very critical in the choice of the objects he observes, but is rather obliged to try what can he done even when circumstances are not particularly favourable. It is moreover a known fact in the history of science that the best results have not always been exactly those, which have been obtained under the most favourable circumstances; and indeed a countinghouse calculation of the proportion of labour bestowed to value received is hardly applicable in science, where so many other factors usually enter into the calculation. I should however in all probability long ago have abandoned the observation of nebulæ, had any such series at that time been systematically carried on at any more favourably situated observatory; but, as this, as far as I am aware, did not appear to be anywhere the case, I considered myself bounden to continue in the path I had entered upon at least untill the results obtained could be considered as forming a respectably intire whole of a tolerably extensive compass, although reduced within considerably narrower limits than those originally intended.

When I first began to occupy myself with the observation of nebulæ, I was far from appretiating the difficulties I should have to meet. I soon satisfied myself, that, with the STEINHEIL 13-foot-refractor that I employed, I ought, at least in a dark field, to be able to see all the objects registered in Sir JOHN HERSCHEL'S northern catalogue (Phil. Transact. 1833) and even with tolerable certainty to determine the positions of the greatest part of them with the parallel-wire-micrometer, if bright spider-lines on a dark field were employed. Thus arose the plan of micrometrically determining the greater part of the small centrally condensed nebulæ occurring in that catalogue, that lie north of the equator. I imagined that such a work, resolutely prosecuted, could not occupy very many years, even though it were of course intended to observe every nebula several times, and micrometrically to determine the most important faint neighbouring stars. But although

I often observed by moonlight, and did not shrink from other unfavourable circumstances, it soon appeared, that the work required so much time, that the original plan could not be carried out. It is then no longer my intention sensibly to extend the compass of the observations beyond the limits of the 500 observed but indiscriminately chosen nebulæ, but only in the course of the following years to endeavour to consolidate the results already attained. In order to avoid being obliged by the above mentioned circumstances entirely to abandon the observation of nebulæ and presserve an in some degree consistent plan of operations, I found it best, as I have elsewhere lately explained, to modify the original plan as regards objects to be observed, so that it has now for some years even included the micrometrical measurement of telescopic clusters, a work which can at least partially be performed in spite of moonlight, and indeed generally under circumstances unfit for the observation of nebulæ.

If then on the one side I have arrived by experience at the discouraging conclusion, that a single person's powers are very limited in turning to account on a more extensive scale, though only in one special direction, the vastly rich material, which the nebular world offers; I have nevertheless on the other hand fully convinced myself that a relatively great accuracy, not in only a few but in fact in most cases, can be attained in the determination of the nebulæ's positions.

From a number of arbitrarily chosen differences between the evening-means of the observed quantities and the respective media of them I have in the following manner approximatively computed the outstanding errors in the results. Considering a number μ of nebulæ, let ε be one of the differences in question and $\Sigma\varepsilon^2$ the sum of the squares of the differences in the μ nebulæ; n the number of nights any one of the nebulæ was observed, Σn the whole number of nights for all the μ nebulæ; and r the mean error in the result of one night, all the results being supposed to have the same weight; we have

$$r = \sqrt{\frac{\Sigma\varepsilon^2}{\Sigma n - \mu}}$$

Meanwhile, as each nebula is here only observed very few nights, the right use of this formula in the present case may seem doubtful, because μ must obtain too great a weight in regard to Σn, for it to be possible that the resulting values should become very trustworthy. Under such circumstances it would no doubt give a nearer approximation to the truth simply to put $\mu = 1$, although in this way r should come out a little too small,

whereas the true value of μ always must make r somewhat too great. Consequently I think it most satisfactory to compute r for both these hypotheses, as in this manner two limits at least are obtained, between which the true value of the quantity sought must lie. The computation of the errors on these principles has given the following numbers, where the objects are divided in 3 groups according to the 5 classes of brightness here adopted, and all the media employed are supposed to have approximately the same weight.

Probable Errors in the Results of One Evening.

$(\Delta\alpha^s . \mathrm{Cos}\delta)$.

Class of Brightness.	μ	Σn	$\Sigma\varepsilon^2 . \mathrm{Cos}^2\delta$	Prob. Error.	
				μ = true value.	$\mu = 1$.
1 and 2	37	101	1^s1788	0^s092	0^s073
3	98	221	2,5122	0,096	0,072
4 and 5	86	198	4,2166	0,130	0,099

Probable Errors in the Results of One Evening.

$(\Delta\delta'')$.

Class of Brightness.	μ	Σn	$\Sigma\varepsilon^2$	Prob. Error.	
				μ = true value.	$\mu = 1$.
1 and 2	37	100	87″41	0″79	0″63
3	95	217	287,87	1,03	0,78
4 and 5	86	197	429,36	1,32	1,00

As I presuppose that my observations are free from sensible constant errors of an instrumental nature, and as the probable errors here given are not based upon the deviations of single observations from the medium of the corresponding evening but are deduced from the deviations between the media of the different nights; they ought, I conceive, to give on an average a very fair idea of the real errors of the positions. Small constant differences will no doubt often be found between definitive positions by different observers, but I am fully convinced, that the indications of greater personal equations, which have been found in nebular observations, depend more on instrumental circumstances than on the observer's person.

Before terminating this introduction it appears desirable, with respect to the conditions necessary for the obtaining of a trustworthy nebular position, to add the following. First it ought to be mentioned, that in my series of observations some cases occur, in which the discrepancies between the various positions are very considerable, although the character of the objects does not appear to be such as to lead us to expect anything of the kind. The true explanation of these discrepancies, whether they are real errors of observation or not, can only be obtained by means of new observations; but the possibility of committing errors of observation to this amount does not appear to me a priori inexplicable, but needs not nevertheless preclude the possibility in general of determining the positions with great accuracy. It is in fact well known, that the rareness of the occasions, on which nebulæ can be advantageously observed, causes the work during the few hours available for the purpose to be too frequently prosecuted with a sort of anxious haste, which may easily account for errors of various kind creeping into the observations. When experience had taught me, that nebula-observations thus hurriedly made could only have a very ambiguous value, I never afterwards attempted to execute an observation in too scanty a time, but in the earlier period of my observations the contrary was not so unfrequently the case. Such discordant positions can even arise from a partially false perception of the objects on occasions very disadvantageous for observation, when, according to my present opinion, nebulæ ought not to be observed at all. Bad atmospheric condition may in fact utterly disfigure the nebula and thus often give the observer an intirely false idea of it. But even, when the atmosphere is unexceptionable, an observer may easily commit a sensible error of pointage, unless he give himself sufficiently time; for it often happens, that the point, which for objective reasons ought to be selected as that, to which the instrument should be directed, is on close inspection pretty conspicuous, but would not strike the eye in a hasty and passing view. In order that a nebula's position may consequently be reliably determined, it is in my opinion generally speaking indispensably necessary, that the observations should be made with all due calmness and in a sufficiently favourable atmosphere. It is moreover perhaps not too much to assume, that the satisfactory carrying out of nebula-observations requires more custom and more practice than those of many other objects.

It belongs lastly, as I have already intimated, to the conditions necessary for the obtaining of a trustworthy nebular position, that the instrumental means are fitted for the purpose, that is generally speaking to say,

that the observations be executed with an instrument of sufficient optical power and with an appropriate eye-piece, and that the method of observation employed or the micrometer be fit for the purpose. As regards the necessary dimensions of the telescope it is of course impossible to offer any general opinion. I know however from my own experience, that with an instrument of the size of the Upsala refractor much may be done; but I have reason to believe it desirable, that nebula-observations should not be undertaken with a much smaller instrument, especially if an illuminated micrometer be employed. I hardly ever found any difficulty in the choice of an eye-piece, a magnifying power of about 200 times appearing to be the most advantageous in almost all determinations of position, though in some cases a power of 300 times could be used with advantage. As regards the choice of the micrometer, as far as I can trust my judgement, the choice would seem wholly to lie between the parallel-line-micrometer with illuminated spider-lines on dark field and the heliometer. With respect on the other hand to the ring-micrometer, I must confess that I can not understand its being otherwise than a totally inappropriate instrument, as a part of the object at the very moment of observation is always necessarily concealed by the ring, and I can not in many cases imagine, how it is possible sharply to point a nebula, without having at the same instant the whole of its central portion before my eye. The ring-micrometer is moreover an instrument, which must, more than any other, leave free room for an observer's idiosyncrasies, and thus almost unavoidably give rise to sensible personal discrepancies in the results obtained, which in all probability would have been very trifling, had the observations been made with an instrument, which does not give such latitude to subjective fancy. The fact, that relatively very excellent results have been attained by the use of the ring-micrometer for the observation of nebulæ, of course only proves, that a skilful observer can effect much with an inferiour or even an unfitting instrument; but it by no means follows from this, that the parallel-hair-micrometer is not preferable, whenever circumstances admit of its application and we have to determine the positions with the highest degree of precision, which the nature of the objects and other circumstances permit.

THE INSTRUMENT.

The observations have been made at the University-Observatory at Upsala with the same 13-foot telescope by v. STEINHEIL, which was used for the measurement of the stargroup 20 Vulpeculæ, and as the instrumental arrangements were in both instances exactly the same, and I have given a detailed account of them in my essay on that cluster (Kongl. Svenska Vetenskaps-Akademiens Handlingar. Band 11, N:o 3), I do not consider it here necessary to repeat all those details. I therefore confine myself to remarking that the equatoreal has always been sufficiently accurately adjusted, both as regards the relative position of the axes and the instrument's orientation with respect to the locality, to insure the outstanding errors being insensible in the differential observations.

In order to give some idea of the magnitude of the outstanding errors in the instrument, I here give a few approximative mediumnumbers for the whole period of the observations. With regard to these numerical data it should however be remarked, that the numbers in the various sections are not comparable with each other, the telescope having during that period been several times dismounted, and the adjustments in the intervals for various reasons altered.

The signs employed in the table below have the following significations.

$\Delta\varphi$ = the angle between the instrument's polar axis and the axis of the earth.
Δa = the angle between the instrumental and celestial meridians.
i = the error of the angle between the instrument's two axes of rotation.
c = the error of collimation of the optical axis.

Periods.	$\Delta\varphi$	Δa	i	c
Autumn 1862—Summer 1863	−0′3	+0′8	−0′4	−2′4
Summer 1863—Autumn 1864	−0,4	+0,2	−0,2	+0,2
Spring 1865—Spring 1866	−0,5	+0,2	+1	−1
1866 Aug.—1867 July	+0,3	−2,4	+0,1	+0,3
1867 July—1868 June	+0,3	+0,5	+0,2	+0,2
1868 July—1870 June	−0,1	+0,2	−0,1	−0,3
1870 Aug.—1874 May	+0,1	+0,3	0,0	+0,2

I ought also to add that the relative flexure of the tube on an average amounts to 0′,2 or 0′,3, and has never sensibly affected the centering of the glasses in the tube, of which I have convinced myself by direct examination with the telescope placed horizontally in opposite positions. A slight

focal-adjustment under different temperatures has, at the different seasons of the year, been found necessary, and I am certain that no serious errors of this kind have presented themselves during the observations.

In the work referred to I have given a detailed account of the two parallel-wire-micrometers employed (I and II), of the reasons for procuring the new micrometer, of the advantages possessed by microm. II over the older one, of the examination of the micrometers, etc. Micrometer I was used up to 1868 May 9 (night 215), and micrometer II was first used in observation Aug. 11 (night 216) of the same year. As regards micrometer I, here, as in the reduction of 20 Vulp., care was taken to determine the coincidences made use of nearly in the same position of the screw as in the corresponding observations. In the reductions of the observations, made with microm. II, the corrections given loc. cit. p. 24 for the different breadth of the threads of the screw have been employed here also. It is tolerably clear, that these corrections depend more on the mounting of the screw than on any really existing difference in the breadth of the threads. It is also shown (loc. cit. pp. 21—23), that the corrections due to the form of the threads are extremely small and accordingly of no practical consequence in the reduction of the observations. The values of a single revolution of the screws here made use of are (loc. cit. p. 26) respectively

For micrometer I . . . . . 17″,230
— micrometer II . . . . . 12″,887.

The observations have not indicated any variation in the value of a revolution for differences of temperature.

The timekeeper used throughout the whole series of observations is a Chronometer by Kessels N:o 1329. It is adjusted to sidereal time and beats half-seconds.

RESULTS OF THE AUTHOR'S OBSERVATIONS.

This section of the work presents the tables of the differential coordinates resulting from each several evening's observations which constitute the main matter in the present communication, preceded by such tables and notices as are necessary for their illustration. These are followed by three notes — note I containing an approximate catalogue of the right ascensions and declinations of the stars of comparison; note II different supplementary details of observation; and lastly note III a table of the nights of observation.

The observed differential coordinates, as has already been intimated, consist of differences of right ascension observed with chronometer with eye and ear, and of differences of declination measured with the micrometer-screw, in which illumination of the wires in dark field always was employed. On many occasions it would have been desirable to determine angles of position and distances, but through the want of a good driving-clock such measures in general were frustrated; and this want was often in the nebula-observations even more sensible than in those of the star-clusters at least of Cl. VII and Cl. VIII. In the first years of observation I indeed exceptionally tried to measure distances, but under the existing circumstances always found this operation too difficult and unsatisfactory to be repeated. Angles of position, as well as it could be done, I have nevertheless on several occasions measured with better success; but in the last years even those measures have been generally avoided and consequently only executed, when the circumstances were more favourable and such measures to be regarded as quite indispensable. The dimensions of the few nebulæ, which on favourable occasions could be measured with passable exactness — as for instance the planetary — were also for the said reason only determined in the directions of the parallel and declination-circle. For the determination of the positions of the axes of elongated nebulæ I have, as often as a relatively exact measure was possible and the time permitted it, observed their angles of position, which of course, even without a driving-clock, could be done with an accuracy quite sufficient for the purpose. The results of those and the before mentioned measures of the sizes of some nebulæ I give in note II below. Concerning the observations in general it ought here to be remarked, that in almost all cases a magnifying power of 210 was employed, wherefore it is not necessary to indicate the few and in general insignificant exceptions from that rule; some such cases are however mentioned in note II. As my eye generally requires strong illumination, 320 m. p. in most cases made the objects too faint, but with 160 m. p. the details of the nebulæ on the other hand did not come out with the precision, which by the use of 210 m. p. was guarantied without any serious sacrifice of brightness.

The differential coordinates are by the application of all astronomical corrections first reduced to the mean equinox for the commencement of the year of observation, although the numbers of reduction — with exception of those for refraction in some cases — were usually exceedingly small; and the results so obtained then by means of precession reduced to the mean equinox of 1865,0.

In these calculations, as has already been intimated, all the instrumental corrections — with exception of those for the screw of micr. II — were negliged as imperceptible; and the reductions for the rest performed in a manner analogous to that employed for 20 Vulp. (l. c., § 5). These reductions were always carried out with 3 and 2 decimals in the numbers of reduction for $\Delta\alpha^s$ and $\Delta\delta''$ respectively, and also the original uncorrected media of $\Delta\alpha^s$ were calculated with 3 decimals of the second, although in the present table (col. 3) only two decimals are given. In the few cases, in which the comparison-star showed a sufficiently proved proper motion, such motion was duly taken account of in the reduction to the general epoch 1865,0 in th 5th and 6th columns of the tables of observation.

As the zero-points of the micrometers were usually left untouched for considerable times, and as the apparent variations of the coincidences of the wires were indeed in general so small, that they evidently must principally depend on the uncertainty of the single coincidences; I have in the reductions not employed the coincidences obtained immediately on every single night, which are given in note III below, but instead of them the coincidence-means, which are communicated in the here following list. During the spring 1863 (nights 1—18), as in these months the zero-point was several times changed, the original coincidences were nevertheless immediately employed. Concerning the coincidences of micr. I it is first to be remarked, that, as this micrometer has no ocular-slide, it was, in order to render it possible to measure distances exceeding half the field, furnished with 3 moveable wires (A, B, C); and secondly, that up to the autumn of 1865 two fixed wires (I and II) were employed alternately. These last distant about 16″ from each other formed the system, that was employed in the observation of Mars during the opposition 1862 and was retained unchanged during the next following years. The coincidence-numbers for micr. I, corresponding to the position of the screw above the centre of the micrometer, are in the following list marked with an asterisk. In the case of micr. II such discrepancies between the coincidences, depending upon the positions of the micrometer-screw, never occurred (l. c., p. 20). The numbers enclosed in parentheses express the number of nights, from which the single coincidences are taken, where also in many instances coincidences of nights are used, when nebulæ were not observed — thus for example is explained the coincidence of 1870 Aug. 8, which reposes on coincidences from 36 evenings, although during this period nebulæ were on no other evening observed. The coincidences of micr. II were always taken with m. p. 320, those of micr. I often also with m. p. 210.

Means of Coincidences for Micrometer I

1863—1864.

A—I	1863 July 28	—	1863 Oct. 24	24ʳ982	(7)*
	1864 April 10			24,989	(1)*
	1863 Aug. 17	—	1863 Nov. 14	24,973	(8)
	1864 March 16	—	1864 May 5	24,978	(4)
A—II	1864 Febr. 19			24,191	(4)*
	1863 Sept. 29			24,178	(2)
	1864 Jan. 24	—	1864 May 4	24,186	(13)
B—I	1863 Aug. 14	—	1863 Sept. 21	33,198	(4)*
	1864 Apr. 13			33,202	(1)*
	1864 Apr. 14	—	1864 May 4	33,186	(5)
B—II	1863 Aug. 2	—	1863 Sept. 2	32,400	(6)*
	1864 Apr. 8	—	1864 Apr. 12	32,410	(4)*
	1863 Sept. 18			32,393	(4)
	1864 March 13			32,401	(3)

1864—1865.

A—I	1864 Oct. 6	—	1865 Jan. 7	30ʳ951	(5)*
	1865 Febr. 27	—	1865 March 30	30,935	(7)*
	1865 Apr. 19	—	1865 Apr. 23	30,948	(3)*
	1864 Aug. 30	—	1864 Oct. 31	30,937	(4)
	1865 Jan. 31	—	1865 Apr. 2	30,923	(8)
	1865 Apr. 3	—	1865 Apr. 24	30,932	(6)
A—II	1864 Aug. 31	—	1865 Jan. 6	30,182	(8)*
	1864 Aug. 14	—	1864 Dec. 24	30,173	(9)
	1865 Jan. 11			30,162	(2)
B—I	1865 March 18	—	1865 March 30	39,151	(2)*
	1865 Apr. 19	—	1865 Apr. 20	39,154	(2)*
	1865 March 30			39,142	(1)
	1865 Apr. 3	—	1865 Apr. 24	39,139	(6)
B—II	1864 Aug. 4			38,177	(1)*
	(zero-point changed)				
	1864 Aug. 4	—	1865 Jan. 5	38,397	(5)*
	1865 Febr. 27	—	1865 Febr. 28	38,375	(2)*
	1864 Aug. 13	—	1864 Aug. 31	38,384	(2)

Means of Coincidences for Micr. I (Contin.)

1865—1866.

A —	1865 Aug. 12	— 1866 March 9	26″448	(12)*
	1866 Apr. 7	— 1866 Apr. 11	26,429	(5)*
	1865 Aug. 11	— 1866 March 16	26,435	(30)
	1866 March 24	— 1866 Apr. 20	26,423	(13)
B —	1865 Aug. 12	— 1865 Dec. 19	34,650	(5)*
	1866 Apr. 11		34,635	(2)*
	1865 Aug. 11	— 1866 March 15	34,632	(6)
	1866 March 24	— 1866 Apr. 13	34,621	(7)
C —	1865 Dec. 10		44,110	(1)
	1866 Apr. 7		44,104	(1)

1866—1867.

A —	1866 Oct. 2	— 1866 Oct. 11	29″055	(5)*
	1866 Oct. 15	— 1866 Oct. 17	29,062	(4)*
	1866 Aug. 19		28,956	(1)
	1866 Sept. 1	— 1866 Oct. 3	29,047	(6)
	1866 Nov. 10		30,200	(1)
	1867 Jan. 5	— 1867 March 9	30,175	(7)
B —	1866 Oct. 4	— 1866 Oct. 8	37,243	(3)*
	1866 Oct. 3		37,238	(2)
	1867 March 9		38,356	(2)

1867—1868.

A —	1867 Aug. 1		28″012	(2)*
	1867 Aug. 4		27,386	(1)*
	1867 Aug. 5		27,338	(1)*
	1867 Aug. 30	— 1867 Sept. 5	27,228	(4)*
	1867 Oct. 21	— 1867 Oct. 25	27,231	(2)*
	1867 Nov. 1		27,237	(3)*
	1867 Nov. 27	— 1868 Febr. 25	27,254	(8)*
	1867 Aug. 6	— 1867 Aug . 7.	27,329	(2)
	1867 Aug. 25	— 1867 Sept. 5	27,223	(9)
	1867 Sept. 25	— 1867 Oct. 30	27,222	(6)
	1867 Nov. 1	— 1867 Nov. 2	27,225	(3)
	1867 Nov. 27	— 1868 March 28	27,235	(12)
	1868 Apr. 8	— 1868 May 9	27,365	(11)

Means of Coincidences for Micr. I (Contin.).

1867—1868.

B —	1867 Aug. 25 — 1867 Sept. 5	35r432	(3)*
	1867 Nov. 2	35,448	(3)*
	1867 Nov. 27 — 1868 Febr. 18	35,460	(7)*
	1868 March 29	35,437	(1)*
	1867 Aug. 25 — 1867 Sept. 4	35,421	(8)
	1867 Sept. 26 — 1867 Oct. 21	35,433	(2)
	1867 Nov. 2	35,426	(1)
	1868 Febr. 16 — 1868 March 28	35,435	(6)
C —	1867 Sept. 25	44,900	(1)*
	1867 Sept. 19	44,893	(1)

Means of Coincidences for Micrometer II.

1868—1874.

1868 Aug. 11—1868 Sept. 22	34r963	(20)
1868 Oct. 9 —1868 Dec. 12	34,970	(10)
1869 Jan. 7 —1869 Apr. 22	34,975	(21)
1869 Aug. 3 —1869 Oct. 10	34,976	(19)
1869 Nov. 1 —1870 May 2	34,981	(26)
1870 July 30	35,034	(1)
1870 Aug. 6	35,221	(1)
1870 Aug. 8	35,000	(36)
1871 Jan. 24—1871 Apr. 28	34,992	(25)
1871 Dec. 26—1872 Jan. 1	34,993	(30)
1872 Febr. 2—1872 Febr. 16	34,997	(7)
1872 Apr. 25—1872 Apr. 28	34,993	(4)
1872 Oct. 1 —1872 Dec. 18	34,990	(9)
1873 Jan. 16—1873 Apr. 16	34,988	(23)
1873 Aug. 24—1873 Oct. 12	34,990	(11)
1873 Nov. 24—1873 Dec. 31	34,994	(21)
1874 Jan. 13 —1874 April 24	34,993	(23)

In recording the coincidences of micr.I I have found unnecessary always to indicate, when the zero-point was changed, as this immediately appears from the numbers themselves. The variations of the zero-point of micr.II are indicated by the horizontal lines.

Before summarily indicating the contents of the columns in the tables of the observations in general, it will yet be necessary somewhat at large to give a relation of the double-col. 7, which contains descriptions of the nebulæ by means of numbers nearly according to a project made for that purpose by Sir JOHN HERSCHEL, but which has never since come into practical use. This mode of description, which is very commodious and to the use of which it is very easy to accustom oneself, offers moreover many advantages, so that in my opinion it is well worth the general acceptance of nebula-observers. The description is in the first place quite independent of the language of the observer, which is here no small advantage; the description is further obtained without the troublesome use of many and often doubtful terms and in a so concise form, that it in an instant can be looked through, which for example in the comparison of the nebulæ with each other is of the greatest consequence; while on the other hand the descriptions are so precise and complete as in the most cases can possibly be desired, whenever the matter before us is not a monographical study of a single object.

For the nebula-observers it is a well known fact, that in his Results of Astronomical Observations made at the Cape of Good Hope (pp. 137—143) Sir J. HERSCHEL has discussed the question of classification of nebulæ and that at pp. 140—143 is proposed the project here adopted by me. He divides the nebulæ in their widest sense into 3 classes:

Class I	containing	*regular nebulæ.*
Class II	—	*irregular nebulæ.*
Class III	—	*irregular clusters.*

To Class I are reckoned all nebulæ, that have a more or less evidently prominent centre of condensation, or in which the nebulosity with some degree of regularity is symmetrically lodged around a certain determinate point. Sir J. HERSCHEL himself describes these objects in the following termes (l. c., p. 140):

"The normal form of what may be called *Regular Nebulæ* is the Elliptic, admitting every degree of ellipticity, from the spherical to the linear form; every law of condensation, from the circumference to the centre of the apparent outline, from a uniform disc to a starlike centre, surrounded with faint and gradually fading nebulosity; every grade of brightness, from an object like ω Centauri distinctly visible to the naked eye, to the feeblest third-class nebula, barely discernible with the best telescopes; and every shade of resolvability, from stars distinctly numerable even in the middle,

to a total absence of any suspicion of stars indicated by the slightest mottling of surface, etc."

To this class, as is well understood, the planetary and the annular among other nebulæ and the globular clusters are consequently to be reckoned. Concerning ClassII and ClassIII Sir JOHN (l. c., pp. 142, 143) says: "We come now to consider the next great division of our nebulous system II. *Irregular nebulæ*, in which class may be comprehended all which, to a want of complete and in most instances even of partial resolvability by the power of the 20-feet reflector, unite such a deviation from the circular or elliptic form, or such a want of symmetry (with that form) as precludes their beeing placed in ClassI, or that of Regular Nebulæ. This second class comprises many of the most remarkable and interesting objects in the heavens, as well as the most extensive in respect of the area they occupy. Such, for instance, as the great nebulæ about θ Orionis, η Argus, etc.; as well as those smaller but still very extensive and most singular objects IV.41, 8Messier, etc.; the partially symmetrical forms M.51, M.27, etc.; etc.".

"The third principal class of subdivision of these objects is that of *Irregular Clusters*. This class will comprehend all the clusters designated by Sir W. HERSCHEL as of the VII and VIII classes, and such of those of his VI class as are irregular in their form and can not be referred to the class of Globular Clusters."

To obtain a convenient form for the descriptions of the nebulæ, these classes are further divided into 5 subclasses in respect to magnitude, brightness, roundness, condensation and resolubility, which especially among the nebulæ of ClassI become of greater consequence, as those objects are capable of being systematically described from those several points of view, what is not the case with the nebulæ of the two other classes, and in the summary description of which Sir JOHN HERSCHEL consequently proposes only to indicate size and brightness. I have with HERSCHEL admitted 5 degrees in respect to the 5 principal points in question and thus for the summary descriptions of the nebulæ of ClassI employed the following scheme.

Sub-Classification of Regular Nebulæ

(Sir John Herschel's Class I).

Sub-Classes in Respect of	Magnitude	Brightness	Roundness	Condensation	Resolubility
1	Extremely or Very Large	Extremely or Very Bright	Nearly Circular	Nuclear	Resolved
2	Large or Considerably Large	Bright or Considerably Bright	Irregularly Round	With a Strong Central Condensation	Partially Resolved
3	Pretty Large	Pretty Bright	Oval	With a Fainter or a Faint Centr. Condensation	Resoluble
4	Pretty or Considerably Small, or Small	Pretty or Considerably Faint, or Faint	Elongated	Missing a Central Condensation	Granulate or Spotted
5	Very or Extremely Small	Very or Extremely Faint	Much Elongated	Annular	Milky

*

For the understanding of this scheme the few following elucidative remarks on its respective columns will probably be sufficient.

Magnitude (col. 1).

1 .....$> 4'$; **2** .....$\begin{matrix}<4'\\>2'\end{matrix}$; **3**..... $1'$ to $2'$; **4** .....$\begin{matrix}<1'\\>1/3'\end{matrix}$; **5**.....$< 1/3'$.

In the case of "much elongated" nebulæ the magnitude is not indicated.

Brightness (col. 2).

1 ..... only a few of the brightest nebulæ of Sir WILLIAM HERSCHEL'S Class I, and Class VI.
2 ..... the brighter nebulæ Class I.
3 ..... the middlebright and fainter nebulæ Class I and the brightest Class II.
4 ..... the faintest nebulæ Class II.
5 ..... Class III.

So far as an immediate comparison of the brightness of nebulæ with that of the stars is possible, I assume the nebulæ of the 1st class of brightness

to be brighter than a star of the 9th magn. (Besselian scale), those of the 5th class to be fainter than a star of 12,5 magn., and the middle-bright (cl. 3) to correspond in brightness to stars from about the 10,5 to about the 11,5 magn. But, as those estimations in many cases must become extremely difficult, it is evident, that here nothing like a higher degree of precision can be expected.

Roundness (col. 3).

3 ..... length not more than double the breadth.
4 ..... length not exceeding 5 times the breadth.
5 ..... length 5 times the breadth or more.

Condensation (col. 4).

In this col. of our scheme a * is inserted in the 6th line to remind the reader of the following notation:

if the nebulæ of the degrees of condensation 2, 3, 4 have a star in the middle, and which in such cases is of course the point observed, I put in the 4th place of their description respectively * 2, * 3, * 4; and, if nothing is said of the condensation, simply a *.

Resolubility (col. 5).

2 ..... resolved part of the nebula quite or nearly free from nebulosity.
3 ..... single stars steadily visible on nebulous ground, or only momentarily seen glimmering through the nebulosity.

Lastly it is to be remarked, that a zero in any of the descriptive columns always indicates. that no description in that respect could be obtained. After the 5 places of description the magnitude of the nucleus or of the central-star (*) is indicated according to the Bessel-Argelandrian scale. When the object observed is a star, its magnitude merely is given in the descriptive column.

This may be best illustrated by a few examples:

321. 1. 3 (10) = nebula, on an average 1′ to 2′ in diameter, of the brighter ones of W. HERSCHEL'S classI; nearly circular; with a nucleus of the 10th magn.; nebulosity with more or less evident traces of resolubility.

242. * 2. 4 (12) = nebula, on an average 2′ to 4′ in diameter, of the faintest of class II (W. H.); irregularly round; strongly condensed; with a central star of 12th magn.; light not equable.

433. * . 0 (11.12) = nebula on an average $^1/_3$′ to 1′ in diameter; middle-bright; a little elongated; with a central star 11.12 magn. — respecting condensation and resolubility nothing remarked.

The description of a nebula ought in all cases also to indicate to which of Sir J. HERSCHEL'S three classes it belongs, and as, in observations such as the present, objects of the third class do not occur, the simplest course appears to be, only to indicate the class, when the object belongs to the second class, it being understood, that, where no class is mentioned, the nebula is of the first class. In the few here occurring objects of the second class I have thought it best, in so summary descriptions as the present, simply to state the brightness of the stars within them observed.

The contents of the different columns of the following tables may now summarily be indicated, as follows.

Col. 1: names of the observed objects, in which the nebulæ are indicated by well known signatures (note II — list of abbreviations employed); and faint stars by means of letters in such a manner, that for instance h131(* a) signifies a star in the nebulosity itself, but * a (h210) a neighbouring star or satellite. The few nebulæ occasionally found and occurring in no catalogue are designated as *Novæ*. The comparison-stars, that were micrometrically determined, are designated by their current numbers in the star-catalogue below.

Col. 2: numbers of the nights of observation (note III).

Col. 3: the immediate means of the observed differences of right ascension in the sense: *nebula—star*. The numbers in parentheses indicate the number of the single passages observed. The few $\Delta\alpha^s$, calculated from observed angles of position, are marked with †.

Col. 4: the immediate means of the observed differences of declination (in the sense: *nebula—star*) expressed in scale-divisions. The numbers in parentheses indicate the numbers of the single measures. The micrometrical numbers obtained with micr. I (up to night 215) and corresponding to the positions of the screw above the centre of the micrometer are, as previously the coincidences of the same species, always marked with an asterisk.

The few micrometrical numbers, which correspond to measurements of distances, are designated with †. In the few cases, in

which double distances or $\Delta\delta$ were observed and the measurements consequently executed without coincidences, this is indicated by the sign $\overset{*}{*}$.

Coll. 5 and 6: the mean $\Delta\alpha^s$ and $\Delta\delta''$ for 1865,0.

Double-Col. 7: descriptions of the nebulæ according to the scheme p. 20.

Col. 8: the comparison—stars indicated by their numbers in the catalogue, and the nebulæ employed as comparison-stars designated in the usual manner.

Col. 9: mean hour-angles, by which is indicated the position of the tube during the observation, and which besides are sufficiently exact for computation of the small occurring corrections of refraction.

TABLES OF THE MICROMETRICAL OBSERVATIONS.

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC.1	163	-107^s64 (5)	$-4''498$ (3)	-107^s63	$-77''7$	441. *. 5	12	h 4	2^h
	217	−107,95 4)	− 6,236 (4)	−107,94	− 80,5	451. *. 5	13	—	22
	218	−108,08 (6)	− 6,061 (4)	−108,07	− 78,3	452. *. 4	13	—	23
h 1	189	− 45,49 (7)	− 3,562 (3)	− 45,49	− 61,4	452. 3. 0		* 2	23
h 2	164	+ 52,97 (7)	+ 3,239 (2)*	+ 52,96	+ 55,9	541. 0. 5		* 1	3
	222	+ 52,37 (6)	+ 4,486 (3)	+ 52,35	+ 57,9	0		—	22
h 4	163	− 65,50 (6)	− 7,560 (3)	− 65,49	−130,4	431. *. 5	11	* 3	2
	217	− 65,58 (6)	−10,057 (4)	− 65,57	−129,8	443. *. 5	12	—	22
	218	− 65,58 (6)	−10,012 (4)	− 65,57	−129,2	431. 1. 3	11	—	23
Nova I	163	− 46,63 (4)	− 6,899 (3)	− 46,62	−119,0	542. 4. 5		* 3	2
	218	− 46,58 (6)	− 9,192 (4	− 46,58	−118,7	541. 4. 5	12	—	0
* a (Nova II)	164	− 71,74 (5)	− 8,876 (3)*	− 71,73	−153,1	10		h 6	2
h 6	164	−192,80 (4)	+ 3 316 (2)*	−192,77	+ 56,9	443. *. 5	13	* 6	1
	222	−193,44 (5)	+ 4,524 (2)	−193,39	+ 58,0	0		—	23
h 7	188	− 79,90 (6)	+20,738 (3)	− 79,89	+357 3	452. 3. 0		* 5	22
h 9	188	− 37,75 (6)	− 9,017 (4)	− 37,74	−155,5	550. *3. 0		* 5	22
* 5	374	+181,95 (10)	− 8,698 (1)	+181,84	−112,0	9,5		* 4	23
	375	+181,96 (8)	− 8,640 (3)	+181,84	−111,2			−	22
h 13	224	+ 4,84 (8)	+ 6,302 (3)	+ 4,84	+ 81,3	431. 1. 3	11.12	* a (11.12)	23
	178	−333,38 (4)	−25,374 (2)*	−333,37	−437,8	431. 1. 4		* 8	0
	223	−334,05 (6)	−33,935 (2)	−334,00	−438,7	031. 1. 0		—	23
	224	−333,84 (5)	−33,947 (3)	−333,79	−438,8	431. 1. 3	11.12	—	22
h 16	165	−106,95 (4)	− 5,617 (3)*	−106,90	− 97,3	441. 1. 5	12	* 9	1
	219	−107,22 (5)	− 7,448 (3)	−107,14	− 97,0	441. *3. 3	12.13	—	21
	220	−106,92 (6)	− 7,359 (3)	−106,83	− 95,8	341. 1. 3	12	—	23
h 17	165	− 94,98 (5)	+10,577 (3)*	− 94,93	+181,8	442. *. 4	12.13	* 9	1
	219	− 95,28 (6)	+13,923 (3)	− 95,20	+178,4	442. 2. 3		—	22
	220	− 95,52 (6)	+13,967 (3)	− 95,44	+178 9	442. 3. 3		—	23
GC.40	165	− 66,17 (3)	+ 3,140 (2)*	− 66,12	+ 53,7	450. 0. 5		* 9	3
	219	− 66,60 (5)	+ 4,456 (3)	− 66,52	+ 56,5	452. *. 5	13	—	0
	220	− 66,42 (6)	+ 4,621 (2)	− 66,34	+ 58,7	352. *. 5	13	—	22

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC.41	165	− 65^{s}67 (3)	− 3^{r}344 (4)*	− 65^{s}62	− 58″1	551. ∗.5	13	∗9	3^h
	219	− 66,40 (5)	− 4,479 (3)	− 66,32	− 58,6	551. ∗.5	12.13	—	23
	220	− 66,15 (6)	− 4,220 (3)	− 66,07	− 55,3	451. ∗.5	12.13	—	22
GC.42	165	− 54,76 (5)	+ 4,977 (3)*	− 54,71	+ 85,4	442. 1.5	12.13	∗9	2
	219	− 54,82 (6)	+ 6,910 (3)	− 54,74	+ 88,2	540. ∗.5	12	—	22
	220	− 54,75 (6)	+ 6,715 (3)	− 54,67	+ 85,7	442. ∗.5	12	—	23
∗9	375	+233,89 (7)	+ 4,223 (2)	+233,78	+ 54,6	7,5		∗7	23
	375	− 42,59 (7)	+25,361 (2)	− 42,57	+326,4			∗11	23
h21	197	+172,37 (6)	+ 2,260 (2)	+172,33	+ 39,2	432. 1.0	12	∗10	1
	227	+172,80 (6)	+ 2,921 (3)	+172,75	+ 37,9	432. 1.4	11.12	—	1
h23	132	—	− 5,340 (3)	—	− 92,1	541. ∗.5	13	h25	23
	134	− 25,07 (6)	− 5,514 (3)	− 25,06	− 95,1	541. ∗3.2	13	—	23
h25	134	+115,13 (4)	+10,672 (2)	+115,11	+184,2	432. 1.5	12	∗12	23
h26	231	− 78,17 (7)	−12,443 (4)	− 78,16	−160,8	441. 3.3		∗13	1
	232	− 78,23 (7)	−12,571 (3)	− 78,22	−162,4	443. 3.3		—	0
	278	− 78,33 (7)	−12,486 (4)	− 78,32	−161,4	451. ∗.4	13	—	3
h32	184	− 5,20 (8)	−14,224 (5)*	− 5,20	−245,2	541. ∗.0	12.13	∗14	23
	277	− 5,63 (8)	−18,793 (3)	− 5,62	−242,6	541. ∗.4	12.13	—	2
GC.80	184	+ 5,25 (4)	+ 1,383 (2)*	+ 5,25	+ 23,9	551. 4.5		h32	0
GC.82	184	− 15,20 (7)	− 5,411 (5)*	− 15,20	− 93,3	441. ∗3.3	13	∗15	23
	277	− 15,73 (6)	− 7,757 (4)	− 15,73	−100,1	450. 0.0		—	2
h33	231	− 16,04 (8)	−16,600 (3)	− 16,04	−214,5	351. ∗3.4	13	∗16	2
	232	− 15,77 (7)	−16,978 (3)	− 15,77	−219,3	452. ∗3.5	13	—	0
	375	− 16,15 (10)	−16,726 (2)	− 16,15	−216,1	452. ∗.0	13.14	—	0
∗16	369	− 9,98 (10)	−26,170 (2)	− 9,98	−338,2	9,5		∗17	0
	375	− 10,12 (10)	−26,181 (2)	− 10,12	−338,4			—	1
h39	229	−207,88 (5)	− 8,468 (2)	−207,87	−109,6	443. ∗.5	11.12	∗24	1
∗a(h37)	333	− 11,17 (10)	+ 3,216 (2)	− 11,17	+ 41,4	12		∗21	23
h40	134	+ 12,44 (5)	−17,855 (3)	+ 12,49	−307,6	542. ∗.5	12.13	∗20	0
	188	+ 12,38 (8)	−17,850 (3)	+ 12,52	−307,1	432. 1.5		—	0
h37	188	− 9,23 (8)	+ 3,256 (4)	− 9,23	+ 56,1	542. ∗.5	12.13	∗21	23
	333	− 9,42 (10)	+ 4,349 (2)	− 9,42	+ 56,1	000. ∗.0		—	22
h41	229	+ 4,83 (3)	+13,512 (4)	+ 4,83	+174,6	542. 1.5	11.12	h39	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
II. 857	134	− 6^{s}93 (4)	+17$'$496 (4)	− 6^{s}93	+301″8	440. 1 . 5	12.13	* 22	1^h
	188	− 6,92 (6)	+17,561 (5)	− 6,92	+302,9	442. 1 . 5		—	1
II. 858	134	+ 16,15 (4)	−31,405 (3)	+ 16,15	−541,6	542. 4 . 5		h40	1
h42	333	+ 16,48(10)	− 4,311 (2)	+ 16,48	− 55,6	400. * . 0		* 21	22
h44	175	− 35,13 (7)	− 9,757 (5)*	− 35,12	−168,3	0		* 23	21
GC.5058	229	− 30,38 (5)	+ 3,151 (2)	− 30,38	+ 40,6	551. * . 5		* 24	1
h51	37	− 55,09 (5)	− 8,190 (4)	− 55,09	−141,1	511. 1 . 0	8,5	* 25	3
	39	− 55,12 (6)	− 8,214 (3)*	− 55,12	−141,5	411. 1 . 0	8,9	—	7
	175	− 55,09 (7)	− 8,197 (5)*	− 55,07	−141,4	0		—	21
GC.119	229	− 25,55 (5)	−13,318 (4)	− 25,55	−172,1	441. 1 . 5		* 24	1
h50	175	−121,81 (7)	− 9,602 (5)*	−121,78	−165,7	0		* 26	21
h59	135	—	−18,350 (3)*	—	−316,4	442. * . 5		* 28	23
	136	−110,24 (5)	−18,375 (4)*	−110,21	−316,8	442. * . 5	13	—	23
	137	−109,98 (4)	−18,369 (2)*	−109,94	−316,7	442. * . 5	13	—	0
h65	328	− 3,68(10)	+16,571 (2)	− 3,68	+213,3	541. 4 . 2		* 27	3
	334	− 3,77(10)	+16,614 (3)	− 3,77	+213,9	550. * . 0	12.13	—	21
h78	69	− 26,26 (7)	− 7,865 (5)	− 26,25	−135,6	441. 1 . 0	13	* 31	2
	72	− 26,19 (8)	− 7,820 (4)	− 26,18	−134,8	0		—	4
	373	− 26,38(10)	−10,495 (2)	− 26,19	−136,6	050. * . 0	12.13	—	23
h79	38	− 10,32 (9)	− 7,118 (5)	− 10,33	+122,7	0		* 31	3
	69	− 10,38 (5)	− 7,018 (4)	− 10,38	+121,0	441. 1 . 0	12.13	—	3
	72	− 10,26 (9)	− 7,103 (2)	− 10 25	+122,4	0		—	4
	373	− 10,86(10)	− 9,615 (2)	− 10,68	+122,7	040. * . 0	12	—	23
* 31	373	+290,12(10)	− 7,874 (3)	+289,95	−100,6	8,3		* 29	22
	373	+ 91,61(10)	+14,919 (3)	+ 91,55	+192,8			* 30	22
* 33	328	+ 57,33(10)	+23,417 (2)	+ 57,30	+302.6	10.11		* 32	6
h84	118	− 1,94 (5)	+ 7,918 (2)	− 1,94	+136,5	432. 0 . 0		h85	2
	126	− 1,86 (4)†	+ 7,662 (3)*	− 1,86	+132,1	032. 3 . 0		—	22
h85	118	−226,53 (3)	− 7,879 (2)	−226,53	−136,2	031. 0 . 0		* 36	1
h86	126	+ 2,76 (5)†	+ 4,766 (4)*	+ 2,76	+ 82,1	030. 1 . 0	11.12	* a (11)	0
	118	+ 7,18 (5)	−14,602 (3)	+ 7,18	−251,7	031. 0 . 0		h85	2
	126	+ 7,22 (5)†	−14,601 (3)*	+ 7,22	−251,7	030. 1 . 0	11.12	—	23
GC.205	123	− 0,87 (5)†	− 1,598 (3)	− 0,87	− 27,5	540. 0 . 0		h86	0
	126	− 1,06 (4)†	− 1,851 (2)*	− 1,06	− 31,9	541. 1 . 0	13	—	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC.207	328	− 15^{s}87 (10)	+ 7$''$628 (2)	+ 15^{s}86	+ 98$''$5	551. 4 . 5		* 33	5^h
	123	− 1,96 (4)†	− 5,464 (3)	− 1,96	− 94,2	041. 4 . 0		GC.208	2
	126	− 2,06 (5)†	− 5,549 (3)*	− 2,06	− 95,6	541. 4 . 0		—	0
h 208	328	+ 18,03 (10)	+15,323 (2)	+ 18,02	+197,9	551. 4 . 5		* 33	5
h 87	190	−154,14 (5)	− 4,276 (1)*	−154,11	− 74,1	541. 1 . 5	12.13	II. 220	21
h 88	193	− 73,62 (6)	− 2,044 (3)	− 73,60	− 35,4	442. *3.4		* 35	21
	377	− 73,32 (10)	—	− 73,26	—	452. * . 0	13.14	—	22
	379	− 73,50 (8)	− 2,776 (3)	− 73,44	− 36,0	552. 3 . 3		—	22
GC.217	387	−152,63 (8)	+ 2,136 (2)	−152,56	+ 27,2	453. 2 . 0		* 37	3
	193	− 6,54 (9)	+ 7,727 (5)	− 6,55	+133,2	543. *3.0		* b (10)	22
h 89	190	− 14,90 (8)	+19,398 (3)	− 14,85	+334,0	432. * . 3	12	* 34	21
	308	− 15,45 (8)	+26,083 (3)	− 15,34	+335,1	030. * . 3	11.12	—	2
* 35	377	−141,82 (10)	—	−141,72	—	8,9		* 38	22
	379	−141,76 (7)	+23,506 (4)	−141,66	+302,0			—	22
Nova III	191	− 22,01 (7)	− 5,336 (3)	− 22,01	− 92,0	552. 3 . 5		II. 220	22
	192	− 22,56 (5)	− 5,325 (2)	− 22,55	− 91,9	452. 2 . 5		—	22
II. 219	192	− 7,68 (6)	0	− 7,67	0,0	550. * . 5	13	II. 220	21
II. 220	190	−123,87 (6)	− 1,478 (3)*	−123,84	− 25,8	441. 1 . 5	12	* 40	22
	192	−123,42 (6)	− 1,684 (3)	−123,39	− 29,3	443. 1 . 4		—	20
Nova IV	191	+ 18,47 (7)	− 7,940 (3)	+ 18,47	−136,8	552. 3 . 5		II. 220	22
	192	+ 18,66 (7)	− 8,027 (2)	+ 18,66	−138,3	452. 2 . 5		—	22
h 90	196	− 43,12 (6)	+ 9,606 (4)	− 43,11	+165,5	441. 2 . 5		* 39	1
GC.250	192	− 40,67 (6)	−10,542 (3)	− 40,66	−181,8	442. * . 5	12	* 41	23
I.108	186	− 12,87 (6)	+ 1,927 (3)	− 12,87	+ 33,2	442. 2 . 0		* 43	0
	269	− 12,74 (7)	+ 2,848 (3)	− 12,74	+ 36,7	442. *3.3	12.13	—	1
	270	− 12,84 (8)	+ 2,626 (3)	− 12,84	+ 33,8	442. *3.4		—	2
III.250	186	− 22,00 (5)	− 1,492 (4)	− 22,00	− 25,8	340. 2 . 5		III. 251	1
	269	—	− 1,724 (4)	—	− 22,3	343. 1 . 5		—	1
	270	− 21,99 (9)	− 1,750 (3)	− 21,99	− 22,7	343. 1 . 5		—	2
III 251	186	+ 43,99 (7)	+26,258 (4)*	+ 43,98	+453,0	531. 1 . 5	10.11	* 43	1
	269	—	+35,100 (3)	—	+453,7	531. 1 . 5	11	—	1
	270	+ 43,41 (7)	+35,126 (2)	+ 43,41	+454,1	531. 1 . 5	11	—	2
h 102	194	− 11,78 (6)	+ 2,222 (3)*	− 11,78	+ 38,3	452. * . 4	13	* 44	1
	366	− 11,53 (10)	+ 2,508 (2)	− 11,52	+ 32,2	452. * . 0	12.13	—	22

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865.0 $\Delta\alpha^s$	Mean-Coordinates 1865.0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
* 44	369	+192^{s}03 (10)	− 0$'$404 (2)	+191^{s}91	− 4$''$4	9,3		* 42	3^h
	371	+192,03 (7)	− 0,411 (2)	+191,91	− 4,5			—	20
h 103	81	− 36,39 (7)	+ 0,816 (5)	− 36,38	+ 14,1	030. * . 0	12	* 45	0
	129	− 36,46 (5)	+ 0,652 (4)	− 36,46	+ 11,2	030. 1 . 0		—	0
	129	− 2,42 (5)†	+ 5,485 (4)†	− 2,42	+ 87,4	030. 1 . 0		* a (11)	23
GC.274	192	− 72,58 (5)	+ 3,742 (3)*	− 72,58	+ 64,6	443. 2 . 5		* 47	0
h 104	195	− 16,76 (5)	−24,500 (3)*	− 16,75	−422,3	343. 3 . 4		* 46	1
III.156	194	− 15,58 (10)	+ 1,874 (3)	− 15,58	+ 32,3	452. 0 . 0		h 106	2
	366	− 15,48 (10)	+ 2,905 (2)	− 15,47	+ 37,4	542. * . 0	12	—	22
	367	− 15,65 (10)	+ 3,106 (2)	− 15,64	+ 40,0	552. 0 . 0		—	2
III.157	367	− 7,29 (10)	− 6,537 (2)	− 7,28	− 84,4	050. * . 0	13	h 106	2
III.158 = h 10 6	193	+ 63,66 (5)	−10,468 (3)	+ 63,65	−180,3	431. 1 . 4	12	* 44	23
	194	+ 63,70 (5)	−10,468 (2)	+ 63,70	−180,3	431. 1 . 4	11.12	—	4
	366	+ 63,27 (10)	−13,982 (2)	+ 63,24	−179,8	432. 1 . 0	11	—	21
	367	+ 63,45 (10)	−13,921 (2)	+ 63,42	−179,0	433. * . 0	11.12	—	2
III.158	193	− 17,12 (5)	+ 1,172 (3)	− 17,12	+ 20,1	431. 1 . 4	12	* a (11)	23
GC.293	189	− 46,35 (6)	−13,715 (3)	− 46,35	−236,6	531. 1 . 5	11	* 48	23
	192	− 46,58 (5)	−13,690 (3)*	− 46,58	−236,1	441. 1 . 5	12	—	0
GC.291	195	+ 16,23 (6)	−17,727 (2)	+ 16,24	−305,5	541. 1 . 5	12	* 46	3
h 108	195	+ 27,77 (6)	− 6,937 (4)	+ 27,76	−119,5	431. 1 . 5	11.12	* 46	2
h 109	195	+ 28,22 (6)	− 1,725 (2)	+ 28,21	− 29,7	542. 2 . 5		* 46	2
Nova V	194	+ 43,96 (5)	+ 7,935 (4)	+ 43,95	+136,9	541. * . 0	12.13	h 106	2
	367	+ 43,70 (10)	+10,568 (2)	+ 43,67	+136,6	543. * . 0	12	—	3
h 113	193	+ 70,05 (4)	+ 4,113 (2)	+ 70,04	+ 71,1	441. 3 . 5		* a (h 106)	0
	194	+ 86,28 (5)	+ 3,091 (2)	+ 86,26	+ 53,5	441. 2 . 5		h 106	4
	367	+ 86,19 (10)	+ 3,956 (2)	+ 86,14	+ 51,4	442. 2 . 5		—	4
h 114	194	+ 92,72 (5)	− 5,877 (2)	+ 92,70	−101,1	542. * . 0	11.12	h 106	4
	367	+ 91,98 (10)	− 8,085 (2)	+ 91,94	−104,6	440. * . 5	12	—	4
	368	+ 92,22 (10)	− 8,035 (2)	+ 92,17	−104,0	0		—	1
h 117	166	−100,64 (5)	+ 1,328 (3)	−100,63	+ 22,7	332. 1 . 3	11.12	* 50	1
	383	−100,72 (10)	+ 1,763 (2)	−100,70	+ 22,3	432. 1 . 3		—	2
GC.308	166	− 33,08 (6)	− 1,870 (5)	− 33,08	− 32,3	551. 1 . 5	13	* 49	1
	383	− 33,37 (9)	− 2,244 (2)	− 33,36	− 29,1	552. 0 . 0		—	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h. 118	232	− 81^{s}34 (5)	+20^{r}093 (3)	− 81^{s}32	+258$''$4	531. 2 . 2		* 51	1^h
	389	− 81,53 (10)	+20,109 (2)	− 81,48	+258,6	441. 2 . 2		—	3
h. 120	232	− 40,22 (6)	+18,712 (4)	− 40,21	+240,7	443. 4 . 5		* 51	1
	389	− 40,38 (10)	+18,652 (3)	− 40,36	+240,0	443. 3 . 4		—	3
GC. 348	221	−150,63 (3)	− 7,345 (4)	−150,61	− 95,2	443. *3.5	13	* 53	21
	388	−150,19 (10)	− 7,342 (2)	−150,10	− 95,6	452. 3 . 0		—	4
GC. 349	221	—	− 8,176 (3)	—	−105,9	443. *3.5	12	* 53	21
	388	−123,73 (10)	− 8,284 (2)	−123,66	−107,4	442. 3 . 3		—	4
D'A. Nova	221	—	+ 5,277 (3)	—	+ 67,9	442. *3.5		* 53	21
	388	−102,40 (10)	+ 5,279 (2)	−102,34	+ 67,6	441. * . 0		—	5
h 131 (* a)	388	− 82,77 (10)	− 0,671 (2)	− 82,72	− 9,0	Class II	11.12	* 53	5
h 133	221	− 44,18 (5)	+31,113 (3)	− 44,18	+400,2	431. 2 . 3		* 53	22
	388	− 44,00 (10)	+31,057 (2)	− 43,98	+399,9	433. 1 . 0	12	—	3
h 142	233	+161,23 (6)	+ 4,176 (4)	+161,20	+ 54,3	431. 3 . 5		* 52	1
	335	+161,09 (8)	+ 4,214 (2)	+161,05	+ 55,1	332. 3 . 4		—	0
II. 253	335	+ 35,07 (10)	−25,514 (3)	+ 35,07	−328,3	045. 1 . 5		* 54	23
h 147	329	− 51,52 (10)	− 8,792 (2)	− 51,50	−113,4	042. * . 0	12.13	* 55	4
	334	− 51,44 (10)	− 8,854 (3)	− 51,42	−114,5	541. 1 . 0	12	—	21
h 149	333	− 33,55 (10)	− 9,015 (2)	− 33,54	−116,5	443. 1 . 0		* 56	23
h 150	280	−126,30 (5)	− 9,824 (4)	−126,26	−127,2	244. 1 . 5		* 58	3
II. 228	133	− 22,21 (9)	+ 5,578 (5)	− 22,21	+ 96,1	442. 0 . 0		II. 229	2
	390	− 22,48 (12)	+ 7,199 (2)	− 22,48	+ 92,6	444. 3 . 0		—	3
II. 229	133	−199,78 (5)	− 4,513 (3)	−199,80	− 78,2	541. * . 5	12	* 59	3
	390	−199,93 (10)	− 5,827 (2)	−199,85	− 76,2	432. 1 . 3	12	—	4
h 152	280	+ 36,68 (10)	−16,875 (3)	+ 36,67	−217,1	443. * . 0	12.13	* 57	3
	333	+ 35,95 (10)	−16,981 (2)	+ 35,95	−218,5	444. * . 0		—	0
	333	− 7,98 (10)	+ 1,333 (2)	− 7,98	+ 17,1	444. * . 0		* a (11)	0
h 169	276	− 85,08 (8)	−14,044 (3)	− 85,04	−181,6	541. * . 0	12	* 62	23
h 175	276	− 21,66 (8)	+ 8,936 (4)	− 21,66	+115,0	443. * . 3	13	* 60	23
* 62	276	+ 13,46 (11)	−23,173 (3)	+ 13,47	−298,2	9,5		* 60	23
	370	+ 13,28 (10)	−23,211 (2)	+ 13,28	−298,8			—	22
GC. 461	128	− 6,09 (5)†	−11,202 (4)	− 6,09	−193,1	041. 0 . 0		h 181	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC.461	219	− 6^{s}18 (6)	−14$''$646 (2)	− 6^{s}18	−189$''$1	440. 1. 5	12	h 181	0^h
h 181	73	+106,94 (7)	+ 3,842 (4)	+106,93	+ 66,3	0		* 61	1
	128	+107,02 (5)	+ 4,060 (4)	+107,01	+ 70,2	031. 1. 0	11.12	—	23
	219	+107,14 (5)	+ 5,167 (3)	+107,14	+ 67,0	331. 1. 5	11.12	—	0
	128	+ 11,63 (4)	—	+ 11,63	—	031. 1. 0	11.12	* a (11.12)	0
h 182	329	− 16,34 (10)	+24,419 (2)	− 16,34	+315,3	441. *. 4	12	* 63	4
h 194	334	−119,86 (10)	+19,484 (3)	−119,81	+250,1	453. 2. 0		* 66	22
	389	−120,01 (7)	+19,501 (2)	−119,93	+250,4	443. 3. 5		—	5
h 193	167	+ 2,61 (7)†	− 2,851 (4)	+ 2,61	− 49,2	531. *. 0	12	* 65	1
	386	+ 2,84 (11)	− 3,648 (2)	+ 2,84	− 47,0	441. *. 0	12	—	2
	167	− 0,25 (5)†	+ 6,067 (4)	− 0,25	+104,6	531. *. 0	12	* a (11)	2
h 197	335	+ 70,52 (10)	−18,018 (2)	+ 70,49	−231,5	450. *. 4	12.13	* 64	21
h 208	279	− 6,80 (9)	+22,542 (3)	− 6,80	+289,2	444. 2. 4		* 68	1
	383	− 6,70 (10)	+22,649 (3)	− 6,71	+289,8	444. 3. 5		—	3
h 210	279	− 4,54 (9)	+21,330 (4)	− 4,55	+274,6	441. 3. 3		* 69	1
	383	− 4,44 (10)	+21,329 (2)	− 4,45	+274,6	442. 3. 5		—	3
* a (h 210)	279	+ 45,64 (7)	+18,010 (2)	+ 45,63	+231,7	12		* 68	2
* 70	282	+295,18 (6)	−18,099 (2)	+295,09	−231,8	8,9		* 67	3
	372	+294,77 (7)	−18,197 (2)	+294,62	−232,2			—	21
h 217	282	+143,97 (7)	+ 7,609 (3)	+143,92	+ 98,8	441. *. 4	12	* 70	4
	372	+143,63 (10)	+ 7,607 (2)	+143,55	+ 99,2	443. 1. 4	12	—	21
h 226	133	− 47,37 (6)	− 6,353 (4)	− 47,36	−109,6	442. 0. 5		* 71	4
	383	− 48,15 (10)	− 8,162 (2)	− 48,13	−105,7	434. 1. 5		—	4
h 231	282	− 67,93 (4)	+22,746 (2)	− 67,91	+292,5	443. 1. 0	12	* 72	5
	283	− 67,59 (7)	+22,711 (3)	− 67,57	+292,1	541. 1. 0	12	—	4
h 233	283	− 50,03 (7)	+24,533 (3)	− 50,02	+315,6	450. 3. 0		* 72	5
h 234	282	− 28,06 (5)	− 5,006 (2)	− 28,05	− 64,7	441. *. 4	12.13	* 72	5
	283	− 27,79 (9)	− 5,208 (3)	− 27,78	− 67,3	541. *. 4	12.13	—	4
h 242	39	− 33,51 (7)	− 3,194 (6)*	− 33,51	− 55,0	030. 1. 0	11	* 73	7
	41	− 33,80 (6)	− 3,126 (5)	− 33,80	− 53,9	530. 1. 0	11.12	—	6
h 257	234	− 46,25 (8)	+14,224 (5)	− 46,24	+182,9	541. *. 0		* 74	3
	278	− 46,36 (7)	+14,463 (5)	− 46,35	+186,0	441. 2. 3		—	2
	281	− 46,48 (8)	+14,499 (2)	− 46,46	+186,5	441. 1. 0	12	—	3

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 260	234	− 11^{s}53 (9)	+28^{r}654 (3)	− 11^{s}54	+368″7	552. ∗.0		∗ 74	4^h
	278	− 12,44 (7)	+28,602 (3)	− 12,45	+368,0	453. ∗.0	12.13	—	3
	281	− 11,83 (12)	—	− 11,84	—	450. 0.0		—	2
II. 619	196	− 29,18 (6)	− 1,799 (5)	− 29,18	− 31,1	350. 3.5		∗ 75	0
h 135 (Centrum)	127	+ 61,35 (6)	+ 3,583 (2)	+ 61,35	+ 61,9	040. 0.0		∗ 76	21
	168	+ 61,67 (6)	+ 3,504 (6)	+ 61,66	+ 60,6	042. 3.3		—	3
	127	+ 3,37 (6)†	− 8,964 (3)	+ 3,37	−154,5	040. 0.0		*R.* ε (9,5)	21
α (h 315)	168	− 3,41 (4)†	− 1,541 (4)	− 3,41	+ 26,6	11.12 (nebs)		*R.* β	5
β (h 315)	168	+ 70,14 (5)	+ 8,614 (3)	+ 70,13	−148,7	10,5		∗ 76	4
h 316	127	− 45,38 (7)	+ 6,500 (3)	− 45,38	+112,1	031. 1.0		∗ 80	22
	373	− 45,96 (10)	+ 8,666 (2)	− 45,97	+111,2	442. 1.0	12	—	0
h 317	127	− 41,92 (5)	+ 7,078 (3)	− 41,92	+122,1			∗ 80	23
	373	− 42,14 (10)	+ 9,498 (2)	− 42,15	+121,9	551. ∗.0	12.13	—	0
h 318	127	+154,50 (3)	− 3,063 (3)	+154,51	− 52,6	030. 1.0		∗ 77	0
	127	− 3,97 (3)†	− 1,692 (3)	− 3,97	− 29,2	030. 1.0		∗ a (11.12)	0
∗ 80	373	+ 52,86 (10)	+26,051 (2)	+ 52,84	+337,3	8,0		∗ 78	23
	373	+ 68,68 (10)	−16,378 (2)	+ 68,69	−210,2			∗ 79	0
h 355 (∗ a)	198	−162,98 (5)	− 4,289 (3)	−162,98	+ 73.1	Class II	11	∗ 81	22
h 359	236	− 26,67 (9)	− 9,406 (3)	− 26,66	−121,4	542. ∗3.0	12	∗ 82	22
	237	− 26,84 (7)	− 9,540 (3)	− 26,83	−123,1	442. ∗3.3	12	—	22
	237	− 10,54 (7)	− 5,048 (3)	− 10,54	− 65,1	442. ∗3.3	12	∗ a (10)	23
h 365	1	+153,67 (5)	− 0,886 (5)	+153,66	− 15,6	0		∗ 83	0
	73	+ 34,03 (6)	−16,285 (4)	+ 34,03	−280,8	532. 1.3		∗ 84	1
	74	+ 34,22 (6)	−16,311 (5)	+ 34,22	−281,2	532. 1.3		—	23
∗ 86	378	+212,17 (8)	+20,982 (2)	+212,16	+273,8	9,3		∗ 85	23
h 368 (∗ a)	169	+ 38,14 (5)	+ 6,297 (3)	+ 38,14	+108,7	Class II		∗ 86	1
	378	+ 37,96 (8)	+ 8.372 (2)	+ 37,96	+108,6	Class II		—	23
h 368 (∗ b)	169	+ 1,29 (5)†	+ 2,815 (3)	+ 1,29	+ 48,6	Class II		∗ a	1
h 368 (∗ c)	169	+ 2,16 (3)†	+ 6,692 (3)	+ 2,16	−115,4	Class II		∗ a	1
h 393	2	− 7,02 (5)	− 1,486 (6)	− 7,02	− 25,6	0		∗ 87	22
	78	− 6,91 (8)	− 1,449 (3)*	− 6,91	− 25,0	020. ∗.0		—	23
h 399 (∗ a)	40	− 8,88 (5)	−19,845 (3)	− 8,88	+342,2	Class II		∗ 88	2
	78	− 8,81 (8)	−19,845 (5)*	− 8,81	+342,1	Class II	10	—	0

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865 0 $\Delta\alpha^s$	Mean-Coordinates 1865 0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
Dunér Nova	336	—	−25″359 (3)	—	−327″6	0		* 90	19h
	337	− 13s98 (10)	−25,229 (2)	− 13s91	−326,0	442. * . 4	12.13	—	19
	336	—	− 8,832 (3)	—	−114,6	0		* 91	19
	337	− 52,52 (10)	− 8,615 (3)	− 52,51	−111,8	442. * . 4	12.13	—	19
h 406	75	− 32,95 (6)	− 1,832 (4)*	− 32,95	− 31,6	540. * . 0		* 92	21
	76	− 32,62 (6)	− 1,654 (4)*	− 32,62	− 28,5	040. * . 0		—	19
h 407	75	− 32,03 (8)	+ 5,318 (2)*	− 32,03	+ 91,6	052. 4 . 5		* 92	21
	76	− 32,12 (5)	+ 5,158 (4)*	− 32,12	+ 88,9	050. 1 . 5		—	20
* 92	75	+ 94,40 (4)	− 9,523 (2)*	+ 94,40	−164,1	11		* 89	22
	76	+ 94,33 (3)	− 9,502 (4)*	+ 94,33	−163,8			—	18
h 434	199	− 95,00 (6)	−14,466 (4)*	− 94,99	−249,8	351. 4 . 3		* 93	23
	200	− 95,52 (6)	−14,660 (3)*	− 95,51	−253,1	351. 3 . 3		—	23
h 444	76	−159,21 (4)	+11,606 (4)*	−159,22	+200,0	0		* 94	21
	128	−158,82 (5)	+11,538 (3)	−158,84	+198,7	0		—	21
h 445	76	+ 2,17 (9)	+ 1,137 (4)*	+ 2,17	+ 19,6	0		h 444	22
	128	+ 1,93 (6)†	+ 0,818 (3)**	+ 1,93	+ 14,1	0		—	20
	130	+ 2,09 (6)†	1,849 (5)†	+ 2,09	+ 15,7	0		—	20
	131	+ 2,15 (5)†	1,832 (4)†*	+ 2,15	+ 15,5	0		—	20
h 447	172	− 36,10 (5)	− 5,173 (4)	− 36,10	− 89,3	551. 0 . 5		h 449	5
h 448	172	− 11,16 (5)	− 8,905 (4)	− 11,16	−153,5	541. 1 . 5		h 449	3
h 449	172	− 56,93 (6)	+20,320 (3)	− 56,94	+350 0	441. 1 . 5	13	* 96	2
h 450	11	− 0,42 (5)	− 5,769 (5)	− 0,42	− 99,5	411. * . 0	8,9	* 95	3
	41	− 79,90 (5)	+ 8,994 (5)	− 79,89	+155,2	020. * . 0	8,9	* 98	4
	79	− 79,92 (5)	+ 9,048 (5)*	− 79,92	+156,0	020. * . 0		—	23
* 96	380	− 42,37 (10)	+13,126 (3)	− 42,39	+168,6	9,5		* 97	17
GC.1537	87	− 83,42 (6)	+ 6,013 (4)*	− 83,41	+103,7	552. 1 . 0		* 99	2
	88	− 83,67 (6)	+ 6,235 (5)*	− 83,66	+107,5	052. 1 . 0		—	1
GC.1538	87	− 78,33 (8)	—	− 78,32	—	552. 1 . 0	13	* 99	2
	88	− 78,38 (8)	+ 5,720 (4)*	− 78,73	+ 98,7	0		—	2
	89	− 78,58 (6)	+ 5,655 (3)*	− 78,58	+ 97,5	0		—	2
h 456	198	− 8,29 (8)	− 2,310 (4)	− 8,29	− 39,8	541. 3 . 4		* 100	22
h 457	170	+ 20,17 (7)	+ 0,762 (2)	+ 20,17	+ 13,2	333. 3 . 3		* 101	1
	198	+ 19,93 (7)	+ 0,660 (4)	+ 19,93	+ 11,5	332. 3 . 4		—	23
II. 616	201	− 39,05 (8)	− 7,696 (3)	− 39,05	−132,8	442. * . 0		* 102	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
II. 616	202	− 39^{s}30 (6)	− 7$''$971 (4)*	− 39^{s}30	−132$''$4	443. 2 . 3		* 102	22^h
	375	− 39,98 (10)	−10,161 (2)	− 39,97	−131,5	443. 3 . 3		—	21
* a (II. 616)	202	+ 4,42 (6)	+ 3,145 (3)*	+ 4,41	+ 54,2	12		II. 616	23
	375	− 35,46 (7)	− 6,009 (2)	− 35,46	− 77,8	12		* 102	22
* 102	375	−368,73 (8)	+15,057 (2)	−368,84	+189,3	9,4		* 103	21
	375	−374,76 (8)	+14,312 (2)	−374,87	+179,6			* 104	21
h 481	199	− 22,09 (7)	+ 8,569 (3)*	− 22,09	+147,6	442. 1 . 5	12	* 105	23
	200	− 22,29 (7)	+ 8,496 (4)	− 22,29	+146,4	442. 1 . 4		—	23
h 483	2	−178,78 (5)	− 1,687 (2)	−178,77	− 28,5	0		* 107	23
	82	−178,73 (7)	− 1,717 (3)	−178,73	− 29,7	431. * . 0	12	—	23
h 482	236	− 50,86 (7)	− 4,758 (3)	− 50,85	− 61,7	442. 2 . 4		* 106	21
	237	− 51,00 (7)	− 4,640 (3)	− 51,00	− 60,1	442. 2 . 3		—	21
* 106	384	−452,78 (10)	−26,878 (2)	−452,84	−352,7	8,5		* 108	20
h 489	235	−182,35 (6)	−12,864 (4)	−182,37	−167,0	542. *3.4		* 109	20
h 494	198	− 77,48 (4)	− 9,325 (3)	− 77,48	−161,0	440. 2 . 0		* 110	23
h 495	337	− 88,29 (10)	+15,820 (2)	− 88,39	+202,8	433. 1 . 0	11	* 111	18
	338	− 88,21 (10)	+15,852 (2)	− 88,31	+203,2	433. 1 . 3	11	—	16
* 112	384	+374,89 (8)	−19,879 (2)	+374,97	−251,5	9,5		* 108	22
h 497	236	+ 9,19 (9)	+21,398 (4)	+ 9,18	+276,2	432. 1 . 4	11.12	* 112	22
	237	+ 9,02 (9)	+21,510 (3)	+ 9,02	+277,6	532. 1 . 3	11	—	22
	384	+ 9,05 (10)	+21,418 (2)	+ 9,04	+276,7	542. 1 . 0	12	—	23
* a (h 497)	384	+ 11,47 (10)	+16,415 (2)	+ 11,46	+211,8	12		* 112	23
h 500	77	− 49,63 (7)	− 3,755 (4)*	− 49,63	− 64,8	041. 0 . 0		* 113	21
	85	− 49,77 (7)	− 3,685 (6)	− 49,77	− 63,6	541. * . 0	12.13	—	23
h 501	77	− 37,24 (8)	−16,687 (4)*	− 37,24	−287,7	040. 0 . 0		* 113	21
	85	− 37,66 (7)	−16,912 (3)*	− 37,65	−291,6	541. * . 0	12.13	—	0
	86	− 37,29 (7)	−16,927 (4)	− 37,28	−291,8	0		—	0
II. 259	241	−166,01 (8)	+17,138 (2)	−166,03	+219,8	432. 1 . 5	11.12	* 114	0
	242	−166,08 (6)	+17,102 (2)	−166,10	+219,4	0		—	23
h 507	241	−129,48 (6)	+12,989 (3)	−129,49	+166,5	542. * . 4	12	* 116	22
	242	−129,50 (8)	+12,971 (2)	−129,51	+166,3	0		—	22
* 116	385	+107,97 (10)	+28,119 (2)	+107,97	+364,5	9,0		* 115	20

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 526	11	+215^{s}81 (5)	+ 6$''$756 (3)	+215^{s}79	+116$''$1	500. *.0	11	*117	4^h
	89	+215,49 (5)	+ 7,092 (3)*	+215,47	+122,3	432. 1.0	12	—	2
h 530	338	− 58,19 (10)	+18,492 (2)	− 58,26	+237,6	432. 1.5	10.11	*118	17
	376	− 58,45 (10)	+18,465 (2)	− 58,53	+237,1	422. 1.0	10	—	17
h 532	10	−227,88 (4)	− 1,341 (3)	−227,84	− 22,8	204. 1.0		*119	3
	83	−227,87 (5)	− 0,982 (4)	−227,87	− 17,1	224. 1.0		—	2
	84	—	− 0,970 (3)	—	− 16,9	0		—	20
*118	376	−298,71 (8)	+16,812 (2)	−298,98	+213,4	8,9		*120	17
	376	−337,70 (8)	+17,439 (2)	−337,99	+220,9			*121	17
h 550	337	+ 28,70 (10)	−13,620 (2)	+ 28,76	−175,1	354. 3.3		*122	19
h 557	199	− 22,34 (7)	+11,678 (3)	− 22,35	+201,2	442. 2.5		*123	23
	200	− 22,13 (8)	+11,872 (3)	− 22,13	+204,6	442. 3.3		—	0
h 565	202	−106,02 (6)	+ 3,967 (3)	−106,03	+ 68,1	352. 3.4		*125	22
h 564	3	− 48,18 (6)	+ 1,291 (7)	− 48,18	+ 22,3	421. *.0	10.11	*124	23
	80	− 48,03 (6)	+ 1,375 (5)*	− 48,04	+ 23,7	030. *.0	11	—	23
h 566	290	− 61,24 (7)	−22,722 (3)	− 61,24	−293,7	442. 3.3		*126	1
	292	− 61,21 (7)	−22,662 (3)	− 61,22	−292,9	442. 3.3		—	2
h 567	292	+ 20,40 (8)	−29,021 (3)	+ 20,42	−373,4	452. 3.3		h 566	2
h 577	170	− 11,05 (6)	+17,716 (4)	− 11,05	+305,4	541. 1.5	12	*128	1
*128	385	+ 57,03 (10)	+11,745 (2)	+ 57,04	+152,2	9,5		*127	21
h 578	170	+ 6,08 (6)	− 9,173 (3)	+ 6,09	−158,1	551. *.5	12.13	*128	1
h 584	337	− 26,04 (8)	—	− 26,05	—	022. 1.0		*129	20
	338	− 26,11 (10)	−10,809 (3)	− 26,12	−139,8	432. 1.0		—	17
h 593	80	+ 29,16 (8)	− 7,529 (5)*	+ 29,17	−129,8	033. *.0	12.13	*130	21
	284	+ 28,79 (9)	−10,113 (3)	+ 28,80	−130,1	431. 1.0	11.12	—	21
h 597	78	− 27,83 (7)	+ 4,311 (4)*	− 27,83	+ 74,3	030. *.0		*132	0
	90	− 27,91 (7)	+ 4,268 (3)*	− 27,90	+ 73,6	0		—	1
	144	−119,16 (5)	+ 1,896 (2)	−119,15	+ 32,6	431. *.5	11	*133	3
h 598	78	− 22,85 (6)	+ 2,588 (4)*	− 22,85	+ 44,6	040. *.0		*132	1
	90	− 22,79 (7)	+ 2,215 (2)*	− 22,79	+ 38,2	0		—	2
	144	+ 4,76 (6)†	− 1,631 (4)	+ 4,76	− 28,1	442. *.5	12	h 597	1
*132	78	− 91,45 (4)	− 2,367 (2)*	− 91,46	− 40,8	11		*133	2
	90	− 91,43 (4)	− 2,450 (2)*	− 91,42	− 42,2	11		—	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h596	338	+ 80^{s}18 (10)	+15^{r}532 (2)	+ 80^{s}25	+201″2	441. 1 . 4	12	* 131	17^h
I. 56	47	+109,94 (5)	−16,496 (7)	+109,93	−284,4	030. 1 . 3		* 134	2
	140	+109,60 (5)	−16,253 (3)	+109,60	−280,1	233. 1 . 4	11	—	22
I. 57	140	+ 1,86 (5)†	+ 3,745 (4)	+ 1,86	+ 64,6	233. 1 . 4	13	I. 56	23
h608	295	− 40,00 (7)	+ 0,139 (3)	− 40,00	+ 1,6	443. 3 . 3		* 136	1
	311	− 16,73 (7)	+12,050 (3)	− 16,73	+155,1	443. 3 . 3		h609	0
	312	− 16,74 (8)	+12,060 (2)	− 16,74	+155,2	0		—	23
h 609	295	− 22,87 (7)	−11,935 (3)	− 22,87	−154,2	443. * . 4	12	* 136	2
	311	− 22,91 (7)	−11,903 (3)	− 22,91	−153,8	542. * . 4	12.13	—	0
	312	− 22,89 (7)	−11,909 (2)	− 22,88	−153,9	0		—	23
* 136	312	+204,96 (8)	+ 0,475 (2)	+204,97	+ 7,3	9,4		* 135	0
h622	171	+ 91,60 (5)	−14,058 (3)	+ 91,61	−242,1	333. 1 . 4	11.12	* 137	1
	203	—	−14,094 (4)	—	−242,7	0		—	20
	205	—	−13,952 (3)	—	−240,2	332. 2 . 3		—	22
	212	+ 91,47 (7)	−13,970 (2)	+ 91,48	−240,6	333. * 3 . 3	11.12	—	2
h624	6	+108,92 (5)	—	+108,91	—	0		* 137	0
	11	+109,18 (4)	+ 2,669 (2)	+109,16	+ 45,9	0		—	4
	171	+109,18 (5)	+ 3,070 (4)	+109,19	+ 53,2	431. 1 . 5	11.12	—	1
	203	—	+ 3,042 (4)	—	+ 52,7	0		—	21
	205	—	+ 3,233 (4)	—	+ 56 0	443. 1 . 0	12	—	22
	212	+109,27 (6)	+ 3,099 (2)	+109,27	+ 53,7	433. * . 4	11.12	—	3
*a (h627)	171	+ 5,71 (4)†	+11,717 (3)	+ 5,71	+202,0	10		h624	3
	205	+ 5,62 (6)	+11,535 (2)	+ 5,62	+198,8	10		—	23
h627	171	+ 19,69 (8)	+10,426 (3)	+ 19,69	+179,7	541. 1 . 5	12.13	h624	1
	205	—	+10,414 (3)	—	+179,5	552. * 3 . 0	13	—	22
	212	+ 19,16 (8)	+10,457 (2)	+ 19,16	+180,3	552. * . 5	12.13	—	3
h630	5	+ 2,04 (5)	+ 1,432 (5)	+ 2,04	+ 24,8	0		* 138	0
	91	+ 2,19 (12)	+ 1,465 (5)*	+ 2,19	+ 25,3	432. * . 0	12	—	1
GC.1916	293	− 9,12 (6)	− 5,922 (2)	− 9,12	− 76,5	452. * . 0	13	h634	2
h634	247	−120,55 (6)	− 9,561 (4)	−120,56	−123,8	550. * . 0	12.13	* 139	0
	293	−119,98 (6)	− 9,326 (2)	−119,99	−120,9	552. * . 4	12.13	—	2
	247	—	+ 4,449 (2)	—	+ 57,3	550. * . 0	12.13	* a (12)	0
	293	− 1,87 (7)	+ 4,282 (2)	− 1,87	+ 55,2	552. * . 4	12.13	—	2
h636	247	+ 25,50 (7)	+21,142 (3)	+ 25,50	+273,1	452. 3 . 3		h634	0
	293	− 94,07 (6)	+12,122 (2)	− 94,08	+155,7	453. 1 . 0		* 139	2
h645	245	+ 57,13 (8)	−13,182 (3)	+ 57,14	−169,5	442. 3 . 2		* 140	22

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 645	245	− 4ˢ25 (8)	+ 2ʳ688 (3)	− 4ˢ25	+ 34″6	442. 3 . 2		* a (10.11)	22ʰ
* a (h 645)	143	+ 61,44 (5)	−11,860 (3)	+ 61,44	−204,3	10.11		* 140	1
M. 82 = IV. 79	59	+ 25,33 (6)	+ 7,807 (4)	+ 25,32	+134,6	134. 3 . 3		* 142	12
h 650	284	− 91,99 (7)	+10,402 (3)	− 92,01	+133.5	541. * . 0	12	* 143	22
	284	−115,13 (3)	+ 5,354 (3)	−115,16	+ 68,5	541. * . 0	12	* 144	22
* a (h 650)	284	—	+19,059 (3)	—	+244,9	9		* 143	22
* b (h 650)	284	—	+ 2,319 (3)	—	+ 29,5	10.11		* 143	22
M. 81 = h 649	59	+139,49 (4)	+ 4,727 (4)	+139,46	+ 81,5	322. 1 . 0	9	* 141	11
	339	+139,42 (10)	+ 6,148 (2)	+139,67	+ 80,3	030. * . 0	10.11	—	15
h 656	5	− 92,68 (6)	+ 4,805 (4)	− 92,68	+ 83,0	0		* 145	1
	92	− 92,39 (5)	+ 4,645 (4)	− 92,39	+ 80,1	043. 1 . 2	12.13	—	1
h 659	12	+ 65,96 (5)	− 0,474 (4)	+ 65,95	− 8,3	0		11°.2131	3
	91	− 21,27 (8)	−13,418 (6)*	− 21,27	−231,4	431. * . 0	12.13	* 147	1
h 658	35	+107,46 (5)	+ 3,982 (5)*	+107,47	+ 68,5	0		* 146	17
	340	+107,60 (8)	+ 5,089 (3)	+107,79	+ 66,4	030. 2 . 4		—	14
	340	+ 22,08 (6)	−14,286 (2)	+ 22,17	−183,8	030. 2 . 4		* 148	14
h 663	286	− 11,29 (8)	+ 0,699 (2)	− 11,29	+ 9,0	433. * . 4	11	* a	0
	309	—	+ 0,710 (3)	—	+ 9,1	443. * . 4	12	—	21
	315	− 11,33 (10)	+ 0,653 (2)	− 11,33	+ 8,4	544. 1 . 4	11.12	—	2
	286	−187,90 (5)	− 3,936 (2)	−187,93	− 51,5	433. * . 4	11	* 149	0
	309	−187,80 (6)	− 4,003 (3)	−187,84	− 52,6	443. * . 4	12	—	21
	315	−187,73 (6)	− 4,005 (2)	−187,76	− 52,6	544. 1 . 4	11.12	—	1
h 668	12	−130,02 (3)	+ 0,662 (2)	−130,01	+ 11,6	224. 1 . 0	9.10	* 150	1
h 682	246	− 39,37 (7)	−12,851 (3)	− 39,37	−166,1	442. 3 . 2		* 153	21
	294	− 38,77 (9)	−13,154 (3)	− 38,77	−169,9	352. 3 . 3		—	1
	246	+ 14,50 (7)	+ 2,068 (2)	+ 14,50	+ 26,6	442. 3 . 2		* a (10)	22
	246	− 3,90 (7)	+ 2,865 (2)	− 3,90	+ 36,8	442. 3 . 2		* b (12.13)	22
* 153	294	+ 65,77 (7)	−23,604 (3)	+ 65,78	−303,6	8,9		* 151	1
h 684	3	+ 32,88 (5)	+ 7,309 (5)	+ 32,88	+126,0	0		* 152	13
	41	+ 33,00 7)	+ 7,247 (4)	+ 33,00	+125,0	031. * . 0	10.11	—	2
	45	+ 32,87 (6)	+ 7,323 (3)*	+ 32,87	+126,3	031. * . 0	10.11	—	2
h 685	3	− 28,10 (5)	+ 0,250 (4)	− 28,10	+ 4,4	0		* 154	0
	41	− 27,83 (6)	—	− 27,83	—	033. * . 0	11.12	—	2
	45	−117,31 (5)	−18,110 (4)*	−117,30	−312,2	032. * . 0	12	* 155	1
	88	−117,11 (5)	−17,998 (3)*	−117,11	−310,3	432. 1 . 0	11.12	—	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC.2054	5	− 20^{s}17 (7)	− 1^{r}152 (2)	− 20^{s}17	− 19$''$9	0		* 157	3^h
	97	− 20,12 (7)	− 1,152 (7)	− 20,12	− 19,9	042. * . 3	13	—	2
h. 692	5	+ 66,33 (4)	−16,693 (3)	+ 66,32	−287,8	0		* 156	2
	50	+ 66,40 (5)	−16,808 (4)	+ 66,39	−289,8	334. * . 0	11.12	—	3
	97	+ 66,26 (5)	−16,679 (3)	+ 66,26	−287,5	034. * . 0	11.12	—	3
h 693	4	+ 85,15 (6)	− 3,847 (5)	+ 85,14	− 66,4	401. * . 0	11	* 156	1
	50	+ 85,37 (6)	− 3,683 (6)	+ 85,36	− 63,6	431. * . 0	12	—	4
	97	+ 85,31 (4)	− 3,752 (3)	+ 85,30	− 64,7	431. * . 0	11.12	—	1
GC 2084_1	145	− 0,55 (7)†	1,355 (4)†	− 0,55	− 22,0	551. * . 5	13	GC.2084_2	2
GC.2084_2	94	− 52,47 (4)	− 2,502 (2)	− 52,46	− 43,1	530. * . 0		II. 28	2
	145	− 56,04 (5)	+ 4,409 (2)	− 56,04	+ 76.0	541. * . 5	12	II. 29	4
II. 28	94	− 3,86(12)	+ 6,935 (4)	− 3,86	+119,5	331. * . 0	11.12	II. 29	1
	145	− 3,70 (6)†	7,510 (5)†	− 3,70	+118,6	331. * . 0	11	—	1
	394	− 3,73(10)	—	− 3,74	—	000. * . 0	11	—	0
II. 29	93	+216,19 (4)	+ 4,720 (3)	+216,18	+ 81,4	331. * . 0	11.12	* 159	1
	394	+215,83 (7)	+ 6,251 (2)	+215,88	+ 82,1	000. * . 0	11	—	0
h 711	18	− 21,42 (5)	+17,251 (4)	− 21,41	+297,4	432. * . 0	10.11	* 161	2
	91	− 21,21 (6)	+17,345 (3)	− 21,21	+299,0	332. * . 0	11	—	2
h. 713	246	+ 33,65 (6)	+22,024 (2)	+ 33,65	+284,5	531. 1 . 5	11.12	* 160	23
	291	+ 33,96 (7)	+21,982 (3)	+ 33,96	+284,0	443. * . 0	12	—	1
h 714	200	−106,48 (6)	+ 8,914 (3)	−106,50	+153,4	334. 1 . 4	12	* 164	0
	204	−106,37 (7)	+ 8,815 (4)	−106,39	+151,7	0		—	22
	310	−106,71 (7)	+11,971 (3)	−106,74	+153,7	543. 1 . 4	12.13	—	23
	204	− 25,41(12)	− 4,370 (5)	− 25,41	− 75,4	0		* 162	23
	310	− 25,20 (7)	− 5,756 (3)	− 25,20	− 74,4	543. 1 . 4	12.13	—	23
Nova VI	139	− 0,20 (5)†	7,814(4)*†	− 0,20	−134,7	542. * . 4		* a	23
* a (Nova VI)	139	− 8,63 (5)†	− 6,596 (4)	− 8,63	−113,7	10		* 165	23
* 165	384	+ 64,12(10)	−28,837 (2)	+ 64,17	−370,8	9,5		* 163	22
h 721	91	+ 59,19 (6)	− 5,243 (3)	+ 59,19	− 90,4	331. * . 0	11.12	* 165	3
	139	+ 59,03 (6)	− 5,101 (4)	+ 59,04	− 87,9	332. * . 4	11.12	—	21
	384	+ 59,09(10)	− 6,870 (2)	+ 59,12	− 88,1	432. 1 . 4	11.12	—	22
h 724	285	− 28,79(10)	− 6,682 (4)	− 28,80	− 86,3	334. 1 . 4		* 166	23
h 728	50	−203,29 (4)	+12,189 (3)	−203,27	+210,4	432. * . 0	11.12	* 167	5
	96	−203,62 (4)	+12,232 (3)	−203,61	+210,8	030. * . 0	11.12	—	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h737	244	+ 10^{s}41 (7)	− 2'548 (3)	+ 10^{s}41	− 32"8	234. 1 . 5		* 168	23^h
	395	+ 10,81 (14)	− 2,638 (2)	+ 10,81	− 33,9	343. 3 . 3		—	22
h739	243	− 69,68 (8)	−11,994 (3)	− 69,69	−155,0	542. *3.0	12.13	* 170	23
	244	− 69,38 (8)	−11,979 (3)	− 69,38	−154,8	442. 1 . 4	11.12	—	21
	243	—	+ 0,731 (5)	—	+ 9,4	542. *3.0	12.13	* a (9.10)	22
	244	− 7,09 (11)	+ 0,846 (2)	− 7,09	+ 10,9	442. 1 . 4	11.12	—	21
* c(h739)	243	—	− 0,660 (2)	—	− 8,5	12		* a	22
	244	-- 5,71 (9)	− 0,709 (3)	− 5,71	− 9,2	12.13		—	22
*b (h739)	243	—	+ 0,941 (2)	—	+ 12,1	11		* a	22
	244	− 2,68 (8)	+ 0,981 (2)	− 2,68	+ 12,6	11		—	22
*a(h739)	243	− 62,29 (7)	−12,848 (3)	− 62,30	−166,0	9.10		* 170	23
	244	− 62,51 (7)	−12,795 (4)	− 62,52	−165,3	10		—	21
h743	8	+ 24,98 (5)	− 1,245 (6)	+ 24,98	− 21,5	501. 1 . 5	10.11	* 169	1
	44	+ 24,96 (7)	− 1,275 (5)*	+ 24,95	− 22,0	030. * . 0	11	—	1
	46	+ 24.89 (8)	− 1,249 (7)*	+ 24,89	− 21,6	030. 0 . 0		—	1
h748	8	+ 59,48 (6)	− 7,123 (3)	+ 59,48	−122,8	0		* 171	5
	147	+ 59,60 (5)	− 6,463 (4)	+ 59,60	−111,4	331. 1 . 4		—	1
h749	8	− 45,90 (5)	− 3,934 (5)	− 45,90	− 67,7	0		* 174	2
	46	− 45,83 (7)	− 3,768 (5)*	− 45,83	− 65,0	020. 1 . 0	9.10	—	2
h754	13	−300,81 (4)	+ 8,194 (3)	−300,78	+141,7	0		* 180	2
	49	−300,83 (4)	+ 8,301 (2)	−300,80	+143.2	530. * . 0	11	—	1
h755	14	+ 77,20 (4)	− 3,876 (3)	+ 77,19	− 66,9	0		* 172	1
	48	+ 77,38 (6)	− 3,910 (4)	+ 77,38	− 67,4	421. 1 . 2	9.10	—	1
h758	14	+104,65 (3)	+ 6,282 (2)	+104,64	+108,1	0		* 172	1
	48	+104,63 (4)	+ 6,289 (3)	+104,62	+108,4	420. 1 . 0	10	—	2
h761	14	+ 37,62 (6)	−10,868 (2)	+ 37,61	−187,3	000. * . 0	13	h757	2
h765	146	−172,17 (3)	− 2,451 (2)	−172,17	− 42,4	433. * . 4	11.12	* 178	5
	393	−172,35 (10)	− 2,888 (2)	−172,47	− 37,9	443. 1 . 4	12	—	22
h766	143	+ 5,34 (5)†	+ 1,684 (4)	+ 5,34	+ 29,0	0		h765	1
	146	+ 5,09 (5)†	4,013(5)†*	+ 5,09	+ 27,6	531. * . 0	12	—	0
	393	+ 5,32 (12)	+ 2,144 (2)	+ 5,32	+ 27,7	541. * . 0	12	—	23
h774	17	+208,99 (4)	+ 7,661 (3)	+208,97	+131,9	400. 1 . 0	10.11	* 173	2
II. 493	146	− 43,86 (5)	+28,619 (4)	− 43,86	+493,3	341. 1 . 5		* 177	4

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta''$ in Scale-Divisions	Mean-Coordinates 1865.0 $\Delta\alpha^s$	Mean-Coordinates 1865.0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 773	148	+ 82^{s}94 (5)	+ 0$''$837 (4)	+ 82^{s}94	+ 14$''$5	431. 1 . 4	11.12	* 175	0^h
	149	+ 83,00 (4)	+ 0,850 (2)	+ 83,00	+ 14,6	431. 1 . 4	11	—	0
h 775	148	− 11,08 (5)†	10,866(3)*†	− 11,08	+117,9	442. 1 . 4	12.13	* a	1
	149	+ 58,10 (4)	− 2,419 (3)	+ 58,10	− 41,7	0		* 176	0
II.494 (* a)	146	− 52,03 (4)	−19,104 (3)	− 52,02	−329,3	444. 0 . 0	11.12	* 178	3
	393	− 52,11 (10)	−25,450 (2)	− 52,14	−328,9	444. 3 . 0	12	—	23
I. 118	146	− 29,88 (6)	− 8,637 (4)	− 29,88	−148,9	332. 3 . 2		* 178	2
	393	− 29,91 (10)	−11,099 (2)	− 29,92	−143,4	343. *3.4	12.13	—	23
h 784	15	− 17,97 (7)	+ 5,045 (4)	− 17,97	+ 87,0	0		* 179	1
	85	− 17,49 (8)	+ 5,509 (5)*	− 17,49	+ 95,0	441. * . 0	13	—	23
h 805	206	−168,56 (5)	+ 8,775 (3)	−168,59	+151,0	332. *2.3	11.12	* 183	23
	207	−168,66 (5)	+ 8,864 (2)	−168,68	+152,5	332. * . 3	11	—	23
h 806	13	+ 33,80 (5)	− 0,661 (5)	+ 33,80	− 11,4	0		* 181	3
	49	+ 33,58 (6)	− 0,640 (5)	+ 33,58	− 11,1	030. 0 . 0		—	2
h 810	149	+ 63,26 (5)	+20,759 (3)*	+ 63,26	+357,8	433. 1 . 5	10.11	* 182	2
	208	+ 63,42 (5)	+20,786 (3)*	+ 63,42	+358,4	003. 1 . 0	11	—	23
h 813	206	+ 32,58 (5)	− 8,909 (3)	+ 32,59	−153,5	044. 3 . 0		* 183	23
	207	+ 32,33 (7)	− 9,244 (3)	+ 32,33	−159,3	344. 3 . 4		—	23
h 815	149	− 93,75 (6)	+ 2,436 (4)*	− 93,75	+ 41,9	441. 1 . 3	12.13	* 185	2
h 818	16	+106,93 (3)	− 0,773 (4)	+106,93	− 13,4	0		* 184	1
	92	+107,19 (4)	− 0,853 (3)	+107,19	− 14,8	020. 1 . 0	10.11	—	1
h 838	338	−147,73 (8)	−26,562 (2)	−147,91	-343,3	241. 4 . 4		* 188	18
h 840	209	− 81,35 (6)	− 6,214 (3)	− 81,35	−107,2	333. * . 5	12	* 187	1
	214	− 81,28 (5)	− 6,313 (3)	− 81,28	−108,9	033. * . 0		—	2
h 843	16	+ 61,50 (6)	+ 8,532 (2)	+ 61.49	+147,1	431. * . 0	11	* 186	3
	42	+ 61,51 (7)	+ 8,597 (3)	+ 61,51	+148,2	431. * . 0	12	—	1
h 844	16	− 8,32 (6)	− 7,250 (4)	− 8,32	−125,0	041. * . 0	11.12	h 845	3
	42	− 8,03 (6)	− 7,332 (4)	− 8,03	−126,4	040. * . 0	13	---	3
	43	−158,96 (4)	−19,377 (3)*	−158,95	−334,0	441. * . 0	13	* 190	1
h 845	7	+149,46 (5)	− 3,316 (3)	+149,45	− 57,2	031. 1 . 0	10	* 186	23
	42	+149,60 (5)	− 3,431 (4)	+149,59	− 59,2	431. 1 . 0	10	—	2
h 846	7	+153,93 (4)	+16,863 (2)	+153,91	+290,6	441. 1 . 0	11.12	* 186	0
	43	+153,67 (5)	+16,870 (2)*	+153.66	+290,8	031. 1 . 0	12	—	1
	42	+ 4,03 (6)	—	+ 4,03	—	031. 1 . 0	11.12	h 845	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h847	338	− 44^{s}28 (10)	−28$'$136 (2)	− 44^{s}34	−363″5	433. 1 . 3	10.11	* 189	17^h
h854	209	− 71,68 (6)	+14,510 (3)	− 71,69	+250,1	034. * . 5	11.12	* 191	0
	213	− 71,65 (6)	+14,520 (2)	− 71,65	+250,3	124. 1 . 3	10.11	—	0
h856	287	−160,35 (6)	+ 2,256 (3)	−160,38	+ 28,8	431. 1 . 0	11	* 194	1
h857	209	+ 7,67 (6)	− 6,363 (3)	+ 7,67	−109,7	234. * . 5	11.12	* 191	23
	213	+ 7,70 (7)	− 6,411 (3)	+ 7,70	−110,5	223. 1 . 3	10.11	—	1
h859	353	− 59,45(10)	−28,529 (2)	− 59,45	−368,7	045. 3 . 0		* 193	23
	395	− 59,55(10)	−28,695 (2)	− 59,56	−370,9	035. 2 . 5		—	22
h861	285	− 31,21 (7)	—	− 31,22	—	533. 1 . 0	11	* 192	0
	286	− 31,20 (7)	+28,507 (4)	− 31,20	+367,0	533. 1 . 5	11.12	—	0
h864	286	−126,73 (7)	+20,527 (3)	−126,74	+264,2	431. 1 . 3	11	* 196	1
	395	−126,43(10)	+20,401 (2)	−126,44	+262,3	433. 1 . 3	10.11	—	23
h871	245	− 91,30 (6)	+12,125 (3)	− 91,33	+156,0	443. 2 . 4		* 198	22
* 198	391	−152,08(10)	− 1,937 (2)	−152,17	− 25,1	8,4		* 199	19
h881	245	+ 66,87 (6)	+19,378 (2)	+ 66,88	+250,3	432. 1 . 0	11.12	* 197	23
h891	15	− 11,20 (4)	—	− 11,20	—	0		* 200	3
	45	− 11,29 (7)	—	− 11,28	—	030. 0 . 0		—	2
	46	− 11,06 (5)	−19,650 (5)*	− 11,06	−338,8	040. 0 . 0		—	3
	246	− 11,57 (7)	−26,340 (2)	− 11,57	−340,2	432. 1 . 5	12	—	23
	316	− 11,40 (7)	−26,457 (2)	− 11,40	−341,7	442. 1 . 0	12	—	1
h890	341	+220,62 (6)	−16,743 (2)	+221,07	−214,9	043. * . 0	12	* 195	16
h892	348	−174,23(10)	− 9,394 (2)	−174,45	−121,8	045. 3 . 4		* 204	19
h893	46	+ 30,17 (6)	+15,165 (3)*	+ 30,16	+261,6	050. 0 . 0		* 200	4
	246	− 34,08 (6)	+ 0,335 (2)	− 34,09	+ 4,3	442. *3.4	12.13	* 202	23
	316	− 33,73 (8)	+ 0,572 (2)	− 33,73	+ 7,3	442. 1 . 4	13	—	1
h894	246	− 88,47 (7)	− 4,429 (2)	− 88,48	− 57,2	343. 1 . 4	12	* 203	23
	316	− 88,57 (7)	− 4,325 (3)	− 88,58	− 56,0	342. *3.3	12.13	—	0
h897	287	+ 78,29 (7)	+22,577 (2)	+ 78,30	+291,7	443. 1 . 5		* 201	2
h898	246	−110,26 (5)	−18,670 (3)	−110,27	−241,2	453. 1 . 5	12.13	* 205	0
h902	285	− 25,40 (8)	−35,126 (3)	− 25,40	−453,8	433. 1 . 3	11.12	* 206	23
h933	289	− 60,23 (6)	− 0,318 (2)	− 60,24	− 4,2	542. 0 . 0		* 208	0

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 938	250	− 60^{s}71 (7)	−30$''$586 (4)	− 60^{s}72	−395$''$1	443. 1 . 3	12	* 209	23^h
	251	− 60,20 (7)	−30,551 (3)	− 60,21	−394,6	443. 1 . 3	12	—	23
h 939	289	− 30,31 (7)	+ 3,830 (3)	− 30,32	+ 49,3	442. 3 . 3		* 208	23
h 940	289	− 29,07 (7)	+14,849 (2)	− 29,08	+191,2	552. * . 0	13	* 208	0
h 943	288	− 64,24 (7)	+ 2,614 (3)	− 64,25	+ 33,6	442. 2 . 3		* 211	23
h 944	250	− 6,44 (8)	+ 4,234 (4)	− 6,44	+ 54,5	542. *3.5	12	* 209	23
	251	− 6,41 (7)	+ 4,263 (4)	− 6,42	+ 54,9	542. *3.4	12.13	—	22
h 945	248	+ 63,51 (7)	+19,556 (3)	+ 63,53	+252,6	443. 2 . 4		* 207	20
h 946	250	+ 25,49 (9)	− 1,823 (4)	+ 25,49	− 23,5	054. 1 . 5	13	* 209	23
	251	− 8,55 (10)	−29,105 (2)	− 8,55	−375,9	053. 1 . 5	13	* 210	23
* 210	251	+ 34,18 (10)	+27,515 (5)	+ 34,18	+355,4	9,5		* 209	21
	404	+ 34,02 (10)	+27,530 (2)	+ 34,04	+355,6			—	1
h 988	356	+133,05 (8)	− 0,286 (2)**	+133,10	− 3,5	433. 2.0		* 212	23
h 1005	355	+116,10 (8)	+26,610 (2)	+116,16	+343,9	422. 1 . 0	10.11	* 213	23
h 1017	346	−183,40 (2)	− 1,852 (2)	−183,62	− 24,3	431. 2 . 4		* 215	18
	347	−183,48 (10)	− 1,910 (2)	−183,70	− 25,0	433. 3 . 3		—	18
* 214	376	−377,53 (7)	+ 3,028 (3)	−378,09	+ 38,7	9,1		* 219	17
h 1029	346	− 21,52 (10)	+ 1,370 (2)	- 21,55	+ 17,6	443. *3.3	12.13	h 1031	17
	347	− 21,35 (10)	+ 1,268 (2)	− 21,38	+ 16,3	442. * . 0	12	—	19
h 1031	346	+ 21,97 (10)	−21,598 (2)	+ 22,01	−278,0	431. 1 . 3	10.11	* 214	17
	347	+ 22,02 (10)	−21,544 (2)	+ 22,06	−277,3	432. 1 . 0	10.11	—	19
h 1047	344	− 25,46 (10)	−28,708 (2)	− 25,48	−370,9	035. 1 . 4	11	* 216	17
h 1049	247	−114,54 (7)	−13,656 (2)	−114,56	−176,3	343. 2 . 3	12	* 218	23
h 1050	342	+ 76,87 (10)	−19,766 (2)	+ 77,00	−254,4	334. 1 . 4	11.12	* 217	15
h 1054	342	−229,98 (6)	−10,709 (2)	−230,36	−138,5	432. *2.3	12	* 222	16
I. 224	345	+ 23,05 (10)	+18,917 (2)	+ 23,07	+244,3	344. 3 . 0		* 220	17
I. 206	345	−323,22 (6)	− 1,994 (2)	−323,55	− 26,2	234. 3 . 0		* 226	18
* 221	381	−464,09 (10)	+19,604 (2)	−464,20	+252,1	9,1		* 230	12
h 1087	343	+ 68,42 (8)	+ 5,496 (2)	+ 68,30	+ 71,0	442. 3 . 0		* 221	18

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1088	345	− 18^{s}03 (10)	− 8$''$956 (2)	− 18^{s}04	−115$''$6	035. 1 . 3	11	* 223	20^h
h 1091	345	+ 25,97 (10)	+ 8,279 (2)	+ 25,99	+106,7	551. 0 . 0		* 223	20
Hind Nova	343	−128,36 (10)	+ 0,167 (2)	−128,60	+ 2,0	432. 1 . 0	11.12	* 225	16
	343	− 24,87 (10)	+ 2,431 (2)	− 24,92	+ 31,3	432. 1 . 0	11.12	65°.870	16
h 1106	47	−160,36 (5)	+ 1,618 (6)	−160,35	+ 27,9	421. 1 . 3	10	* 228	1
	53	−160,50 (5)	+ 1,511 (4)	−160,49	+ 26,1	420. 1 . 2	10	—	2
h 1115	351	+ 52,96 (10)	+ 7,560 (2)	+ 52,98	+ 97,6	442. 3 . 0		* 224	22
	396	+ 53,11 (10)	+ 7,607 (2)	+ 53,13	+ 98,2	442. *3 . 3	12	—	22
h 1125	351	− 27,84 (10)	+ 7,147 (2)	− 27,85	+ 92,1	055. 3 . 0		* 229	23
h 1132	299	−133,23 (6)	− 0,032 (3)**	−133,25	− 0,4	035. 1 . 3	11.12	* 235	0
	322	−133,10 (7)	+ 0,023 (3)**	−133,12	+ 0,3	035. *2 . 0	12.13	—	1
h 1140	314	+ 5,94 (9)	−16,474 (4)	+ 5,95	−212,3	431. 1 . 3	11.12	* 231	22
h 1146. α	152	− 15,87 (5)†	+10,238 (2)	− 15,87	+176,5	000. * . 0	11	* 234	4
	391	− 15,45 (12)	+13,506 (3)	− 15,46	+174,2	330. *2 . 5	12	—	20
h 1146. β	152	+ 1,58 (5)†	− 1,480 (3)	+ 1,58	− 25,5	000. * . 0	12.13	h 1146. α	3
* 234	391	+204,07 (10)	+ 5,990 (2)	+204,22	+ 77,4	9,5		* 227	20
	404	+204,05 (10)	+ 5,981 (2)	+204,20	+ 77,3			—	1
h 1147	292	−100,50 (7)	+ 4,731 (3)	−100,51	+ 61,0	441. 1 . 3		* 237	0
	296	−100,73 (7)	+ 4,715 (2)	−100,73	+ 60,8	443. 1 . 4	12	—	0
	292	+ 7,84 (7)	−10,416 (2)	+ 7,84	−134,2	441. 1 . 3		* a	1
	296	+ 7,71 (7)	−10,393 (3)	+ 7,77	−133,9	443. 1 . 4	12	—	0
h 1148	47	+ 41,22 (5)	+ 1,248 (5)	+ 41,22	+ 21,5	034. 1 . 3	10.11	* 232	3
	54	+ 41,34 (5)	+ 1,199 (6)	+ 41.34	+ 20,7	224. 1 . 2	10	—	1
h 1153	95	+ 43,76 (7)	+ 3,145 (3)	+ 43,76	+ 54,2	033. 2 . 0		* 233	0
	297	+ 43,30 8)	+ 4,124 (2)	+ 43,30	+ 53,2	443. * . 4	13	—	0
h 1161	95	− 35,96 (7)	− 8,546 (4)	− 35,96	−147,4	031. 1 . 0	12	* 238	1
	297	− 35,94 (7)	−11,395 (3)	− 35,94	−147,1	442. 1 . 0	12.13	—	0
h 1159	95	− 44,87 (6)	+26,161 (5)	− 44,87	+451,2	033. * . 0		* 240	2
h 1162	355	− 64,33 (10)	+ 0,336 (2)	− 64,35	+ 4,3	443. 3 . 0		* 241	23
h 1168	142	+ 33,32 (6)	− 2,559 (4)	+ 33,31	− 44,1	330. 1 . 0	11.12	* 236	22
	210	+ 33,37 (7)	− 2,606 (3)	+ 33,38	− 44,9	432. 1 . 3	11.12	—	22
h 1171	303	− 83,86 (8)	+ 3,972 (3)	− 83,96	+ 50,5	434. 1 . 0	10.11	* 243	3

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1171	320	− 83^{s}90 (9)	+ 3′955 (2)	− 84^{s}02	+ 50″2	0		* 243	0^h
	303	− 13,09 (7)	+ 3,690 (2)	− 13,09	+ 47,5	434. 1 . 0	10.11	* a	3
h 1173	152	− 61,80 (6)	+ 7,105 (4)	− 61,80	+122,5	331. 1 . 2	11.12	* 245	0
	402	− 61,91 (10)	+ 9,472 (2)	− 61,93	+122,1	332. 1 . 3		—	23
	152	− 6,89 (5)†	+ 3,025 (4)	− 6,89	+ 52,2	331. 1 . 2	11.12	* a	1
h 1178	9	+ 43,40 (3)	+ 3,122 (4)	+ 43,40	+ 53,9	0		* 242	1
	103	− 59,38 (5)	− 2,353 (2)	− 59,38	− 40,6	040. * . 0	13	h 1187	2
	150	− 59,40 (5)	− 1,963 (3)	− 59,40	− 33,8	551. * . 0	13	—	0
h 1177	51	− 25,28 (9)	+12,021 (6)	− 25,28	+207,3	040. * . 0	13	* 244	2
	100	− 24,83 (6)	+12,324 (2)	− 24,83	+212,5	443. 1 . 2	13	—	2
h 1176	52	− 48,72 (5)	+15,389 (3)	− 48,72	+265,5	432. 1 . 0	10.11	* 248	3
	100	− 48,87 (5)	+15,393 (5)	− 48,87	+265,4	333. 1 . 2	11.12	—	23
h 1179	323	−146,03 (7)	+ 8,425 (2)	−146,05	+108,6	530. 1 . 0	11	* 252	1
	402	−146,01 (8)	+ 8,448 (2)	−146,05	+109,1	521. 1 . 0	10.11	—	0
h 1180	100	− 35,86 (5)	+19,885 (3)	− 35,86	+342,9	040. * . 3	13	* 248	1
h 1178. a	10	−166,13 (5)	− 4,801 (4)	−166,10	− 82,7	0		* 254	2
	141	− 9,40 (6)	−12,549 (3)	− 9,40	−216,4	432. * . 0	11.12	h 1183	0
	150	—	−12,459 (2)	—	−215,0	531. 1 . 5	12	—	4
GC. 2849	51	− 15,38 (5)	+20,338 (4)	− 15,38	+350,8	443. * . 0	13	* 247	4
	100	− 15,03 (5)	+20,262 (3)	− 15,03	+349,5	043. 0 . 0		—	3
h 1178. b	103	− 83,15 (4)	− 5,455 (3)	− 83,15	− 94,0	032. 1 . 0	12.13	* 250	2
	150	− 83,33 (4)	− 5,448 (2)	− 83,32	− 93,9	432. 1 . 0	11.12	—	0
h 1183	9	+ 77,70 (5)	− 3,801 (4)	+ 77,72	− 65,5	0		* 254	2
	103	− 25,67 (4)	− 8,925 (3)	− 25,67	−153,9	033. 2 . 2		h 1187	1
	141	− 25,82 (6)	− 9,012 (4)	− 25,82	−155,4	330. 1 . 0	11.12	—	23
	150	− 25,60 (5)	− 8,784 (3)	− 25,60	−151,6	332. 1 . 3	11.12	—	4
h 1185	142	+168,22 (6)	− 1,318 (3)	+168,23	− 22,7	233. * . 0	11	* 236	22
	210	+168,30 (5)	− 1.436 (2)	+168,32	− 24,7	333. 1 . 3	11	—	23
h 1186	142	−185,88 (4)	+ 6,530 (2)	−185,89	+112.6	421. 1 . 0	10	* 256	23
	210	−186,92 (5)	+ 6,519 (3)	−186,96	+112,3	421. * . 5	10	—	0
	211	−186,77 (7)	+ 6,477 (2)	−186,79	+111,6	0		—	23
	212	−186,42 (6)	+ 6,532 (2)	−186,44	+112,7	0		—	3
h 1187	103	−131,90 (4)	+16,559 (2)	−131,89	+285,6	030. * . 0	11.12	* 254	0
	141	−132,02 (5)	+16,630 (3)	−132,03	+286,9	322. 1 . 0	11	—	22
	150	−131,97 (3)	+16 579 (2)	−131,97	+286,0	330. 1 . 3	10.11	—	3

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1188	142	+ 14^{s}72 (5)†	+ 6$'$363 (4)	+ 14^{s}72	+109$''$7	531. ∗ . 0	11	h 1186	23^h
	210	+ 14,12 (6)	+ 6,300 (3)	+ 14,12	+108,6	431. ∗ . 5	11	—	0
	212	+ 14,28 (9)	+ 6,412 (2)	+ 14,28	+110,5	531. ∗ . 0	11	—	3
∗ 249	399	+ 59,85 (10)	+22,149 (2)	+ 59,85	+286,1	8,3		∗ 246	0
∗a (h 1196)	298	+ 14,59 (7)	+20,985 (2)	+ 14,59	+271,1	10.11		∗ 249	1
h 1196	298	+ 16,39 (7)	+15,411 (3)	+ 16,39	+199,0	443. ∗ . 0	12.13	∗ 249	1
h 1204	349	−223,09 (10)	−15,567 (2)	−223,20	−201,1	035. 1 . 3		∗ 261	21
h 1201	103	− 51,63 (4)	+15,719 (2)	− 51,63	+271,2	042. ∗ . 0	13	∗ 254	3
	141	− 51,82 (6)	+16,116 (3)	− 51,83	+277,9	442. ∗2.0	12	—	22
	150	− 51,74 (5)	+16,149 (3)	− 51,75	+278 5	442. 1 . 5	12.13	—	2
h 1202	298	+ 55.21 (7)	−18,596 (2)	+ 55,21	−239,5	231. 1 . 0	11	∗ 249	1
	298	+115,07 (7)	+ 3,669 (2)	+115,07	+ 47,3	231. 1 . 0	11	∗ 246	1
GC.2892	103	+ 32,36 (4)	−12,172 (2)	+ 32,35	−210,0	032. ∗ . 0	12	∗ 254	3
	150	+ 32,78 (5)	−11,934 (3)	+ 32,77	−205,9	432. 1 . 5	11.12	—	0
h 1213	296	− 23,37 (6)	− 2,503 (3)	− 23,37	− 32,3	050. 0 . 0		h 1222	1
	321	− 23,31 (7)	− 2,561 (2)	− 23,31	− 33,0	452. ∗ . 0	13	—	1
h 1215	296	− 12,87 (6)	−11,804 (3)	− 12,87	−152,4	050. 0 . 0		h 1222	1
	321	− 13,00 (7)	−11,422 (2)	− 13,00	−147,5	452. ∗ . 0	13	—	1
h 1222	51	− 36,55 (4)	+22,943 (3)	− 36,55	+396,0	442. ∗ . 0	12	∗ 259	4
	52	− 36,53 (4)	+22,798 (4)	− 36,52	+393,1	443 ∗ . 0	12	—	1
	100	− 36,67 (4)	+22,726 (3)	− 36,66	+392,1	442. ∗ . 0	12	—	3
	296	− 36,58 (9)	+30,617 (3)	− 36,58	+394,2	441. ∗ . 0	12.13	—	2
	321	− 36,49 (7)	+30,585 (2)	− 36,49	+393,8	441. ∗ . 4	12	—	0
∗a (h 1222)	321	− 36,04 (7)	+23,736 (2)	− 36,05	+305,6	11.12		∗ 259	1
h 1212	330	+ 93,23 (10)	− 6,804 (2)	+ 93,25	− 87,7	442. ∗3.0	12	∗ 253	2
	331	+ 93,17 (10)	− 6,936 (3)	+ 93,19	− 89,4	0		—	1
h 1221	330	+115,98 (10)	−14,965 (2)	+116,01	−192,7	334. ∗2.0	11	∗ 253	2
	331	+115,95 (10)	−14,990 (2)	+115,98	−193,0	0		—	1
h 1232	95	−171,44 (4)	+16,162 (3)	−171,43	+278,9	421. 1 . 0	10.11	∗ 264	3
	303	−171,57 (7)	+21,736 (2)	−171,57	+280,0	431. 1 . 3	11	—	1
h 1237	54	+ 31,46 (5)	+ 2,245 (3)	+ 31,46	+ 38,7	312. 1 . 0	9	∗ 260	3
	304	+ 31,69 (11)	+ 2,920 (2)	+ 31,69	+ 37,7	020. 1 . 3	10.11	—	1
h 1239	147	− 9,52 (5)	+22,924 (3)	− 9,52	+395,2	421. 1 . 5	10	∗ 262	1
	399	− 9,43 (12)	+30,723 (2)	− 9,43	+395,5	432. 1 . 4	11	—	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1228	150	− 12^{s}30 (5)†	11$'$932 (4)†	− 12^{s}30	+ 93$''$0	432. 1 . 5	11.12	* 263	1^h
	298	− 12,51 (7)	+ 7,277 (3)	− 12,51	+ 93,8	442. 2 . 3		—	0
* 262	399	+242,57 (10)	−30,172 (2)	+242,63	−388,4	9,5		* 251	23
h 1242	297	+103,70 (6)	+ 5,879 (3)	+103,71	+ 75,8	323. 1 . 3	10.11	* 258	1
	297	− 9,99 (7)	+ 9,757 (2)	− 9,99	+125,7	323. 1 . 3	10.11	* a	2
* b (h 1242)	297	− 7,84 (7)	+13,422 (2)	− 7,84	+172,8	12		* a	1
h 1250	54	+ 69,75 (4)	−13,793 (3)	+ 69,74	−237,8	442. * . 0	12.13	* 260	2
	55	+ 69,64 (5)	−13,816 (6)	+ 69,63	−238,2	443. 1 . 3	12	—	1
h 1244	55	+145,38 (5)	− 1,711 (2)	+145,37	− 29,5	343. 3 . 2		* 257	3
	85	+145,34 (4)	− 1,535 (3)*	+145,33	− 26,5	443. 1 . 0		—	23
h 1251	297	+ 31,59 (7)	+ 6,244 (3)	+ 31,59	+ 80,6	443. * . 0	12	h 1242	1
h 1253	54	+ 99,68 (4)	+14,527 (2)	+ 99,67	+250,6	311. 1 . 0		* 260	4
	55	+ 99,50 (4)	+14,624 (3)	+ 99,49	+252,3	020. 0 . 0		—	4
	304	+ 99,71 (7)	—	+ 99,72	—	020. 1 . 0	10.11	—	1
h 1258	319	+227,28 (6)	− 7,644 (2)	+227,35	− 98,5	035. 1 . 0	11.12	* 255	3
h 1267	85	− 16,81 (9)	− 3,482 (4)*	− 16,81	− 60,0	443. * . 0	12.13	* 267	0
h 1271	351	−122,32 (6)	− 8,435 (2)	−122,34	−108,8	333. 1 . 0	11	* 275	1
	352	− 5,43 (10)	+22,745 (2)	− 5,43	+292,8	432. 2 . 3		* 268	23
	352	− 3,69 (10)	− 8,191 (2)	− 3,69	−105,7	432. 2 . 3		* 266	23
h 1268	102	− 26,20 (4)	− 1,373 (2)*	− 26,19	− 23,6	453. 0 . 0		h 1276	3
h 1274	47	− 67,33 (4)	− 1,957 (3)	− 67,32	− 33,8	420. * . 0	10.11	* 273	3
	48	− 67,44 (5)	− 2,132 (4)	− 67,44	− 36,8	431. * . 0	11	—	1
h 1272	102	− 36,30 (5)	− 6,073 (2)*	− 36,30	−104,7	451. * . 0	13.14	* 271	2
h 1275	47	− 62,35 (4)	−16,822 (3)	− 62,35	−290,2	433. 0 . 0		* 273	4
	48	− 62,38 (5)	−16,587 (4)	− 62,38	−286,0	033. * . 0	12	—	2
h 1276	56	− 23,85 (4)	−11,042 (3)	− 23,85	−190,4	432. * . 0	11.12	* 271	3
	101	− 23,66 (5)	−11,121 (4)	− 23,66	−191,7	432. * . 0	12.13	—	3
h 1279	350	−149,68 (10)	+24,333 (2)	−149,70	+313,5	433. 1 . 3	10.11	* 277	0
h 1286	325	−134,00 (7)	+ 2,073 (2)	−134,00	+ 26,8	030. 1 . 0	11.12	* 278	1
	325	− 95,87 (7)	+17,566 (2)	− 95,87	+226,3	030. 1 . 0	11.12	* 276	1
h 1287	294	− 5,74 (7)	+16,318 (3)	− 5,74	+210,1	443. * . 0	12.13	h 1290	0
h 1288	293	+ 26,11 (7)	+ 7,599 (3)	+ 26,12	+ 98,1	331. 1 . 4	11.12	* 272	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865.0 $\Delta\alpha^s$	Mean-Coordinates 1865.0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1288	319	+ 26^{s}24 (7)	+ 7$''$611 (2)	+ 26^{s}25	+ 98$''$2	431. 1 . 0	11.12	* 272	1^h
	293	− 5,50 (8)	+ 7,915 (3)	− 5,50	+102,0	331. 1 . 4	11.12	* 274	2
	319	− 5,87 (7)	+ 7,937 (2)	− 5,87	+102,3	431. 1 . 0	11.12	—	1
h 1290	294	+ 15,56 (7)	+26,458 (3)	+ 15,56	+341,7	433. 1 . 0	11.12	* 273	0
h 1292	99	− 98,47 (4)	+16,782 (3)	− 98,47	+289,4	430. * . 0	12	h 1305	1
	317	− 98,93 (8)	+22.264 (2)	− 98,93	+286,8	541. 1 . 5	12	—	2
	318	− 98,90 (7)	+22,493 (2)	− 98,90	+289,7	531. 1 . 5	11.12	—	2
Struve Nova	98	− 15,94 (7)	− 1,648 (2)	− 15,94	− 28,4	552. 0 . 0		h 1294	0
II. 630	293	− 22,69 (8)	− 5,466 (3)	− 22,69	− 70,5	454. 1 . 5		h 1295	2
	319	− 22,90 (7)	− 5,633 (2)	− 22,90	− 72,7	452. * . 0	13	—	2
h 1293	98	−117,88 (4)	+ 6,815 (3)*	−117,88	+117.6	042. 2 . 3		* 279	1
	317	−117,64 (7)	+ 9,772 (3)	−117,65	+126,0	441. 1 . 5	13	—	3
	318	−117,81 (7)	+ 9,862 (2)	−117,82	+127,0	443. 1 . 5		—	0
h 1294	98	+138,52 (4)	− 2,877 (4)	+138,51	− 49,7	012. 1 . 2		* 265	23
	99	+138,36 (3)	− 2,852 (3)	+138,36	− 49,3	110. 1 . 2		—	3
	317	+138,49 (7)	− 3,924 (2)	+138,49	− 50,7	221. 1 . 2	10	—	0
	318	+138,50 (6)	− 3,812 (2)	+138,51	− 49,2	221. 1 . 2	9.10	—	3
	317	− 3,54 (9)	0	− 3,55	0,0	221. 1 . 2		* a	0
II. 114	292	+110,13 (7)	− 5,039 (3)	+110,14	− 65,0	332. 1 . 3		* 270	1
	294	+110,25 (6)	− 5,052 (2)	+110,26	− 65,2	332. 1 . 3	11.12	—	2
h 1295	293	+ 53,64 (7)	+24,977 (3)	+ 53,65	+322,6	443. 1 . 4	11.12	h 1288	2
	319	+ 53,86 (7)	+25,034 (2)	+ 53,87	+323,3	433. 1 . 0	12	—	1
h 1296	56	+101,95 (4)	+ 0,272 (3)	+101,94	+ 4,7	442. * . 0	12	* 271	2
	101	− 18,35 (5)	+ 4,049 (3)	− 18,35	+ 69,8	432. * . 0	13	h 1298	3
II. 115	292	−136,72 (6)	− 6,089 (2)	−136,73	− 78,5	332. 1 . 3		* 283	3
	294	−136,85 (6)	− 5,984 (2)	−136,87	− 77,1	332. 1 . 3	11.12	—	2
h 1298	56	+120,13 (4)	− 3,923 (4)	+120,12	− 67,7	432. * . 0	10.11	* 271	4
	102	+120,26 (4)	− 3,900 (4)*	+120,25	− 67,3	432. * . 2	12	—	0
	215	+120,46 (5)	− 3,803 (3)	+120,46	− 65,7	0		—	2
II. 116	292	+ 16,26 (7)	−16,603 (3)	+ 16,26	−213,8	453. 1 . 0		II. 115	3
	294	+ 16,33 (7)	−16,771 (3)	+ 16,33	−216,0	453. 1 . 5	13	—	2
h 1301	56	+152,24 (5)	+ 9,132 (3)	+152,23	+157,4	211. 1 . 2	10	* 271	1
	215	+152,76 (5)	+ 9,171 (3)	+152,76	+158,0	0		—	2
h 1305	98	+ 73,50 (5)	+16,194 (3)*	+ 73,50	+279,2	042. * . 0	12.13	h 1294	1
	317	+ 73,35 (8)	+21,245 (2)	+ 73,36	+274,4	441. * . 4	12.13	—	2
	318	+ 73,23 (7)	+21,188 (2)	+ 73,23	+273,6	441. * . 4	12.13	—	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1307	297	− 9^{s}96 (7)	−26″829 (2)	− 9^{s}96	−346″5	431. 1 . 4	12	* 280	3^h
	313	− 10,29 (9)	−26,749 (2)	− 10,29	−345,5	432. 1 . 3	11.12	—	20
h 1312	147	− 5,50 (5)	+17,701 (3)	− 5,50	+305,2	224. 1 . 0	10.11	* 282	2
	396	− 5,48 (10)	+23,585 (2)	− 5,48	+303,6	035. 1 . 0	11	—	23
* 282	398	+261,98 (10)	+22,229 (2)	+261,92	+287,0	9,4		* 269	22
h 1313	351	+158,17 (6)	+10,183 (2)	+158,19	+131,4	432. 1 . 0	11	* 275	0
	352	+158,19 (10)	+10,116 (2)	+158,22	+130,5	433. 1 . 0	11	—	0
h 1322	99	− 13,33 (8)	−20,114 (2)*	− 13,33	−346,8	541. * . 0		* 285	0
	318	− 13,20 (8)	−26,955 (2)	− 13,20	−348,2	553. 3 . 5		—	0
* h 1329	98	− 27,35 (5)	− 7,698 (3)*	− 27,35	−132,8	023. 1 . 0	11	* 289	3
	318	− 27,46 (8)	−10,220 (2)	− 27,47	−131,9	334. 1 . 3	12	—	23
	99	+ 29,48 (5)	+ 4,461 (3)*	+ 29,48	+ 76,9	224. 1 . 2	10.11	* 286	23
	313	+ 29,90 (7)	+ 6,045 (3)	+ 29,90	+ 78,0	224. 1 . 3	10.11	—	23
	318	+ 29,55 (8)	+ 6,006 (2)	+ 29,55	+ 77,5	334. 1 . 3	12	—	23
h 1331	351	− 31,57 (10)	+ 5,683 (2)	− 31,58	+ 73,3	532. 1 . 0	11.12	* 290	0
h 1345	147	− 34,90 (6)	− 3,584 (3)	− 34,90	− 61,8	430. 0 . 0		* 294	2
	396	− 34,62 (10)	− 5,827 (2)	− 34,63	− 75,2	333. 1 . 3	11.12	—	22
h 1343	207	− 98,36 (5)	+ 8,014 (3)	− 98,39	+138,3	433. * . 5	11	* 297	23
h 1349	207	+ 7,63 (7)	+ 9,040 (3)	+ 7,63	+155,9	433. * . 5		h 1343	0
h 1348	102	+ 92,43 (4)	− 6,127 (2)*	+ 92,42	−105,7	020. 1 . 0		* 288	3
	207	+ 92,66 (5)	− 6,101 (2)	+ 92,68	−105,3	422. 1 . 4	10	—	0
	207	− 7,28 (6)	− 1,437 (2)	− 7,29	− 24,8	422. 1 . 4	10	* a	1
* 294	396	+ 63,48 (10)	+ 5,934 (2)	+ 63,50	+ 76,5	11		* 291	22
h 1356	252	+ 26,80 (8)	+ 8,638 (2)	+ 26,72	+111,5	433. *2 . 0	12	* 293	22
	253	+ 26,94 (7)	+ 8,603 (3)	+ 26,87	+111,0	433. 1 . 3	11	—	22
	310	+ 27,01 (7)	+ 8,588 (2)	+ 26,91	+110,8	533. * . 0	12	—	0
	253	+ 8,05 (6)	− 9,714 (3)	+ 8,05	−121,1	433. 1 . 3	11	* a	23
	253	− 10,04 (7)	+13,689 (2)	− 10,04	+176,3	433. 1 . 3	11	* b	23
h 1357	297	+ 37,03 (7)	+ 1,402 (3)	+ 37,04	+ 18,1	035. 1 . 5	12.13	* 292	3
	313	+ 36,71 (7)	+ 1,456 (3)	+ 36,73	+ 18,8	035. 1 . 4	12.11	—	21
h 1358	252	− 15,42 (9)	—	− 15,42	—	0		* 296	22
	253	− 14,98 (6)	− 0,768 (3)	− 14,98	− 9,9	343. 2 . 5		—	0
	310	− 14,70 (7)	− 0,989 (2)	− 14,70	− 12,7	443. 3 . 5		—	0
*a(h 1357)	313	+ 41,91 (7)	+ 6,813 (2)	+ 41,93	+ 87,9	12		* 292	21

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1359	253	− 13ˢ11 (7)	− 6ʳ131 (3)	− 13ˢ11	− 79″1	343. 2 . 5		* 296	0ʰ
	310	− 13,41 (7)	− 6,325 (2)	− 13,42	− 81,6	443. 3 . 5		—	0
* 296	253	+ 68,45 (6)	−18,292 (2)	+ 68,45	−235,6	11		12°.2490	0
	310	+129,82 (6)	−11,392 (2)	+129,83	−146,8	10		* 290	0
M. 90	48	+ 46,16 (5)	− 4,145 (4)	+ 46,16	− 71,5	020. * . 0	10	* 295	2
	294	+ 46,37 (7)	− 5,588 (2)	+ 46,38	− 72,1	035. 1 . 5	11.12	—	3
h 1361	300	+225,56 (5)	− 1.740 (2)	+225,56	− 22,6	035. 1 . 3	10.11	* 284	0
	300	+199 04 (5)	−10,139 (2)	+199,04	−130,8	035. 1 . 3	10.11	* 287	1
h 1368	151	− 13,22 (6)	−19,900 (3)	− 13,22	−343,1	322. 1 . 2	10	* 300	23
	310	− 12,89 (7)	−26,376 (3)	− 12,89	−340,7	431. 1 . 3	10.11	—	22
* 300	310	+ 44,31 (7)	+26,467 (3)	+ 44,32	+341,8	9,4		* 298	23
h 1371	353	+ 36,30 (10)	+ 7,927 (2)	+ 36,30	+102,3	442. * 2 . 0	11.12	* 299	0
h 1378	312	−116,84 (7)	−17,243 (2)	−116,86	−222,6	432. 1 . 3	11	* 305	0
	316	−116,56 (7)	−17,208 (2)	−116,57	−222,1	431. 1 . 4	11	—	1
h 1383	312	− 39,56 (8)	−23,116 (4)	− 39,57	−298 5	433. 1 . 3	12	* 305	0
	316	− 39,30 (7)	−23,183 (2)	− 39,31	−299,4	431. 1 . 0	11.12	—	1
	312	+ 6,30 (7)	− 2,611 (2)	+ 6,30	− 33,6	433. 1 . 3	12	* a	0
	316	+ 6,04 (7)	− 2,526 (2)	+ 6,04	− 32,6	431. 1 . 0	11.12	—	2
h 1384	300	− 15,14 (7)	−15,496 (2)	− 15,14	−200,1	441. * . 4	12	* 304	1
* a (h 1384)	300	− 11,34 (9)	−15,570 (2)	− 11,35	−201,1	10.11		* 304	1
h 1386	144	− 98,05 (6)	+19,846 (2)	− 98,05	+342,2	322. 1 . 0	10	h 1408	1
	151	+ 21,32 (5)	−24,224 (4)	+ 21,32	−417,6	322. 1 . 2	10	* 303	23
	254	+ 21,33 (7)	−32,483 (2)	+ 21,33	−418,1	030. 0 . 0		—	2
h 1399	302	+ 10,83 (7)	+30,342 (3)	+ 10,83	+392,1	231. 1 . 3	11.12	* 306	1
	302	− 42,56 (7)	+18,861 (2)	− 42,56	+242,9	231. 1 . 3	11.12	* 307	1
h 1402	151	− 52,86 (5)	−22,844 (3)	− 52,86	−393,8	534. * . 0	11.12	h 1408	0
	254	− 52,94 (7)	−30,506 (4)	− 52,95	−394,0	030. 0 . 0		—	0
	254	+ 8,93 (8)	−20,562 (3)	+ 8,92	−264,7	030. 0 . 0		* a	0
h 1404	288	+107,90 (5)	− 5,772 (3)	+107,91	− 74,5	431. 1 . 4	11	* 302	0
	332	+107,91 (10)	− 5,881 (2)	+107,92	− 75,9	334. 1 . 3	11	—	1
	288	+ 4,00 (7)	−10,474 (3)	+ 4,00	−135,0	431. 1 . 4	11	* a	0
h 1405	144	− 7,47 (5)†	+ 6,197 (3)	− 7,47	+106,9	342. * . 4	12.13	h 1408	2
	151	− 7,48 (5)†	+ 6,219 (5)	− 7,48	+107,2	343. 2 . 2		—	1
h 1408	151	−150,03 (4)	+ 4,421 (2)	−150,02	+ 76,3	212. 1 . 2	9.10	* 311	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1408	254	−149^{s}88 (6)	+ 5^{r}954 (3)	−149^{s}89	+ 76″9	020. 0 . 0		* 311	1^h
h 1409	353	−198,63 (7)	−27,327 (2)	−198,68	−352,8	433. 1 . 0	11.12	* 312	23
h 1414	212	−101,18 (5)	− 1,863 (3)	−101,20	− 32,0	234. 3 . 5		* 309	4
h 1417	253	− 96,82 (5)	+ 9,189 (2)	− 96,82	+118,5	431. 1 . 0	10.11	* 310	2
	254	− 96,88 (6)	+ 9,171 (3)	− 96,89	+118,3	030. 0 . 0		—	2
h 1419	302	+ 67,16 (7)	− 3,479 (3)	+ 67,16	− 44,9	332. 1 . 3	11.12	* 308	0
	302	+ 4,53 (7)	− 4,675 (2)	+ 4,53	− 60,4	332. 1 . 3	11.12	* a	1
* 309	392	+292,83 (10)	+16,026 (2)	+293,01	+206,7	9,2		* 301	20
* 313	400	−402,81 (8)	−21,757 (2)	−402,91	−280,3	9,3		* 319	23
h 1436	357	− 38,89 (10)	+ 6,432 (2)	− 38,89	+ 83,0	433. 1 . 0	11	* 314	23
h 1441	357	+ 41,85 (10)	− 1,727 (2)	+ 41,86	− 22,3	045. 1 . 3		15^6.2511	22
	400	+135,24 10)	−23,787 (2)	+135,27	−306,4	035. 1 . 3		* 313	23
h 1451	288	− 73,71 (7)	− 2,488 (3)	− 73,73	− 32,0	433. 1 . 3	10.11	* 315	23
	312	− 73,83 (7)	− 2,385 (3)	− 73,85	− 30,7	032. 1 . 3	11	—	22
*a (h 1466)	255	+ 14,77 (9)	− 4,003 (2)	+ 14,77	− 51,6	10.11		* 316	1
h 1466	253	+ 14,71 (9)	+ 6,026 (3)	+ 14,71	+ 77,8	035. 1 . 0	11	* 316	3
	254	+ 15,03 (8)	+ 5,972 (3)	+ 15,03	+ 77,1	030. 0 . 0		—	3
	255	+ 15,00 (7)	+ 5,994 (3)	+ 15,00	+ 77,4	0		—	0
	255	—	+ 9,907 (4)	—	+127,9	0		* a	23
	255	− 11,56 (8)	+ 3,090 (4)	− 11,56	+ 39,8	0		* b	0
h 1469	355	+ 37,28 (10)	+14,258 (2)	+ 37,28	+184,1	432. 1 . 0	12	* 317	23
h 1462	253	− 23,47 (7)	+29,422 (3)	− 23,47	+378,8	431. 1 . 0	10.11	* 320	2
	254	− 23,44 (7)	+29,396 (3)	− 23,44	+378,5	030. 0 . 0		—	3
	255	+ 22,36 (7)	—	+ 22,36	—	0		* a	1
h 1475	348	+ 87,22 (10)	−15,381 (2)	+ 87,26	−198,2	443. 3 . 0		* 318	19
	403	+ 87,03 (10)	−15,135 (2)	+ 87,07	−195,2	433. 2 . 3		—	23
	348	− 0,75 (5)†	− 7,837 (2)	− 0,75	−101,2	443 3 . 0		* a	19
h 1486	323	+126,63 (7)	+27,053 (2)	+126,67	+349,2	223. 1 . 3	10	* 321	1
	323	− 13,09 (7)	−15,340 (2)	− 13,09	−198,0	223. 1 . 3	10	* a	1
h 1500	349	− 19,64 (8)	−24,841 (2)	− 19,65	−320,8	452. * . 0	13	* 322	21
h 1498	354	− 30,88 (10)	−31,825 (2)	− 30,89	−411,0	035. 1 . 0	11.12	* 323	23
	401	− 30,68 (10)	−31,860 (2)	− 30,69	−411,5	044. * . 0	12	—	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1502	349	− 32^{s}69 (10)	− 4^{r}833 (2)	− 32^{s}71	− 62″3	442. 0 . 0		h 1507	22^h
* 323	401	− 49,03 (10)	−26,096 (2)	− 49,05	−336,9	9,0		* 324	23
h 1507	349	+ 35,16 (10)	−25,078 (2)	+ 35,17	−322,8	441. 1 . 0	12	* 322	22
h 1522	354	−181,48 (10)	− 5,527 (2)	−181,56	− 70,6	543. * . 0	12.13	* 325	22
h 1547	330	+ 51,12 (10)	−26,738 (2)	+ 51,14	−344,2	334. 1 . 0	11	* 326	2
h 1601	289	+ 54,48 (5)	− 1,797 (2)	+ 54,48	− 23,3	432. * . 0	11.12	* 327	23
	332	+ 54,91 (10)	− 1,723 (2)	+ 54,91	− 22,4	040. *3.0	11.12	—	1
h 1622	138	−124,85 (6)	− 7,712 (4)	−124,90	−132,8	222. 1 . 4	10.11	* 328	3
	138	+ 7,07 (5)†	+ 3,139 (4)	+ 7,07	+ 54,1	222. 1 . 4	10.11	* a	3
h 1623	138	+ 7,00 (5)†	+14,763 (4)	+ 7,01	+254,5	422. * . 4	10.11	h 1622	3
h 1650	325	+195,86 (5)	− 5,734 (2)	+195,87	− 74,1	030. 1 . 0	12	* 329	1
	325	+193,30 (5)	−14,686 (2)	+193,31	−188,9	030. 1 . 0	12	* 330	2
h 1664	358	− 90,93 (10)	−16,453 (2)	− 90,98	−212,1	432. 1 . 3	11.12	* 331	22
*a (h 1684)	120	− 0,12 (5)†	− 1,364 (3)	− 0,12	− 23,5	12.13		h 1684	11
h 1684	120	+140,84 (5)	+ 4,164 (3)	+140,85	+ 71,5	020. 1 . 0	9.10	* 332	9
	363	+140,62 (10)	+ 5,422 (2)	+140,80	+ 69,1	332. 1 . 4	11	—	9
h 1701	322	− 69,04 (7)	+22,218 (2)	− 69,04	+286,4	453. 3 . 0		h 1703	1
	324	− 68,66 (7)	+22,234 (3)	− 68,66	+286,6	050. 0 . 0		—	1
h 1703	322	− 13,26 (7)	−10,346 (2)	− 13,26	−133,5	333. *2.4	11	* 333	1
	324	− 13,24 (9)	−10,283 (2)	− 13,25	−132,7	032. 1 . 3	11	—	23
	322	− 76,59 (7)	+ 4,423 (2)	− 76,59	+ 57,3	333. *2.4	11	* 334	1
	324	− 76,36 (8)	+ 4,525 (2)	− 76,37	+ 58,6	032. 1 . 3	11	—	0
h 1705	322	−104,85 (4)	−27,870 (2)	−104,86	−359,7	342. 3 . 0		* 335	2
	324	−105,46 (8)	−27,609 (2)	−105,47	−356,3	442. 4 . 0		—	0
h 1748	122	+ 4,37 (5)†	+ 6,393 (3)	+ 4,37	+110,3	030. * . 0		* 336	9
h 1776	296	+ 21,53 (7)	−10,982 (3)	+ 21,53	−141,5	030. 1 . 0	11.12	* 339	2
* 341	396,397	+284,41 (8)	+21,177 (2)	+284,45	+271,7	9,2		* 337	23
h 1778	298	− 15,74 (7)	+16,427 (2)	− 15,74	+211,7	453. 1 . 5		h 1779	1
h 1779	298	+ 87,08 (6)	+10,376 (2)	+ 87,09	+133,6	531. 1 . 0	11	* 338	1
	298	+ 36,03 (6)	−20,928 (2)	+ 36,03	−269,7	531. 1 . 0	11	* 340	2
	298	− 6,57 (6)	+ 0 962 (2)	− 6,57	+ 12,4	531. 1 . 0	11	* a	2

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 1782	355	+ 51^{s}03 (8)	+ 6^{r}275 (2)	+ 51^{s}03	+ 80$''$7	433. *. 0		* 341	23^h
	396	+ 50,87 (10)	+ 6,434 (2)	+ 50,88	+ 82,7	543. 3. 0		—	23
h 1783	355	+ 58,60 (8)	+15,627 (2)	+ 58,61	+201,5	422. 1. 0	10.11	* 341	23
	396.397	+ 58,75 (10)	+15,601 (2)	+ 58,76	+201,0	522. 1. 0	10.11	—	23
h 1854	352	+ 60,88 (10)	+12,007 (2)	+ 60,89	+154,6	432. 1. 0		* 342	23
	352	− 68,85 (10)	−27,069 (2)	− 68,87	−349,3	432. 1. 0		* 343	23
h 1857	358	− 43,96 (10)	− 0,557 (2)	− 43,96	− 6,9	332. 2. 0		* 344	23
	402	− 43,92 (10)	− 0,560 (2)	− 43,92	− 6,8	433. 2. 3		—	23
h 1863	358	+ 44,72 (10)	− 8,389 (2)	+ 44,72	−108,5	543. *. 0	12	h 1857	23
	402	+ 44,74 (10)	− 8,525 (2)	+ 44,74	−110,3	543. *. 0	12	—	23
h 1874	299	+ 9,76 (8)	−19,959 (2)	+ 9,76	−257,1	035. 1. 3	12	* 345	23
	299	− 79,96 (7)	+17,728 (2)	− 80,01	+228,8	035. 1. 3	12	* 346	23
h 1875	358	− 82,47 (10)	+15,632 (2)	− 82,47	+202,1	443. 2. 0		* 347	22
	402	− 82,19 (10)	+15,704 (3)	− 82,19	+203,1	433. 2. 3		—	0
h 1894	299	− 71,23 (7)	+10,887 (2)	− 71,23	+140,7	443. 1. 0	12.13	* 348	0
	401	− 70,86 (8)	+11,257 (2)	− 70,87	+145,3	0		—	0
h 1896	299	−101,74 (7)	+25,227 (2)	−101,74	+325,4	431. 1. 3	11.12	* 349	1
	401	−101,58 (8)	+25,274 (2)	−101,57	+325,8	0		—	0
* 349	401	−330,59 (10)	− 1,157 (2)	−330,60	− 14,1	9,0		* 351	22
	401	−363,88 (10)	−16,999 (2)	−363,89	−218,7			* 352	23
II. 541	300	− 61,76 (5)	+ 8,718 (2)	− 61,76	+112.7	442. 1. 5	12.13	h 1901	1
	301	− 62,05 (6)	+ 8,844 (2)	− 62,05	+114,3	442. 1. 3		—	23
II. 542	301	+ 9,16 (9)	+22,704 (3)	+ 9,16	+293,3	432. 1. 3	11	* 350	0
III. 511	300	− 28,12 (6)	+ 8,213 (2)	− 28,12	+106,0	531. 1. 5	11	h 1901	1
	301	− 28,50 (7)	+ 8,189 (2)	− 28,50	+105,7	531. 1. 5	11	—	23
*b (h 1901)	301	+ 9,01 (9)	− 4,342 (2)	+ 9,01	− 56,0	12.13		* a	23
h 1901	299	−114,15 (8)	− 6,306 (2)	−114,16	− 80,8	331. 1. 0	11.12	* 351	1
	300	−114,17 (6)	− 6,101 (2)	−114,17	− 78,2	331. 1. 3	11	—	1
	301	−114,20 (6)	− 6,082 (2)	−114,20	− 77,9	332. 1. 3	11	—	23
	301	+ 9,10 (7)	− 1,398 (2)	+ 9,10	− 18,1	332. 1. 3	11	* a	23
h 1902	300	+ 38,10 (5)	−17,675 (2)	+ 38,10	−227,9	441. 1. 5	12.13	h 1901	0
	301	+ 37,96 (7)	−17,742 (2)	+ 37,95	−228,8	442. 1. 3	12	—	23
II. 751	207	− 50,12 (5)	+17,978 (3)	− 50,12	+310,1	442. 1. 5	12.13	* 353	23
	403	− 50,44 (10)	+24,239 (2)	− 50,44	+312,6	542. 2. 0		—	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
II. 752	207	− 42^{s}89 (7)	+14$'$687 (3)	− 42^{s}89	+253$''$3	453. 4 . 5		* 353	0^h
	403	− 42,66 (10)	+19,928 (2)	− 42,66	+257,0	453. 2 . 0		—	23
II. 647	263	− 6,58 (8)	+ 8,420 (3)	− 6,57	+108,5	542. 1 . 0	12.13	* 355	6
	264	− 6,67 (7)	+ 8,185 (3)	− 6,67	+105.5	0		—	7
* 355	264	+178,65 (6)	−21,114 (3)	+178,64	−272,9	9,4		* 354	5
h 1961	262	− 89,03 (4)	−12,815 (4)	− 89,03	−164,9	442. 3 . 3		* 357	6
	263	− 88.88 (6)	−12,890 (3)	− 88,89	−165,9	443. 3 . 3		—	5
	264	—	−12,784 (3)	—	−164,5	0		—	6
* 358	260	+ 88,76 (7)	−27,577 (3)	+ 88,74	−355,6	11		* 356	5
II. 753	257	+ 11,10 (8)	− 2,444 (5)	+ 11,10	− 31,6	040. 0 . 0		* 358	4
	260	+ 10,86 (11)	− 2,961 (3)	+ 10,85	− 38,2	443. 1 . 3		—	4
h 1967	64	−173,41 (5)	−11,882 (4)*	−173,39	−204,9	452. * . 5	13	* 360	6
	65	−173,59 (4)	−11,764 (3)*	−173,57	−202,8	452. * . 5	13	—	6
h 1966	185	−119,78 (5)	+25,728 (4)	−119,76	+444,1	050. * . 0	12.13	* 359	6
	362	−118,39 (10)	+34,473 (2)	−118,37	+445,0	052. * . 0	13.14	—	4
h 1969	61	− 27,64 (5)	+15,684 (3)	− 27,64	+270,5	443. * . 0	12.13	* 361	6
	63	− 27,75 (6)	—	− 27,75	—	0		—	6
* a (h 1970)	306	− 43,05 (8)	+23,651 (3)	− 43,04	+304,8	12		* 362	4
h 1970 (Margo preced)	305	− 34,41 (7)	—	− 34,40	—	513. 4 . 3		* 361	4
	306	− 17,81 (7)	—	− 17,82	—	0		* c	3
h 1970 (Margo follow.)	305	− 33,71 (7)	—	− 33,70	—	513. 4 . 3		* 361	4
	306	− 16,97 (7)	—	− 16,98	—	0		* c	3
h 1970 (Margo north)	57	—	+17,754 (3)*	—	+306,2	523. 4 . 4		* 361	5
	305	—	+23,764 (4)	—	+306,2	513. 4 . 3		—	4
h 1970 (Margo south)	57	—	+17,214 (3)*	—	+296,9	523. 4 . 4		* 361	5
	305	—	+22,965 (4)	—	+295,9	523. 4 . 4		—	4
h 1970 (Geometrical Centrum	57	− 34,15 (6)	+17,447 (4)*	− 34,15	+300,8	523. 4 . 4		* 362	5
	58	− 34,24 (5)	+17,436 (4)	− 34,24	+300,7	020. 1 . 3		—	3
	107	− 34,08 (5)	+17,479 (5)	− 34,08	+301,5	511. 1 . 5		—	5
	216	− 34,06 (8)	+23,318 (5)	− 34,06	+300,8	513. 3 . 3		—	4
	305	− 34,09 (7)	+23,322 (3)	− 34,07	+301,0	513. 4 . 3		—	4
	107	+ 8,94 (7)	− 0,230 (6)	+ 8,94	− 4,0	511. 1 . 5		* a	4
	216	+ 8,84 (8)	0,338 (4)**	+ 8,84	− 4,4	513. 3 . 3		—	4
	107	− 5,32 (6)	+ 8,471 (4)	− 5,32	+146,1	511. 1 . 5		* b	4
	306	—	−10,790 (3)	—	−139,2	0		* c	3

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
IV.50	32	+ 35^{s}15 (6)	− 3$'$639 (5)	+ 35^{s}14	− 62$''$7	0		* 363	7^h
	62	+ 35,23 (6)	− 3,691 (5)	+ 35,23	− 63,6	431. 1 . 0	11.12	—	5
	68	+ 35,25 (8)	− 3,449 (5)*	+ 35,25	− 59,5	431. 1 . 0		—	5
M.92	32	− 32,93 (3)	− 1,747 (2)	− 32,93	− 30,2	112. 1 . 1		* 364	8
	33	− 32,90 (4)	− 1,907 (4)	− 32,90	− 32,9	0		—	7
	110	− 32,59 (8)	− 1,825 (6)	− 32,58	− 31,4	0		—	4
	326	− 32,67 (11)	− 2,620 (3)	− 32,68	− 33,5	0		—	8
h 1989	156	+ 7,94 (8)	− 5,189 (4)	+ 7,94	− 89,5	523. * . 0	11	* a	4
IV.37 (Geometrical Centrum)	33	+ 25,84 (5)	− 3,660 (4)	+ 25,83	− 63,1	411. 4 . 4		* 365	8
	68	+ 25,90 (7)	− 3,633 (5)*	+ 25,89	− 62,6	413. 4 . 3		—	5
	239	+ 25,50 (10)	− 4,864 (5)	+ 25,49	− 62,8	0		—	13
	240	+ 25,30 (10)	− 4,879 (5)	+ 25,29	− 63,1	0		—	13
	249	+ 25,46 (10)	− 4,934 (5)	+ 25,46	− 63,7	0		—	16
	249	− 39,91 (8)	−16,282 (5)	− 39,95	−210,0	0		* a	16
	249	+ 12,55 (6)	− 8,274 (2)	+ 12,54	−106,7	0		* b	16
IV.37 (Margo preced.)	239	+ 24,38 (10)	—	+ 24,37	—	0		* 365	13
	240	+ 24,12 (5)	—	+ 24,11	—	0		—	13
IV.37 (Margo follow.)	239	+ 27,21 (10)	—	+ 27,20	—	0		* 365	12
	240	+ 26,70 (5)	—	+ 26,69	—	0		—	13
IV.37 (Margo north)	239	—	− 4,218 (5)	—	− 54,4	0		* 365	13
IV.37 (Margo south)	239	—	− 5,623 (5)	—	− 72,5	0		* 365	13
II.902	268	− 54,14 (11)	− 5,092 (4)	− 54,13	− 65,4	350. 3 . 0		* 366	3
	360	− 53,95 (10)	− 5,025 (2)	− 53,95	− 64,2	452. 3 . 0		—	3
h 2000	24	− 14,36 (7)	− 1,867 (6)	− 14,36	− 32,2	500. 1 . 0	7.8	* 367	2
	57	− 14,30 (6)	− 1,849 (5)*	− 14,30	− 31,9	512. * . 5		—	1
	104	− 14,32 (5)	− 1,831 (3)**	− 14,32	− 31,6	0		—	1
	105	− 14,33 (6)	− 1,855 (4)**	− 14,33	− 32,0	0		—	3
	106	− 14,31 (7)	− 1,846 (4)**	− 14,31	− 31,8	0		—	2
	104	− 13,98 (5)	+ 6,008 (4)**	− 13,98	+103,6	0		* a	2
	105	− 13,94 (5)	—	− 13,94	—	0		—	2
	106	− 13,85 (6)	+ 6,055 (4)**	− 13,85	+104,4	0		—	2
II.907	259	− 16,45 (8)	+ 5,286 (4)	− 16,44	+ 68,2	552. 2 . 4		* 369	3
	260	− 16,97 (6)	+ 5,343 (3)	− 16,96	+ 68,9	452. 1 . 4		—	4
* 369	259	+ 20,81 (7)	−22,872 (3)	+ 20,80	−294,5	9,0		* 368	3
	260	+ 20,90 (7)	−22,840 (3)	+ 20,89	−294,1			—	4
h 2023 (Geometr. Centr.)	25	+349,05 (4)	—	+349,02	—	0		32°3214	3
	26	+348,90 (2)	+ 3,077 (4)	+348,87	+ 53,8	0		—	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 2023 (Geometr. Centr.)	108	− 62^{s}12 (6)	−13$''$254 (5)*	− 62^{s}11	−228$''$5	323. 5 . 3		* 370	4^h
	177	− 62,08 (6)	−13,230 (6)*	− 62,08	−227,7	0		—	2
	307	− 61,97 (7)	−17,678 (5)	− 61,97	−227,6	0		—	2
	108	− 5,22 (5)	—	− 5,22	—	323. 5 . 3		* a	4
	176	—	− 0,221(16)	—	− 3,8	0		—	2
	177	− 4,86 (7)	− 0,183(11)	− 4,86	− 3,1	0		—	2
h 2023 (Margo preced.)	307	− 7,19 (12)	—	− 7,19	—	0		* a	4
h 2023 (Margo follow.)	307	− 2,45 (13)	—	− 2,45	—	0		* a	4
h 2023 (Margo north)	108	—	+ 1,607 (5)*	—	+ 27,7	323. 5 . 3		* a	2
	307	—	+ 2,040 (5)	—	+ 26,3	0		—	2
h 2023 (Margo south)	108	—	− 1,765 (5)*	—	− 30,4	323. 5 . 3		* a	2
	307	—	− 2,346 (5)	—	− 30,2	0		—	2
* a (h 2023)	108	− 56,90 (6)	−12,966 (4)*	− 56,90	−223,3	12		* 370	1
	176	—	−13,015 (5)	—	−224,2			—	1
	177	− 57,05 (6)	—	− 57,05	—			—	1
	307	− 56,93 (7)	−17,432 (5)	− 56,93	−224,6			—	2
GC. 4473	25	+ 44,36 (9)	+ 5,590 (6)	+ 44,35	+113,7	342, 3 . 2		* 371	1
	60	+ 44,78 (6)	+ 6,531 (5)	+ 44,78	+112,7	340. 3 . 2		—	2
	61	+ 45,22 (5)	+ 6,908 (5)*	+ 45,22	+119.2	340. 4 . 2		—	2
h 2037	154	− 57,90 (7)	+12,403 (5)	− 57,89	+214,0	332. 3 . 5		* 372	1
	155	− 57,98 (6)	+12,795 (4)	− 57,98	+220,8	341. 4 . 3		—	2
* a (h 2037)	155	− 54,16 (5)	+15,143 (3)	− 54,15	+261,4	13		* 372	3
h 2043 (* b)	225	− 15,97 (7)	+20,594 (3)	− 15,96	+265,3	442. 4 . 5		* 373	2
h 2043 (* a)	159	− 15,00 (5)	+16,000 (4)*	− 15,00	+275,9	441. 4 . 5		* 373	2
	225	− 15,09 (7)	+21,345 (3)	− 15,08	+274,9	442. 4 . 5		—	2
h 2050	68	+175,93 (3)	− 2,940 (3)	+175,91	− 50,6	421. *4.0	10	* 374	6
	121	+176,03 (4)	− 2,898 (3)	+175,99	− 50,1	421. *4.0	10	—	5
h 2062	265	− 35,14 (7)	−14,967 (3)	− 35,14	−193,0	442. 4 . 5		* 375	2
h 2072	117	−194,24 (5)	− 1,171 (3)	−194,21	− 20,1	040. 5 . 2		* 376	2
	162	−194,52 (6)	− 1,141 (3)*	−194,48	− 19,3	040. 5 . 2		—	2
* B (h 2075)	359	+ 54,43 (10)	+ 5,173 (2)	+ 54,42	+ 66,2	10		* 377	1
	360	+ 54,39 (10)	+ 5,182 (3)	+ 54,38	+ 66,3	10		—	2
h 2075	20	+ 54,80 (7)	+ 1,374 (5)*	+ 54,80	+ 23,8	400. 3 . 0		* 377	23
	28	+ 54,33 (7)	—	+ 54,33	—	000. 1 . 0		—	1
	62	+ 54,37 (6)	+ 1,011 (5)	+ 54,36	+ 17,5	503. 1 . 0		—	4

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 2075	66	+ 54^{s}47 (6)	+ 1^{r}167 (6)*	+ 54^{s}46	+ 20″2	401. *3.2	13	* 377	2^h
	359	+ 54,80(10)	+ 1,248 (2)	+ 54,79	+ 15,6	441. 3.4		—	1
	360	+ 54,67(10)	+ 1,269 (2)	+ 54,66	+ 15,8	441. 3.4		—	2
* A (h 2075)	359	+ 0,93 (5)†	− 6,504 (2)	+ 0,93	− 83,9	11.12		* B (h 2075)	2
	360	+ 0,87 (5)†	− 6,403 (2)	+ 0,87	− 82,6	11.12		—	2
*D (h 2075)	359	− 1,06 (5)†	+ 4,643 (2)	− 1,06	+ 59,9	12		*C (h 2075)	2
	360	− 1,12 (5)†	+ 4,642 (2)	− 1,12	+ 59,8	12		—	2
* C (h 2075)	359	+ 4,37 (5 †	− 4,355 (2)	+ 4,36	− 56,2	11		* B (h 2075)	2
	360	+ 4,25 (5)†	− 4,395 (2)	+ 4,24	− 56,7	11		—	2
Nova VII	113	− 25,92 (5)	− 2,597 (3)*	− 25,92	− 44,8	531. 1.5	12	* 378	1
	179	− 26,12 (6)	− 2,663 (5)	− 26,12	− 45,9	530. *.0	11	—	0
h 2081	26	+ 7,22 (6)	+ 0,787 (4)	+ 7,22	+ 13,6	401. 3.2		* 378	1
	62	+ 7,43 (7)	+ 0,731 (6)	+ 7,43	+ 12,6	431. 1.0	13	—	3
	113	+ 7,53 (8)	+ 0,902 (4)*	+ 7,53	+ 15,5	000. 3.2		—	1
	179	+ 7,42 (6)	+ 0,638 (5)	+ 7,42	+ 11,0	221. 3.1		—	0
h 2086 (* a)	161	− 22,05 (2)	−15,804 (3)*	− 22,05	−272,5	10.11		* 379	1
h 2097	26	− 85,62 (6)	− 7,637 (6)	− 85,61	−131,8	401. 1.0		* 380	2
	60	− 85,18 (5)	− 7,523 (5)	− 85,18	−129,8	431. 1.0	11	—	1
	111	− 85,30 (6)	− 7,443 (5)*	− 85,29	−128,3	030. 3.2		—	1
*a (h 2102)	114	+ 32,43 (6)	−11,225 (5)*	+ 32,43	−193,5	9,5		* 381	2
	361	+ 32,40 (10)	−15,088 (2)	+ 32,38	−194,5	10		—	1
h 2102	361	+ 34,28 (10)	−23,338 (2)	+ 34,26	−300,7	443. *.3	12.13	* 381	1
	114	+ 1,70 (6)†	− 6,156 (4)*	+ 1,70	−106,1	040. 3.4		* a	1
	361	+ 1,66 (5)†	− 8,250 (2)	+ 1,65	−106,3	443. *.3	12.13	—	2
III. 209	226	−127,25 (6)	− 7,633 (3)	−127,23	− 98,1	452. 3.3		* 382	1
	227	−127,46 (7)	− 7,623 (3)	−127,44	− 98,0	452. 3.3		—	1
h 2112	159	−106,13 (6)	−12,540 (4)*	−106,12	−216,1	441. *.0	13	* 383	1
	160	−106,37 (6)	−12,683 (3)*	−106,35	−218,5	540. *.0	12.13	—	1
	257	−106,17 (6)	−16,930 (2)	−106,15	−218,2	442. *3.0		—	1
	258	−106,72 (5)	−16,883 (3)	−106,71	−217,6	542. *3.4	13	—	1
*a (h 2112)	160	− 99,74 (5)	−12,572 (3)*	− 99,73	−216,6	10.11		* 383	2
	257	− 99,80 (6)	−16,781 (2)	− 99,79	−216,3	10		—	2
* 383	257	−110,81 (8)	+ 1,534 (4)	−110,80	+ 20,1	9,5		* 384	1
	258	−110,90 (7)	+ 1,537 (4)	−110,88	+ 20,2			—	1
h 2120	19	− 70,87 (4)	+ 6,558 (3)*	− 70,88	+113,0	410. 1.0		* 386	22

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
h 2120	31	− 70^{s}92 (5)	+ 6^{r}586 (5)	− 70^{s}92	+113″5	210. 1 . 2		* 386	2^h
h 2121	261	− 20,08 (6)	+ 5,142 (3)	− 20,08	+ 66,4	552. 3 . 0		* 387	1
	267	− 19,90 (10)	+ 5,161 (4)	− 19,89	+ 66,6	0		—	0
* a (h 2121)	267	− 18,61 (8)	+ 1,653 (3)	− 18,61	+ 21,4	11.12		* 387	1
* 387	261	+ 56,71 (7)	− 2,482 (3)	+ 56,71	− 32,2	11		* 385	1
	266	+ 56,70 (7)	− 2,354 (3)	+ 56,69	− 30,6	11		—	0
h 2125	21	+ 37,62 (5)	+12,180 (5)*	+ 37,62	+209,9	313. 1 . 2		* 388	22
	31	+ 37,99 (7)	+11,900 (6)	+ 37,99	+205,1	213. 1 . 2		—	1
h 2132	227	−143,61 (7)	−26,570 (3)	−143,59	−342,8	453. 3 . 3		* 389	2
	228	—	−26,628 (3)	—	−343,5	451. 3 . 2		—	1
	229	−143,94 (8)	—	−143,91	—	0		—	1
h 2135	274	− 10,98 (8)	+30,194 (4)	− 10,97	+388,8	451. 3 . 3		* 390	0
	362	− 10,25 (10)	+30,437 (2)	− 10,24	+392,0	453. 3 . 5		—	0
h 2139	29	− 53,80 (7)	+ 2,840 (6)	− 53,80	+ 48,9	401. 1 . 0	12.13	* 391	0
	61	− 53,82 (5)	+ 2,861 (6)	− 53,82	+ 49,3	431. * . 0	11.12	—	2
	109	− 53,78 (6)	+ 2.881 (7)*	− 53,78	+ 49,7	031. * . 0	12.13	—	1
	109	+ 0,98 6)	+ 4,840 (5)*	+ 0,98	+ 83,5	031. * . 0	12.13	* a	2
h 2149	112	− 12,99 (7)	+ 5,371 (6)*	− 12,99	+ 92,6	030. 1 . 2		* a	1
	180	− 12,97 (6)	+ 5,266 (5)	− 12,96	+ 90,8	331. 1 . 2	11	—	22
h 2149	62	− 18,82 (5)	—	− 18,82	—	441. * . 0	13	* 392	4
	180	− 19,37 (7)	−24,596 (4)*	− 19,37	−423,9	331. 1 . 2	11	—	23
	22	−327,43 (4)	− 3,010 (2)*	−327,46	− 52,5	401. * . 0	12	30°4650	22
* c (h 2160)	259	− 4,44 (6)	− 3,194 (2)	− 4,44	− 41,2	12.13		* a	3
h 2160	258	−123,95 (4)	+ 9,165 (2)	−123,94	+118,3	541. 1 . 0		* 393	2
	259	−123,96 (5)	+ 9,331 (3)	−123,95	+120,4	541. 1 . 4	12.13	—	1
	259	− 3,60 (7)	−10,493 (3)	− 3,60	−135,5	541. 1 . 4	12.13	* a	2
	259	− 5,90 (6)	− 5,512 (3)	− 5,90	− 71,1	541. 1 . 4	12.13	* b	2
* 393	259	−118,75 (6)	+28,405 (3)	−118,73	+365,9	9,0		* 394	1
h 2168	21	− 55,18 (6)	+ 5,130 (1)*	− 55,19	+ 88,4	0		* 395	23
	159	− 54,90 (5)	+ 4,842 (4)*	− 54,90	+ 83,5	532. 1 . 5	11.12	—	1
h 2169	225	+ 6,55 (14)	+11,942 (4)	+ 6,55	+154,1	441. * . 0	12	* 397	1
	226	+ 6,51 (9)	+11,773 (3)	+ 6,51	+151,9	441. 2 . 4		—	1
h 2170	197	− 34,91 (8)	−16,788 (3)	− 34,91	−289,3	442. 3 . 3		* 398	1
	261	− 35,19 (8)	−22,375 (4)	− 35,19	−288,9	453. 3 . 4		—	1

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
Nova VIII	116	− 15^{s}53 (7)	−10^{r}014 (5)	− 15^{s}53	−172″6	542. ∗ . 0	13	h2172	1^h
h2171	274	− 52,70 (7)	−23.383 (4)	− 52,70	−301,9	553. 1 . 0	13	∗ 399	1
	362	− 52,38(10)	−23,415 (2)	− 52,38	−302,1	453. 3 . 5		—	1
h2172	20	+ 97,45 (4)	− 4,575 (4)*	+ 97,46	− 78,7	0		∗ 396	23
	162	+ 97,52 (5)	− 4,772 (4.*	+ 97,50	− 82,2	020. 1 . 0	10	—	1
	327	+ 97,18 (9)	− 6,445 (3)	+ 97,13	− 83,4	0		—	3
h2173	23	−151,91 (3)	+ 0,802 (3)*	−151,92	+ 13,6	304. 1 . 0	10.11	∗ 400	22
	66	−151.98 (4)	+ 0,862 (4)*	−151,97	+ 14,7	234. 1 . 0	11.12	—	3
	67	−151,93 (2)	+ 0,869 (4)*	−151,92	+ 14,9	234. 1 . 0	11.12	—	1
Nova IX	115	− 7,95 (4)	− 2,094 (2)	− 7,95	− 36,1	552. ∗ . 0		h2174	2
h2174	114	—	+ 6,242 (4)*	—	+107,6	050. 0 . 0		h2172	1
	115	+ 15,14 (5)	+ 6,592 (2)	+ 15,14	+113,6	452. 0 . 0		—	1
Rosse C	118	− 15,76 (5)	+ 2,868 (3)	− 15,76	+ 49,4	553. 4 . 5		∗ a	1
Rosse E	115	+ 23,08 (4)	− 8,995 (2)	+ 23,07	−155,0	542. ∗ . 0		h2172	3
	116	—	− 9,104 (3)	—	−156,9	0		—	1
h2175	23	+ 22,22 (6)	− 2,405 (4)*	+ 22,22	− 41,4	0		h2173	23
	67	+ 21,99 (8)	− 2,622 (2)*	+ 21,99	− 45,2	453. 4 . 2		—	1
	66	−130,34 (7)	− 1,682 (2)*	−130,34	− 29,1	453. 3 . 3		∗ 400	2
∗ a (h2172)	118	+ 34,16 (5)	+10,353 (4)	+ 34,16	+178,5	10		h2172	2
Rosse D	115	+ 40,00 (4)	− 1,355 (2)	+ 40,00	− 23,3	542. ∗ . 0	12.13	h2172	3
∗ a (h2183)	265	− 42,06 (5)	+ 5,044 (2)	− 42,06	+ 65,1	10.11		∗ 403	3
h2183	196	− 38.97 (6)	+ 1,816 (4)*	− 38,96	+ 31,4	442. ∗ . 5	12.13	∗ 403	1
	265	− 39,07 (7)	+ 2,085 (3)	− 39,07	+ 26,9	542. ∗3.4	12.13	—	0
h2184	196	+ 7,92 (9)	+18,638 (4)*	+ 7,92	+321,3	343. 2 . 5		h2183	1
	265	+ 8,18 (9)	+24,998 (3)	+ 8,18	+322,9	541. ∗3.5	12.13	—	1
∗ 403	265	+194,05 (6)	+ 5,465 (3)	+194,03	+ 70,1	9,5		∗ 401	2
	265	+198,82 (6)	−29,750 (3)	+198,80	−384,1			∗ 402	3
h2189	272	−156,37 (6)	+ 8,208 (3)	−156,35	+106,1	444. 1 . 3		∗ 404	0
	273	−156,08 (6)	+ 8,383 (3)	−156,07	+108,3	443. 2 . 3		—	0
h2199	23	− 99,93 (4)	+ 5,755 (3)*	− 99,93	+ 99.1	440. 0 . 0		∗ 407	0
	69	− 99,34 (5)	+ 5,725 (5)	− 99,33	+ 98,6	443. 1 . 3	12.13	—	1
	70	− 99,11 (8)	+ 5,586 (3)	− 99,11	+ 96,3	040. 0 . 0		—	1
	113	− 10,28 (6)	+ 1,587 (4)*	− 10,28	+ 27,4	040. 4 . 3		∗ a	0

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
II. 249	29	− 18^{s}59 (7)	− 2^{r}023 (5)	− 18^{s}59	− 34″9	400. 0 . 0		* 406	1^h
	70	− 18,34 (7)	− 1,787 (5)	− 18,34	− 30,8	442. * . 0	13	—	3
h 2201	159	− 43,18 (5)	− 5,251 (4)*	− 43,17	− 90,5	430. *2.0	11.12	* 408	2
	160	—	− 5,305 (2)*	—	− 91,4	—		—	1
	178	− 43,57 (7)	− 5,397 (4)	− 43,57	− 93,0	330. * . 5	11.12	—	22
h 2200	271	− 8,47 (9)	−28,842 (3)	− 8,47	−372,7	442. * . 0	12	* 405	0
	275	− 7,89 (7)	−29,001 (3)	− 7,89	−374,8	551. *3.0	13	—	0
h 2202	67	− 9,10 (8)	+ 3,446 (3)*	− 9,10	+ 59,4	453. 1 . 0		h 2203	3
	69	− 9,03 (6)	+ 3,445 (5)	− 9,03	+ 59,4	453. 1 . 0	13.14	—	2
	113	− 9,02 (6)	+ 3,723 (4)*	− 9,02	+ 64,2	003. 1 . 0	13	—	1
D'A Nova	113	− 7,60 (7)	+ 1,959 (4)*	− 7,60	+ 33,8	553. 1 . 0	13.14	h 2203	1
h 2203	27	+ 17,47 (7)	+ 2,100 (4)*	+ 17,47	+ 36,2	501. 1 . 0	12.13	* 407	1
	67	+ 17,49 (7)	+ 1,958 (5)*	+ 17,49	+ 33,8	441. 1 . 0	12.13	—	2
	69	+ 17,47 (6)	+ 2,096 (3)	+ 17,47	+ 36,1	441. 1 . 0	12.13	—	2
	113	+ 17,48 (5)	+ 1,917 (3)*	+ 17,48	+ 33,1	031. 1 . 0	12	—	2
h 2204	275	− 13,43 (7)	−24,483 (4)	− 13,43	−316,2	542. * . 0	12	* 409	0
	361	− 13,30 (10)	−24,482 (2)	− 13,30	−316,2	541. 1 . 5	12	—	0
	361	− 14,62 (10)	+18,630 (2)	− 14,61	+240,0	541. 1 . 5	12	* a (9)	1
h 2205 (* c)	119	− 3,04 (4)†	− 5,671 (3)	− 3,04	− 97,8	334. 1 . 0	13	* a	2
h 2205 (* b)	30	− 97,62 (6)	+ 2,548 (5)*	− 97,62	+ 43,8	334. 1 . 0	12.13	* 410	23
	119	− 0,24 (5)†	− 4,396 (3)	− 0,24	− 75,8	334. 1 . 0		* a	2
h 2205 (* a)	119	− 97,15 (4)	+ 7,029 (3)	− 97,15	+121,1	334. 1 . 0	11.12	* 410	2
h 2214	164	− 31,32 (6)	−12,040 (4)*	− 31,31	−207,6	441. 1 . 4	12.13	* 411	0
	272	− 31,94 (7)	−15,883 (3)	− 31,94	−205,0	441. 1 . 3	12	—	2
	272	−107,50 (7)	−21,113 (3)	−107,50	−272,6	441. 1 . 3	12	* 412	2
	272	+ 6,27 (7)	+ 6,091 (3)	+ 6,27	+ 78,6	441. 1 . 3	12	* a	2
* 411	272	− 75,84 (7)	− 5,060 (3)	− 75,84	− 65,2	9,4		* 412	2
* b (h 2216)	73	− 38,05 (3)	− 1,717 (1)	− 38,05	− 29,6	0		* a	3
h 2215	65	−105,70 (5)	+ 2,663 (3)*	−105,70	+ 45,9	552. * . 0	13.14	* 416	1
	269	−105,89 (7)	+ 3,212 (3)	−105,88	+ 41,5	453. * . 5	13	—	0
	111	− 17,23 (6)	− 6,696 (3)*	− 17,23	−115,5	552. 1 . 0	13.14	* a (h 2216)	2
h 2217	184	− 45,34 (7)	+ 0,491 (4)	− 45,34	+ 8,5	541. * . 0	12	* 413	22
h 2216	65	− 96,52 (6)	—	− 96,51	—	244. 4 . 0		* 416	2
	269	− 96,83 (7)	+13,004 (3)	− 96,82	+167,6	344. 3 . 0		—	0

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
	65	—	+ 9.048 (3)*	—	+155.9	12		* 416	2h
* a (h 2216)	73	− 88.16 (4)	+ 9,193 (2)	− 88.15	+158,3	12		—	2
	269	− 88,30 (6)	+12,164 (2)	− 88,29	+156,8	11.12		—	0
h 2218	186	− 12,78 (8)	+ 2,452 (4)	− 12,77	+ 42,3	450. * . 5	13	h 2219	23
	270	− 12,84 (8)	+ 3,352 (4)	− 12,84	+ 43,2	452. 3 . 0		—	0
h 2219	64	−158,72 (6)	−10,562 (2)*	−158,71	−182,1	441. * . 0	12.13	* 418	2
	186	−158,67 (6)	−10,620 (4)	−158,65	−183,1	441. * . 5	12	—	23
	270	−158,78 (6)	−14,444 (3)	−158,76	−186,4	443. 1 . 3	11.12	—	1
D'A Nova	186	−157,74 (5)	+ 6,100 (2)	−157,72	+105,1	450. 0 . 0		* 418	0
	270	−157,80 (6)	+ 8,224 (3)	−157,77	+106,1	452. 3 . 4		—	1
* 413	184	−321,28 (5)	−11,404 (2)*	−321,25	−196,7	10		* 423	23
GC. 4917	178	− 17,95 (4)	+ 4,160 (3)	− 17,95	+ 71,7	450. 0 . 0		h 2224	0
h 2222	180	− 17,98 (6)	+21,656 (4)	− 17,98	+373,4	441. * . 5	12.13	* 414	23
	181	− 18,09 (8)	+21,715 (4)	− 18,09	+374,4	451. * . 5	12.13	—	23
Nova X	73	− 26,43 (3)	+ 1,733 (2)	− 26,43	+ 29,9	0		* 416	3
	111	− 26,29 (7)	+ 1,800 (4)*	− 26,28	+ 31,0	543. * . 0	13	—	1
	269	− 26,57 (7)	+ 2,084 (2)	− 26,57	+ 26,9	542. * . 5	12	—	1
Nova XI	111	+ 3,80 (8)	+ 5,201 (4)*	+ 3,80	+ 89,7	542. * . 0	13	Nova X	1
	269	+ 4,13 (9)	+ 7,225 (3)	+ 4,13	+ 93,3	542. * . 0	12.13	—	1
h 2224	178	− 21,71 (7)	+20,628 (5)	− 21,71	+355,7	431. 1 . 5	11	* 415	23
	179	− 21,73 (7)	—	− 21,73	—	0		—	23
	274	− 21,78 (6)	+27,512 (4)	− 21,78	+354,2	531. 1 . 0	11.12	—	1
h 2223	180	− 8,74 (7)	+ 2,095 (5)	− 8,74	+ 36,1	441. * . 5	12.13	* 414	23
	181	− 9 00 (7)	+ 2,116 (4)	− 9,00	+ 36,5	441. * . 0	12	—	23
* 414	181	−333,14 (5)	+ 0,392 (3)	−333,13	+ 6,7	10		* 425	1
	182	−333,08 (5)	+ 0,416 (3)	−333,07	+ 7,2	10		—	22
* 415	274	− 74,70 (6)	+16,402 (3)	− 74,69	+211,4	9,0		* 417	2
III. 238	181	− 89,14 (5)	+18,320 (3)	− 89,14	+315,9	452. 2 . 5		* 420	0
* 419	382	−497,69 (7)	−27,465 (2)	−497,59	−355,7	9,1		* 428	2
	383	−497,80 (8)	−27,563 (2)	−497,70	−357,0			—	2
GC. 4934	123	− 11,37 (6)	+16,743 (4)	− 11,37	+288,7	043. 1 . 0	12.13	* 422	0
D'A Nova	125	+ 95,75 (4)	+ 5,640 (2)	+ 95,74	+ 97,3	041. 1 . 0	12	* 419	0
	365	+ 95,69 (8)	+ 7,481 (2)	+ 95,67	+ 96,3	542. 1 . 4	11.12	—	23

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
D'A Nova	124	-5^s67 (5)†	$-8''210$ (3)	-5^s67	$-141''5$	551. 1 . 0	13	h 2230	1^h
h 2230	34	+ 68,30 (5)	− 3,168 (5)*	+ 68,30	− 54,6	401. * . 0	11.12	* 421	1
	71	+ 68,42 (5)	− 3,213 (4)	+ 68,41	− 55,4	030. * . 0	11.12	—	2
	72	+ 68,53 (6)	− 3,193 (5)	+ 68,53	− 55,0	0		—	1
	267	+ 68,57 (6)	− 4,338 (3)	+ 68,56	− 56,0	431. *2.4	12	—	0
h 2231	125	− 94,98 (4)	−11,568 (3)	− 94,97	−199,5	541. 1 . 0	12.13	* 426	1
	364	− 95,22 (7)	—	− 95,21	—	542. 1 . 0	11.12	—	23
	365	− 95,03 (8)	−16,277 (2)	− 95,01	−210,0	542. * . 5	12	—	22
	125	+ 3,41 (5)†	− 1,406 (2)	+ 3,41	− 24,3	541. 1 . 0	12.13	* a	1
h 2232	181	− 26,74 (7)	− 5,201 (5)*	− 26,74	− 89,7	442. 3 . 3		* 424	21
	364	− 26,29 (10)	− 7,046 (2)	− 26,29	− 90,9	442. * . 4	12	—	22
	365	− 26,46 (10)	− 7,204 (2)	− 26,45	− 92,9	442. 3 . 3		—	21
h 2233	34	+ 96,77 (4)	− 0,961 (4)*	+ 96,77	− 16,5	443. * . 0	12.13	* 421	2
	71	+ 96,26 (5)	− 1,085 (4)	+ 96,25	− 18,7	430. 1 . 0	12.13	—	3
	72	+ 96,40 (5)	− 1,086 (3)	+ 96,39	− 18,7	0		—	2
	267	+ 96,33 (6)	− 1,466 (2)	+ 96,33	− 19,0	431. *2.4	12	—	0
*a (GC 4943)	124	+ 34,90 (4)	− 2,268 (3)	+ 34,90	− 39,1	12		h 2233	2
GC. 4943	364	− 63,65 (10)	− 2,019 (2)	− 63,65	− 25,9	454. 3 . 0		* 427	23
	365	− 63,90 (10)	− 2,260 (2)	− 63,89	− 29,0	453. 3 . 0		—	0
	124	+ 9,33 (5)†	+ 2,476 (3)	+ 9,33	+ 42,7	453. 1 . 0	13	* a	2
* 426	365	+236,43 (7)	−26,689 (2)	+236,39	− 344,2	9,5		* 419	23
h 2241 (Geometrical Centrum)	105	− 44,01 (7)	− 4,713 (3)**	− 44,02	− 81,3	411. 0 . 4		* 429	19
	173	− 44,14 (7)	− 4,728 (6)*	− 44,14	− 81,5	0		—	21
	174	− 43,91 (5)	− 4,694 (6)*	− 43,90	− 80,9	0		—	20
	178	− 44,02 (6)	− 4,683 (3)	− 44,01	− 80,8	0		—	21
	238	− 43,91 (7)	− 6,282 (4)	− 43,90	− 80,9	421. 5 . 3		—	7
	256	− 43,99 (7)	− 6,284 (6)	− 43,97	− 81,1	0		—	21
h 2241 (Margo preced.)	256	− 44,96 (7)	—	− 44,94	—	0		* 429	21
h 2241 (Margo follow.)	256	− 43,01 (7)	—	− 43,00	—	0		* 429	21
*b (h 2241)	173	− 39,85 (8)	− 3,346 (4)*	− 39,84	− 57,7	12.13		* 429	21
	178	− 40,05 (6)	− 3,345 (4)	− 40,04	− 57,7	0		—	21
	238	− 39,90 (7)	− 4,442 (5)	− 39,89	− 57,2	12.13		—	7
*c (h 2241)	238	− 35,96 (7)	−16,771 (5)	− 35,95	−216,0	12.13		* 429	7
h 2242	163	+ 9,41 (6)†	+ 0,812 (3)*	+ 9,41	+ 14,0	541. * . 5	12.13	* 430	0
	183	+ 8,78 (11)	+ 0,898 (4)	+ 8,78	+ 15,5	540. * . 3	12	—	22

Nebula	Night	Observed $\Delta\alpha^s$	Observed $\Delta\delta$ in Scale-Divisions	Mean-Coordinates 1865,0 $\Delta\alpha^s$	Mean-Coordinates 1865,0 $\Delta\delta''$	Description		Star of Comparison	Hour-Angle
GC. 4967	183	− 13^{s}61 (9)	− 1$'$761 (4)	− 13^{s}61	− 30″4	542. ∗. 5	13	∗ 431	23^h
∗ 431	183	+ 34,08 (6)	−14,973 (3)	+ 34,08	−258,2	11		∗ 430	0
h 2257	230	− 75,05 (6)	+ 4,332 (3)	− 75,04	+ 55,9	541. ∗. 5	11.12	∗ 433	0
	230	+ 2,81 (7)	+ 2,798 (4)	+ 2,81	+ 36,1	541. ∗. 5	11.12	∗ a	0
	230	− 10,03 (7)	+15,298 (4)	− 10,03	+197,1	541. ∗. 5	11.12	∗ 432	1
	263	− 10,20 (9)	+15,438 (3)	− 10,20	+198,9	541. 1. 0	12.11	—	0
h 2264	182	—	−17,587 (3)	—	−303,2	0		∗ 434	23
	187	+234,72 (5)	−17,635 (3)	+234,71	−304,0	431. ∗. 0	11	—	22
	264	+235,27 (6)	−23,733 (3)	+235,72	−305,6	431. 1. 3	11	—	1
	264	− 4,97 (9)	+ 1,313 (3)	− 4,97	+ 16,9	431. 1. 3	11	∗ a	2
h 2265	261	+ 92,32 (6)	+13,195 (3)	+ 92,32	+170,4	431. 1. 4	11.12	∗ 435	1
	263	+ 92,13 (6)	+13,239 (3)	+ 92,14	+170,9	431. 1. 3	11.12	—	1
	261	+ 5,59 (7)	+11,018 (2)	+ 5,59	+142,3	431. 1. 4	11.12	∗ a	1
∗ b (h 2265)	261	+ 7,43 (6)	+ 6,793 (2)	+ 7,43	+ 88,0	11.12		∗ a	1
h 2274	36	+ 56,26 (5)	+ 3,861 (5)	+ 56,26	+ 66,6	401. ∗. 0	12.13	∗ 437	2
	155	+ 56,91 (7)	+ 4,108 (5)	+ 56,91	+ 70,9	431. ∗. 5	12	—	0
GC. 5021	155	− 2,32 (5)†	− 3,074 (3)	− 2,32	− 53,0	552. ∗. 5	13	h 2275	2
h 2275	36	+111,28 (6)	+ 0,977 (5)	+111,28	+ 16,9	0		∗ 436	3
	155	+ 21,73 (8)	− 8,090 (5)	+ 21,73	−139,4	444. 1. 5	12.13	h 2274	1
h 2290	157	−238,96 (5)	—	−238,94	—	541. ∗. 5	13	∗ 438	23
	158	−238,40 (4)	+16,576 (2)	−238,38	+285,6	0		—	23
	268	−238,92 (6)	+22,030 (3)	−238,88	+283,4	441. ∗. 3	12	—	1
	157	+ 2,16 (5)†	+ 3,868 (4)*	+ 2,16	+ 66,7	541. ∗. 5	13	∗ a	22
	268	+ 2,72 (9)	+ 5,332 (4)	+ 2,72	+ 68,8	441. ∗. 3	12	—	2
h 2297	153	− 52,78 (8)	+ 4,193 (5)	− 52,77	+ 72,3	332. 2. 3		∗ 439	22
	157	− 52,94 (5)	+ 4,397 (4)*	− 52,94	+ 75,8	442. 3. 2		—	22
Nova XII	157	—	+ 6,585 (2)*	—	+113,5	0		∗ 438	0
	158	− 0,46 (5)†	+ 6,812 (2)	− 0,46	+117,4	443. ∗. 3	13	—	0
h 2300	157	+ 33,47 (6)	+16,086 (2)*	+ 33,46	+277,3	0		∗ 438	0
	158	− 59,86 (5)	+13,247 (4)	− 59,85	+228,3	444. 3. 3		∗ 440	0
	268	− 59,42 (5)	+17,047 (3)	− 59,41	+219,5	344. ∗. 5		—	2
∗ 440	268	+ 93,16 (7)	+ 4,643 (4)	+ 93,14	+ 60,0	9,3		∗ 438	0

NOTES TO THE PRECEDING TABLES.

NOTE I.

Approximate mean positions of the Stars of Comparison for the year 1865,0.

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
1	32.1	8,5	0 0 57,7	+32 39 56	h 2
2	3.8	9,0	2 34,6	+ 3 52 31	h 1
3	26.4	9,3	3 12,8	+27 0 50	h 4, etc.
4	30.10	8,9	3 37,5	+30 16 17	* 5
5	30.20	9,5	6 39,4	+30 14 25	h 7, etc.
6	32.20	8,8	7 0,8	+32 35 6	h 6
7	21.24	8,5	12 3,6	+21 37 9	* 9
8	16.22	9,2	14 6,7	+16 41 58	h 13
9	21.33	7,5	15 57,3	+21 38 6	h 16, etc.
10	28.51	9,0	16 3,2	+28 27 12	h 21
11	21.35	8,5	16 40,1	+21 32 38	* 9
12	2.54	7,7	20 24,7	+ 2 4 1	h 25
13	1.76	8,8	24 33,3	+ 1 23 27	h 26
14	23.82	7,0	29 3,6	+23 16 53	h 32
15	23.84	6,0	30 1,2	+23 16 21	GC. 82
16	1.102	9,5	30 32,9	+ 1 15 29	h 33
17	1.104	9,4	30 42,9	+ 1 21 7	* 16
18	1.108	7,5	31 4,5	+ 2 1 15	* 22
19	0.96	8,2	31 53,3	+ 0 12 23	* 24
20	2.84	7,5	32 9,7	+ 2 22 48	h 40
21	2.85	9,0	32 31,7	+ 2 34 23	h 37, etc.
22	1.114	9,5	32 34,1	+ 1 58 18	II. 857
23	40.142	8,8	33 37,5	+40 59 23	h 44
24	0.109	9,0	35 46,5	+ 0 9 11	h 39, etc.
25	40.150	9,5	36 15,8	+40 9 50	h 51
26	40.154	9,0	37 24,8	+40 34 26	h 50
27	31.122	8,2	42 34,1	+31 28 55	h 65
28	26.131	5,3	42 38,7	+26 58 27	h 59
29	29.147	7,5	45 50,4	+29 36 55	* 31
30	29.155	8,8	49 8,7	+29 32 2	* 31
31	29.158	8,3	50 40,2	+29 35 15	h 78, etc.
32	31.180	7,2	58 46,2	+31 27 31	* 33
33	* a (GC. 207)	10.11	59 43,5	+31 32 34	GC. 207
34	β Androm.	2.3	1 2 10,9	+34 54 15	h 89
35	38°202	8,9	2 14,4	+38 55 56	h 88

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
36	31.192	8,2	1 3 38,6	+31 47 54	h 85
37	31.194	9,2	4 19,7	+32 1 24	GC. 217
38	38.210	9,1	4 36,1	+38 50 54	* 35
39	31.197	7,3	5 24,9	+31 21 29	h 90
40	32.204	7,8	5 34,0	+32 26 25	II. 220
41	32.223	6,5	8 47,1	+32 24 10	GC. 250
42	32.230	7,8	11 21,5	+32 48 17	* 44
43	2.190	8,0	12 25,1	+ 2 34 48	I. 108, etc.
44	32.240	9,3	14 33,4	+32 48 13	h 106, etc.
45	4.230	8,0	15 23,3	+ 4 32 43	h 103
46	32.245	7,7	15 38,0	+32 35 2	h 104, etc.
47	8.218	7,9	16 2,1	+ 8 28 56	GC. 274
48	8.223	9,2	16 38,1	+ 8 24 29	GC. 293
49	8.228	9,1	18 21,2	+ 9 0 22	GC. 308
50	8.234	9,0	19 23,2	+ 8 49 39	h 117
51	33.232	7,8	19 24,0	+33 56 16	h 118, etc.
52	15.233	8,7	26 46,4	+15 4 36	h 142
53	29.265	9,2	27 40,9	+29 58 45	h 133, etc.
54	12.215	8,8	35 14,8	+13 3 12	II. 253
55	28.292	7,8	37 29,6	+28 3 2	h 147
56	27.284	9,0	40 22,4	+27 14 29	h 149
57	26.301	8,2	42 1,6	+27 1 55	h 152
58	26.303	8,0	42 22,8	+26 47 34	h 150
59	21.250	8,3	45 40,6	+21 19 10	II. 229
60	32.354	8,2	50 4,1	+32 30 55	h 175, etc.
61	18.252	8,8	50 8,4	+18 19 53	h 181
62	32.355	9,5	50 17,5	+23 25 57	h 169
63	30.317	8,5	52 43,7	+30 41 10	h 182
64	38.417	9,1	2 0 47,7	+38 36 55	h 197
65	10.293	8,8	1 5,9	+10 21 57	h 193
66	38.423	8,8	2 35,9	+38 3 51	h 194
67	32.405	8,0	6 42,2	+32 41 1	* 70
68	13.364	8,2	9 57.0	+13 50 13	h 208, etc.
69	13.366	8,0	10 43,8	+13 50 30	h 210
70	32.419	8,9	11 37,0	+32 37 9	h 217
71	36.507	8,9	23 20,8	+36 33 46	h 226
72	32.468	7,8	27 9,7	+32 16 15	h 231, etc.
73	38.536	8,8	32 31,5	+38 29 41	h 242
74	31.472	7,5	35 53,9	+31 47 48	h 257, etc.
75	24.419	8,8	52 17,9	+24 42 21	II. 619
76	34.883	6,8	4 20 21,2	+34 57 46	h 315
77	0.763	9,1	21 13,6	+ 0 34 59	h 318
78	0.772	9,3	23 20,8	+ 0 23 34	* 80
79	0.773	9,0	23 36,8	+ 0 14 26	* 80
80	0.778	9,5	24 29,5	+ 0 20 3	h 316, etc.
81	34.1087	8,4	5 25 14,3	+34 7 9	h 355
82	31.1032	9,4	29 27,9	+31 56 8	h 359
83	9.914	8,8	32 9,3	+ 9 1 13	h 365

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
84	9.926	9,1	5 34 8,2	+ 9 5 33	h 365
85	— 0.1059	8,1	35 38,4	− 0 5 4	* 86
86	— 0.1077	9,3	39 10,6	− 0 0 30	h 368
87	10.1171	8,2	6 25 21,7	+10 15 41	h 393
88	8.1430	8,9	31 56,4	+ 8 45 26	h 399
89	33.1404	9,2	37 23,1	+33 45 23	* 92
90	61.910	8,5	38 10,5	+61 4 56	Dunér Nova
91	61.912	8.5	38 48,1	+61 1 16	Dunér Nova
92	* a (h 406)	11,5	38 57,5	+33 42 39	h 406. etc.
93	19° 1629	8,0	7 2 1,2	+19 3 27	h 434
94	29.1535	7,6	19 42,0	+29 41 41	h 444
95	21.1610	8,0	21 11,7	+21 12 45	h 450
96	34 1621	9,5	21 13,7	+34 2 4	h 449
97	34.1623	9,1	21 56,1	+33 59 15	* 96
98	21.1616	9,1	22 31,1	+21 8 30	h 450
99	0.1975	8,9	24 19,4	+ 0 2 5	GC. 1537, etc.
100	35.1658	9,0	28 12,4	+35 32 57	h 456
101	39.1979	7,3	28 39,3	+39 10 35	h 457
102	32.1616	9,4	36 17,0	+32 0 18	II. 616
103	31.1676	7,0	42 25,9	+31 57 8	* 102
104	31.1677	8,1	42 31,8	+31 57 20	* 102
105	16.1614	9,0	54 19,5	+16 2 17	h 481
106	23.1877	8,5	55 56,0	+23 46 55	h 482
107	9.1864	9,0	58 2,2	+ 9 47 36	h 483
108	23.1905	8,3	8 3 28,9	+23 52 48	* 106, etc.
109	34.1774	8,9	4 38,2	+34 23 44	h 489
110	21.1798	9,1	7 36,5	+21 48 21	h 494
111	58.1112	6,0	9 30,6	+58 9 39	h 495
112	23.1929	9,5	9 43,8	+23 48 37	h 497
113	21.1822	9,1	13 20,2	+21 34 14	h 500, etc.
114	22.1931	8,9	17 42,6	+22 55 12	II. 259
115	22.1950	8,9	24 37,2	+22 51 56	* 116
116	22.1955	9,0	26 25,2	+22 58 1	h 507
117	19.2097	8,2	38 4,1	+19 32 18	h 526
118	51.1458	8,9	44 48,1	+51 45 11	h 530
119	33.1782	9,3	48 5,2	+33 55 50	h 532
120	51.1465	8,5	49 47,0	+51 41 38	* 118
121	51 1466	9,0	50 26,2	+51 41 28	* 118
122	61.1101	7,7	56 23,7	+61 3 49	h 550
123	21.1978	9,0	9 0 54,6	+21 55 39	h 557
124	7.2082	9,5	3 57,6	+ 7 34 43	h 564
125	19.2174	9,1	4 47,9	+19 13 34	h 565
126	35.1960	7,8	5 3,8	+35 39 33	h 566
127	20.2293	8,5	8 24,7	+20 37 58	* 128
128	20.2299	9,5	9 21,7	+20 40 30	h 577, etc.
129	51.1499	8,3	13 7,3	+51 35 10	h 584
130	35.1989	7,7	15 34,3	+35 7 33	h 593.
131	63.842	9,0	17 38,8	+63 1 8	h 596

N:o	N:o in the Bonn-Zones.	Magn.	Right Ascension	Declination	Nebula observed with the Star
			h m s	° ′ ″	
132	* a(h597)	10	9 18 51,8	+11 59 31	h597, etc.
133	12°2039	8,7	20 23,3	+12 0 12	h597, etc.
134	22.2098	8,9	22 41,2	+22 10 18	I.56
135	10.2011	7,7	23 46,1	+10 44 47	* 136
136	10.2019	9,4	27 11,0	+10 44 54	h608, etc.
137	32.1915	9,3	33 22,1	+32 31 42	h622, etc.
138	Sf.50	9,2	35 42,2	− 3 5 29	h630
139	22°2131	9,0	41 13,8	+22 40 13	h634, etc.
140	34.2038	7,8	41 58,9	+34 13 45	h645
141	69.540	9,4	42 1,4	+69 40 35	h649
142	70.587	9,0	44 3	+70 21,2	IV.79
143	29.1963	9,0	45 50,0	+29 49 49	h650
144	29.1964	8,5	46 13,0	+29 50 57	h650
145	4.2269	7,5	49 48,0	+ 4 53 2	h656
146	69.550	7,9	50 36,1	+69 21 48	h658
147	11.2133	9,0	51 16,0	+11 3 59	h659
148	69.552	7,8	52 1,0	+69 26 2	h658
149	25.2199	8,5	57 47,2	+25 22 32	h663
150	Sf.56	7,9	10 0 40,7	− 7 4 5	h668
151	23°2190	8,3	5 34,5	+23 32 2	* 153
152	4.2297[a]	9,4	6 12,5	+ 4 3 23	h684
153	23.2196	8,9	6 40,2	+23 26 59	h682
154	4.2302	9,5	7 42,5	+ 4 8 1	h685
155	4.2306	8,5	9 12,0	+ 4 13 14	h685
156	22.2197	8,0	9 32,0	+22 35 9	h692, etc.
157	Sf.61	9,6	10 31,3	+22 22 4	GC.2054
158	22°2200	7,4	10 31,3	+22 38 33	Sf.61
159	γ_1 Leonis	2	12 31,6	+20 31 24	II.29
160	23°2227	8,7	19 45,4	+23 27 18	h713
161	29.2048	9,1	20 4,2	+29 6 37	h711
162	30.2024	8,8	22 10,3	+30 11,8	h714
163	29.2058	9,0	23 19,0	+29 19 58	* 165
164	30.2028	9,0	23 30,0	+30 8 17	h714
165	29.2059[a]	9,5	24 23,2	+29 13 47	h721
166	38.2160	9,0	28 56,4	+38 2 47	h724
167	22.2255	8,6	32 57,6	+22 31 20	h728
168	14.2284	8,5	34 46,9	+14 27 32	h737
169	12.2248	9,5	36 24,9	+12 24 59	h743
170	25.2290	8,8	37 16,1	+25 40 21	h739, etc.
171	14.2293	9,3	38 25,9	+14 29 32	h748
172	13.2303	9,3	39 24,1	+13 18 37	h757, etc.
173	14.2298	8,5	40 15,6	+14 5 23	h774
174	12.2256	9,6	40 23,8	+12 32 50	h749
175	28.1931	6,3	42 28,3	+28 41 9	h773
176	28.1933	8,9	43 0,5	+28 50 19	h775
177	33.2047	7,5	44 33,3	+33 20 42	II.493
178	33.2049	7,5	45 10,0	+33 42 24	h765, etc.
179	4.2390	8,8	45 17,1	+ 4 28 57	h784

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
180	14.2310	9,3	10 45 33,7	+14 39 25	h754
181	14.2326	9,5	52 37,8	+14 37 37	h806
182	18.1961	8,3	54 48,0	+28 35 56	h810
183	29.2116	7,0	55 50,1	+29 39 23	h805, etc.
184	0.2730	8,2	57 6,5	+ 0 41 46	h818
185	28.1966	9,0	58 17,0	+28 45 2	h815
186	18.2465	9,3	11 7 18,0	+18 48 14	h843, etc.
187	13.2379	6,7	8 54,6	+13 34 58	h840
188	55.1453	8,7	9 25,6	+55 50 47	h838
189	59.1367	7,8	11 15,2	+59 37 26	h847
190	18.2473	9,1	12 18,4	+18 50 45	h844
191	13.2389	9,0	13 4,4	+13 45 37	h854, etc.
192	3.2488	8,3	13 51,5	+ 3 36 6	h861
193	14.2379	9,1	14 12,4	+14 26 7	h859
194	19.2444	9,4	15 37,9	+19 5 12	h856
195	67.696	8,8	15 57,4	+67 23 32	h890
196	3.2493	9,3	16 16,7	+ 3 54 6	h864
197	39.2441	8,9	16 18,0	+39 26 5	h881
198	38.2236	8,4	16 50,4	+38 27 40	h871
199	38.2241	8,6	19 22,6	+38 28 5	* 198
200	17.2359	9,5	19 37,2	+17 41 58	h891, etc.
201	26.2213	8,5	19 45,4	+26 19 19	h897
202	17.2360	9,4	20 40,9	+17 46 11	h893
203	18.2499	9,3	22 8,5	+17 58 54	h894
204	57.1325	8,6	22 47,6	+57 39 11	h892
205	17.2363	8,5	22 55,7	+17 43 48	h898
206	10.2294	9,0	23 33,6	+10 8 49	h902
207	37.2205	8,2	33 6,2	+37 13 28	h945
208	18.2517	8,7	33 46,9	+18 27 36	h933, etc.
209	25.2403	8,4	34 10.9	+25 33 15	h938. etc.
210	25.2404	9,5	34 45,1	+25 39 11	h946
211	12.2367	8,9	35 4,3	+12 12 34	h943
212	27.2055	8,7	39 55,9	+27 46 21	h988
213	37.2219	9,0	43 57,2	+37 38 34	h1005
214	56.1556	9,1	50 32,1	+56 16 57	h1031
215	55.1502	9,3	52 28,6	+55 53 0	h1017
216	51.1718	9,0	52 51,9	+51 48 59	h1047
217	62.1209	9,5	53 11,9	+62 43 5	h1050
218	20.2671	9,0	55 31,9	+20 52 30	h1049
219	56.1560	9,4	56 50,2	+56 16 18	* 214
220	51.1725	8,2	58 7,1	+51 2 6	I.224
221	67.732	9,1	58 47,9	+67 53 40	h1087
222	62.1216	9,4	59 5,8	+62 55 37	h1054
223	43.2187	6,8	12 0 29,3	+43 50 56	h1088, etc.
224	20.2689	8,9	3 24,3	+20 53 58	h1115
225	65.871	9,0	3 27,7	+65 55 35	Hind Nova
226	51.1733	7,8	4 3,8	+51 18 0	I.206
227	37.2255	8,2	5 43,3	+37 0 31	* 234

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
228	19.2541	9,0	12 5 53,6	+19 17 11	h 1106
229	11.2439	8,0	6 19,4	+11 36 10	h 1125
230	67.735	8,3	6 32,1	+67 49 28	* 221
231	34.2301	7,8	8 10,3	+34 0 24	h 1140
232	13.2511	9,5	8 20,5	+13 53 40	h 1148
233	8.2582	9,2	8 57,1	+ 8 11 49	h 1153
234	37.2262	9,5	9 7,5	+37 1 48	h 1146
235	15.2436	5.1	9 8,9	+15 39 4	h 1132
236	30.2252	9,5	10 14,9	+30 22 12	h 1168, etc.
237	7.2530	9,2	10 41,7	+ 7 8 5	h 1147
238	8.2585	8,8	10 50,1	+ 8 24 55	h 1161
239	15.2442	7,0	10 52,5	+15 53 48	* 241
240	7.2531	9,4	11 1,7	+ 7 48 57	h 1159
241	Sf.105	10,3	11 24,3	+16 4 24	h 1162
242	6°2585	9,7	11 45,1	+ 6 6 38	h 1178, etc.
243	28.2106	6,5	12 43,6	+28 54 42	h 1171
244	6.2588	7,8	12 54,0	+ 6 47 27	h 1177
245	15.2446	8,8	12 59,9	+15 7 58	h 1173
246	5.2613	9,3	13 6,5	+ 5 12 35	h 1202, etc.
247	6.2591	9,3	13 11,1	+ 6 40 3	GC. 2849
248	6.2592	9,0	13 18,5	+ 6 30 4	h 1176, etc.
249	5.2616	8,3	14 6,3	+ 5 17 21	h 1196, etc.
250	6.2594	8,9	14 19,4	+ 6 14 21	h 1178. b
251	15.2453	9,2	14 28,8	+15 30 31	* 262, etc.
252	15.2454	9,0	15 5,1	+15 35 55	h 1179
253	17.2469	6,8	15 12,2	+17 29 43	h 1212, etc.
254	6.2599	7,1	15 40,2	+ 6 3 23	h 1178. a, etc.
255	32.2239	8,0	15 56,2	+31 59 55	h 1258
256	30.2264	9,5	16 26,4	+29 59,7	h 1186
257	13.2528	9,0	16 29,6	+13 23,7	h 1244
258	18.2603	9,5	16 51,7	+18 55 5	h 1242
259	6.2606	8,1	17 18,5	+ 6 43 17	h 1222
260	13.2530	9,4	17 41,6	+13 37 27	h 1237, etc.
261	30.2267	8,7	18 29,5	+30 42 6	h 1204
262	15.2462	9,5	18 31,4	+15 24 2	h 1239
263	5.2626	8,9	18 37,2	+ 5 38 50	h 1228
264	8.2602	8,1	20 27,4	+ 7 59 20	h 1232
265	8.2604	8,1	20 36,7	+ 8 45 39	h 1294
266	11.2464	8,7	20 39,4	+11 53 3	h 1271
267	13.2535	9,5	20 40,1	+13 30 3	h 1267
268	11.2465	9,2	20 40,9	+11 46 26	h 1271
269	15.2467	9,2	20 54,0	+15 0 7	* 282
270	14.2503	9,2	21 8,9	+14 11 39	II. 114
271	13.2539	9,2	21 27,1	+13 5 39	h 1268, etc.
272	14.2505	9,2	21 44,0	+14 41 54	h 1288
273	13.2540	9,0	21 57,7	+13 50 9	h 1274, etc.
274	14.2507	8,6	22 16,2	+14 41 50	h 1288
275	11.2467	8,8	22 37,9	+11 53 8	h 1271, etc.

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
276	4.2622	7,5	12 23 41,5	+ 4 15 17	h 1286
277	10.2432	8,0	23 42,3	+10 27 53	h 1279
278	4.2624	8,1	24 19,6	+ 4 18 39	h 1286
279	8.2610	9,3	24 43,9	+ 8 32 9	h 1293
280	26.2359	8,2	24 50,4	+26 37 1	h 1307
281	30.2287	7,3	25 8,4	+30 3 40	* 256
282	15.2473	9,4	25 15,9	+15 4 54	h 1312
283	14.2512	9,3	25 29,3	+14 24 15	II. 115
284	8.2614	8,9	26 16,6	+ 7 59 47	h 1361
285	8.2616	7,5	26 33,7	+ 8 41 38	h 1322
286	8.2617	7,1	26 42,0	+ 8 25 22	h 1329
287	8.2618	8,6	26 43,3	+ 8 1 39	h 1361
288	13.2548	9,4	27 18,3	+13 19 48	h 1348
289	8.2619	7,1	27 39,0	+ 8 28 54	h 1329
290	12.2488	7,8	27 48,1	+12 2 48	h 1331, etc.
291	15.2479	9,5	28 9,4	+15 14 25	* 294
292	26.2367	9,5	29 1,7	+26 43 33	h 1357
293	12.2492	8,3	29 10,9	+12 9 9	h 1356
294	* a (h 1345)	11	29 12,9	+15 15 42	h 1345
295	13°2554	9,5	29 14,7	+13 55 38	M. 90
296	* a (h 1358)	10	29 57,9	+12 0 21	h 1358, etc.
297	12°2494	8,5	30 19,7	+12 55 35	h 1343
298	12.2495	8,0	30 23,2	+12 33 41	* 300
299	5.2654	7,3	30 59,4	+ 5 1 58	h 1371
300	12.2497	9,4	31 7,5	+12 39 24	h 1368
301	32.2269	9,1	34 12,7	+32 51 42	* 309
302	2.2567	9,2	34 38,8	+ 2 44 21	h 1404
303	12.2501	9,0	34 52,2	+12 30 18	h 1386
304	8.2632	9,0	34 57,3	+ 8 6 46	h 1384, etc.
305	11.2485	5,2	35 3,0	+10 58 50	h 1378, etc.
306	3.2681	8,5	35 45,8	+ 3 19 12	h 1399
307	3.2685	8,3	36 38,9	+ 3 21 40	h 1399
308	3.2687	8,6	37 6,3	+ 3 48 26	h 1419
309	32.2273	9,2	39 5,7	+32 55 9	h 1414, etc
310	11.2491	9,4	39 18,0	+11 53,5	h 1417
311	12.2510	9,5	39 21,6	+12 16 17	h 1408
312	17.2536	9,1	40 15,7	+17 13 48	h 1409
313	16.2421	9,3	40 38,8	+15 59 11	h 1441
314	Sf. 158	9,1	42 46,5	− 5 5 13	h 1436
315	26°2397	8,6	45 3,3	+26 14 42	h 1451
316	11.2505	9,2	45 54,2	+11 56 35	h 1466, etc.
317	2.2594	8,7	45 59,1	+ 2 51 2	h 1469
318	29.2334	7,7	46 41,5	+29 43 33	h 1475
319	16.2435	8,8	47 21,7	+16 3 51	* 313
320	12.2525	9,1	47 26,5	+12 2 43	h 1462
321	22.2521	8,5	47 59,0	+22 19 1	h 1486
322	28.2171	7,3	53 1,5	+28 47 45	h 1500, etc.
323	15.2530	9,0	53 14,0	+15 0 54	h 1498

N:o	N:o in the Bonn-Zones.	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
324	15.2531	8,3	12 54 3,0	+15 6 31	* 323
325	28.2184	8,4	13 0 21,4	+28 55 57	h 1522
326	37.2371	7,5	3 48,0	+37 52 30	h 1547
327	2.2673	9,1	18 32,4	+ 2 48 39	h 1601
328	47.2066	7,2	26 15,8	+47 55 48	h 1622
329	9.2778	8,9	27 33,7	+ 9 35 44	h 1650
330	9.2779	8,7	27 35,7	+ 9 37 34	h 1650
331	36.2406	8,0	37 38,4	+36 23 42	h 1664
332	60.1492	8,8	42 23,3	+60 50 31	h 1684
333	5.2815	8,0	49 33,5	+ 5 57 15	h 1703
334	5.2816	9,0	50 37,0	+ 5 54 4	h 1703
335	5.2820	7,8	51 10,1	+ 5 46 43	h 1705
336	55.1656	9,4	59 56,0	+55 30 33	h 1748
337	3.2874	7,5	14 8 32,7	+ 3 45 35	* 341
338	4.2848	8,6	12 5,8	+ 4 31 5	h 1779
339	37.2517	9,0	12 24,8	+37 9 23	h 1776
340	4.2850	8,5	12 56,8	+ 4 37 47	h 1779
341	3.2880	9,2	13 16,9	+ 3 50 7	h 1782, etc.
342	5.2898	9,1	31 27,3	+ 5 54 14	h 1854
343	6.2924	8,7	33 36,4	+ 6 2 45	h 1854
344	0.3222[a]	9,4	34 0,7	+ 0 17 43	h 1857
345	2.2858	8,5	37 55,7	+ 2 35 52	h 1874
346	2.2862	4,0	39 25,5	+ 2 27 50	h 1874
347	0.3243	8,5	40 39,1	+ 0 17 29	h 1875
348	2.2900	8,5	54 21,3	+ 2 23 26	h 1894
349	2.2904	9,0	56 2,4	+ 2 8 51	h 1896
350	2.2911	8,5	58 27,7	+ 2 32 40	II. 542
351	2.2918	9,0	15 1 32,9	+ 2 9 5	h 1901
352	2.2919	7,9	2 6,3	+ 2 12 28	* 349
353	20.3068	8,7	2 9,8	+20 1 57	h 751, etc.
354	39.2989	8,5	16 20 10,3	+39 43 51	* 355
355	39.2998	9,4	23 9,0	+39 39 18	II. 647
356	20.3284	8,6	24 47,0	+20 13 26	* 358
357	39.3003	9,1	25 30,0	+39 53 46	h 1961
358	* a(II. 753)	11	26 15,9	+20 7 31	II. 753
359	39°3022	9,0	33 53,3	+39 10 36	h 1966
360	36.2762	9,1	34 39,4	+36 31 54	h 1967
361	37.2793	9,5	38 44,4	+37 0 51	h 1969
362	23.2990	7,3	39 24,3	+23 58 10	h 1970
363	47.2381	7,9	42 36,0	+47 47 9	IV. 50
364	43.2712	9,0	17 13 32,6	+43 17 37	M. 92
365	66.1065	9,2	58 11,2	+66 39 20	IV. 37
366	17.3458	8,6	18 2 4,5	+17 36 13	II. 902
367	6.3651	9,0	5 47,4	+ 6 50 2	h 2000
368	39.3431	8,3	25 8,6	+39 50 12	* 369
369	39.3436	9,0	25 29,5	+39 45 18	II. 907
370	32.3250	9,0	49 36,3	+32 55 32	h 2023, etc.
371	0.4123	8,9	19 3 35,0	+ 0 46 48	GC. 4473

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
372	6.4091	9,1	19 12 50,3	+ 6 14 1	h 2037, etc.
373	8.4129	8,5	25 23,3	+ 8 52 9	h 2043
374	50.2847	6,3	38 13,6	+50 12 47	h 2050
375	33.3695	8,9	57 11,0	+33 12 24	h 2062
376	30.3987	8,8	20 14 11,6	+30 9 23	h 2072
377	19.4377	8,4	15 27,4	+19 40 11	h 2075
378	6.4572	9,0	27 27,4	+ 6 56 29	h 2081, etc.
379	12.4448	9,4	37 52,9	+12 6 12	h 2086
380	15.4318	8,6	56 36,5	+15 41 43	h 2097
381	29.4286	8,5	57 16,7	+29 26 53	h 2102, etc.
382	13.4651	9,2	21 9 27,1	+13 2 51	III. 209
383	25.4511	9,5	14 22,7	+25 56 25	h 2112, etc.
384	25.4521	8,8	16 13,6	+25 56 5	* 383
385	1.4490	8,3	23 56,7	+ 1 53 21	* 387
386	11.4583	7,0	24 37,9	+11 32 46	h 2120
387	* b (h 2121)	11	24 53,4	+ 1 52 50	h 2121, etc.
388	− 1°4173	9,4	25 52,6	− 1 28 39	h 2125
389	21.4630	8,0	44 21,6	+21 37 49	h 2132
390	2.4439	9,0	47 53,7	+ 2 11 53	h 2135
391	17.4676	9,2	55 9,0	+17 4 39	h 2139
392	30.4615	9,4	22 2 9,2	+30 49 8	h 2149
393	15.4655	9,0	21 58,0	+15 25 45	h 2160
394	15.4657	8,2	23 56,7	+15 19 39	* 393
395	4.4878	8,9	28 13,3	+ 4 51 7	h 2168
396	33.4542	9,1	29 16,8	+33 44 21	h 2172
397	19.4965	6,3	29 19,1	+19 34 51	h 2169
398	20.5189	8,6	30 34,6	+21 0 17	h 2170
399	9.5075	8,1	31 39,0	+ 9 54 55	h 2171
400	23.4586	8,6	33 30,3	+23 5 41	h 2173, etc.
401	10.4823	9,4	40 31,2	+10 59 34	* 403
402	10.4824	8,9	40 35,3	+10 52 3	* 403
403	10.4834	9,5	43 50,0	+10 53 12	h 2183, etc.
404	0.4939	6,8	48 5,2	+ 0 20 47	h 2189
405	1.4673	8,8	54 43,1	+ 1 7 58	h 2200
406	15.4746	9,0	54 44,1	+15 40 24	II. 249
407	15.4748	8,4	55 2,0	+15 13 48	h 2199, etc.
408	29.4838	9,0	55 16,1	+29 26 46	h 2201
409	8.4984	8,2	56 41,1	+ 8 14 11	h 2204
410	11.4931	9,5	59 48,9	+11 34 59	h 2205
411	11.4958	9,4	23 6 33,9	+12 0 12	h 2214
412	11.4962	8,4	7 50,0	+12 1 17	* 411, etc.
413	* a (h 2217)	10	8 36,2	+22 56 50	h 2217
414	* a (h 2223)	10	9 18,6	+12 27 3	h 2222, etc.
415	5°5152	9,0	9 28,1	+ 5 51 14	h 2224
416	3.4843	8,6	9 28,2	+ 3 45 10	h 2216, etc.
417	5.5155	9,0	10 42,8	+ 5 47 43	* 415
418	18.5133	9,0	11 12,4	+18 16 35	h 2219, etc.
419	7.5003	9,1	11 18,9	+ 7 48 41	* 426, etc.

N:o	N:o in the Bonn-Zones	Magn.	Right Ascension	Declination	Nebula observed with the Star
	°		h m s	° ′ ″	
420	12.4966	9,2	23 11 27,4	+12 39 32	III.238
421	7.5006	9,0	12 15,9	+ 7 29 1	h 2230, etc
422	7.5009	7,3	12 58,0	+ 7 14 43	GC.4934
423	22.4810	5,0	13 57,5	+23 0 7	h 2217, etc.
424	16.4912	7,9	14 12,0	+16 30 46	h 2232
425	12.4974	8,7	14 51,7	+12 26 56	* 414
426	7.5014	9,5	15 15,3	+ 7 42 57	h 2231
427	7.5016	9,1	15 40,8	+ 7 29 9	GC.4943
428	7.5030	7,1	19 36,5	+ 7 54 35	* 419
429	41.4780	7,4	20 9,6	+41 48 57	h 2241
430	11.5002	9,5	20 22,2	+11 43 15	h 2242, etc.
431	* a (GC.4967)	11	20 56,3	+11 38 57	GC. 4967
432	1°4744	6,8	29 30,1	+ 1 21 11	h 2257
433	1.4747	9,5	30 35,0	+ 1 23 34	h 2257
434	10.4978	9,0	33 29,0	+10 6 10	h 2264
435	9.5265	7,5	35 56,8	+ 9 8 16	h 2265
436	19.5159	9,1	42 42,0	+19 21 21	h 2275
437	19.5160	9,3	43 16,2	+19 22 50	h 2274
438	19.5197	7,5	56 30,4	+19 55 14	h 2290, etc.
439	15.4933	9,5	57 12,8	+15 22 21	h 2297
440	19.5202	9,3	58 3,4	+19 56 14	h 2300

In the reductions are the following annual proper motions employed:

	s	″
* 9 . . .	+0 0192	−0 204
* 20 . . .	+0,0510	+0,287
* 28 . . .	+0,0060	—
* 31 . . .	+0,0193	−0,117
* 34 . . .	+0,0182	−0,092
* 68 . . .	—	−0,200
* 159 . . .	+0,0200	−0,140
* 243 . . .	−0,0145	−0,125
* 254 . . .	−0,0110	−0,070
* 293 . . .	−0,0180	—
* 346 . . .	−0,0120	—
* 428 . . .	—	−0,257

NOTE II.

Besides several other remarks, various details are here communicated with respect to the size, brightness and general appearance of the nebulæ, the positions of their axes, etc., that are intended to form not unimportant additions to the summary descriptions in the preceding tables.

In these notes, as is commonly the custom in descriptions of nebulæ, I for conveniance's sake employ certain abbreviations and signs, which indeed in general agree with those used by Sir JOHN HERSCHEL and Lord ROSSE, but the significations of which it is however necessary here to indicate.

Abbreviations and Signs employed.

ab	about
al.	a little
alm	almost
am	among
att	attached
AR	right ascension
bet	between
biN	binuclear
B	bright, brightness
Br	brighter
br.	broad
C	centre, central
Cl	W. Herschel's classes of nebulæ
cl.	cluster
comp	compare! (vide!)
condens	condensation, condensed
decl	declination
def	defined, defination
diagr	diagram
diam	diameter
diff	difference, differing, differs, etc.
diffic	difficult
dist	distance
dNeb	double nebula
E	elongated, elongation
e	extremely
equil	equilateral
F	faint
Fr	fainter
f	following, follows, etc.
fig	figure
G	great
Gr	greater
geometr	geometrical
gr	group
inv	involved
irreg	irregular
isosc	isoscel
l	little
lg.	long
m	margo
M.	middle
min	minut of time
magn. p. 210 .	a magnifying power of 210 times employed
neb	nebula, nebulæ
nebs	nebulous
neby	nebulosity
n	north
nf.	north following
np	north preceding
nr.	near, nearly
N.	nucleus, nuclear
obl	oblong
obs	observation, observed, observe, etc.

obtuseang . . .	obtuseangular
p	preceding, precedes, etc.
par	equator-parallel
pos	angle of position
rectang	rectangular
r	resoluble, resolubility, resolves, etc.
rr	partially resolved
rrr	well resolved
Rd	round
sev	several
S	small
Sr	smaller
s	south
sf	south following
sp	south preceding
stell	stellar
trap	trapezium
var	variable
v	very
vv	very much

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*	a star
*	stars
d*	double star
trip*	triple star
comp*	comparison-star
◯	planetary nebula
⊕	globular cluster
Δ	triangle; forms a triangle
$\Delta\alpha$	difference of right ascension
$\Delta\delta$	difference of declination

AN . . . . . "Astronomische Nachrichten".
BB . . . . . Bonner Beobachtungen.
D'A . . . . . D'Arrest.
GC . . . . . General Catalogue of Nebulæ by J. Herschel, Phil. Trans. 1864.
H . . . . . Sir W. Herschel, Catalogue of Nebulæ by Auwers in the Königsberger-Beobachtungen, Vol. 34.
h . . . . . Sir J. Herschel, Catalogue of Nebulæ in Phil. Trans. 1833, Cape-Catalogue, GC .
Ph. Tr. 33 . . J. Herschel's Catalogue of Nebulæ.
Ph. Tr. 61 . . Catalogue of Nebulæ by Earl of Rosse in Phil. Trans. 1861.
Ph. Tr. 64 . . GC in Phil. Transactions 1864.
R . . . . . Earl of Rosse, Rossian; Catalogue of Nebulæ in Phil. Trans. 1861.
Sf . . . . . Schönfeld, Beobachtungen von Nebelflecken und Sternhaufen, Mannheim 1862.
SN . . . . . Siderum Nebulosorum Observationes etc., Auctore D'Arrest, Havniæ 1867.

Supplementary Details of Observation and Remarks.

GC.1 f in ⊿ of 2 *... 11^m and * 10^m.
h4: d* 11^m12 p in the par. *Nova I* f $^1/_3$ min nr in the par of *h4*; ◯ in F neby.

h1 has * 11^m12 np, 2 *... 12^m nf.

R. Nova according to *R* p ab 1 min nr in the par of *h2*, as *Nova* no doubt is the Fr of the two. In vain sought for 1866 Oct. 16.
h2 in np angle of trap with 3 *... 12^m; neb a trifle Fr than the stars.
Nova II p *h6* 72^s ab 2',5 s; * a (10^m) f, att.
h6 E nr n and s; biN; v badly def; diffic.

h7 bet 2 *...: 12^m np, 12^m13 sf; S *... att s? F neb $^1/_2$' n in the same decl-circle?
h9 l Br than *h7*; * 12^m nr. An eF neb suspected np bet * 30°20 and a F * n?

h13 r M; *a 11^m12 sp.

h16 r; *h17* r, not bicentral; np in trap with 3 *... 10^m.
GC.40 f *h17* $^1/_2$ min and 2' s; vvF.
GC.41 2' s nr in the decl-circle of *GC.40*; * 10^m11 p bet them.
GC.42 f *GC.40* $^1/_5$ min al s; 2 *... l Br than the neb p in the par; a S nr rectang ⊿ of *... 12^m f nr.

Rem. Several more neb suspected in this region. Of the three last mentioned neb the first was observed as Nova 1866 Oct. 17, the other two as Novæ *D'A*. These objects are here respectively designated as *GC.40, 41, 42*, although no certain identification possible.

h21 F neb of ClI, not Gr than $^1/_2$'; has a B stell N; * 10^m11 np.

h23: d* 11^m12 sp.

Rem. The positions of the 3 *R* neb in the vicinity are probably nr right in *GC;* the objects, as scarcely discernible, are not observable with the wiremicrometre. Comp *SN!*

h26 no doubt S ⊕, glimpses of *... seen; Br * in nf part; C condens v indistinct; * 11^m sf and *12^m nf in the same decl-circle.

h32 has sev *... nr the C?
GC.80 eF.
GC.82 is probably S ⊕, and seems sometimes to be divided into two separate objects.

h33 is Gr than h 26; * 12^m sp; Fr * nf (nebs?).

h37: *a 12^m immediately sp.
h42 not much Fr than *h37*.

h 39 E in the decl-circle.
h 41 ○.
h 43 an indistinct but not eF object; not obs.

h 44, *h 50* only once obs; obs of small value, the sky being too light.

II. 857 less F than *GC. 103* (seen but not observed).
h 40 inconsiderably Br than *h 23*, but no doubt Br than *II. 857* and *II. 858*.
II. 858 less F than *GC. 103*, nr as *II. 857*.

GC. 5058: * a 11^m immediately sp, which makes the obs of that vvF object e diffic.
GC. 119 in straight line bet 2 *...: * 11^m np = *a (*GC. 5058*), * 10^m11 sf.

h 59 has * 10^m nf — neb 1′ n from that *? Neb also suspected p in the par of *h 59* and also immediately nf.

Rem. The other neb of this vicinity could not be verified with certainty, although the state of the air at least once pretty favourable. They are no doubt eF. Comp *SN!*

h 65 rr with magn. p 210; no essential C condens, but Br * nr M.

h 78 △ with 2 *... 10^m sf and 12^m nf.
R. Novæ not in my working-list, and not noticed. Comp *SN!*

h 84; h 85; h 86 = *R.* α, β, γ; S and nr ○; *h 86* the brightest.
GC. 205 = *R.* γ' considerably Fr and Sr than the 3 np; *a 11^m sp.
GC. 207; GC. 208 = *R.* σ, ε; S and ○; pretty good for obs even in moonlight.
GC. 209; GC. 210 = *R.* δ, θ; not found. Comp *GC* and *SN!*

GC. 199 long and carefully searched for in vain 1867 Oct. 30. Strong aurora.
GC. 217 not v diffic but for 2 *... 10^m sf.

h 87 and nf *GC. 215* nr equal — *GC. 215* at the time not obs.
GC. 213 identical with *GC. 215*. Comp *SN!*

h 89 r M. State of the air always infavourable.

Nova III sp *II. 220*: $\Delta\alpha$ = 0,4 min, $\Delta\delta$ = 1′,5; 2 *... p : 12^m n and 11^m s.
II. 219 p exactly in par of *II. 220*: $\Delta\alpha$ = 0,1 min; stell. The eF neby several times verified is indubitable.
II. 220: N v indistinct; r?
Nova IV sf *II. 220*: $\Delta\alpha$ = 0,3 min, $\Delta\delta$ = 2′,3; 2 *... 11^m f.

Rem. The identification here hypothetically adopted agrees with *GC* and with *H:s* remark (*II. 219*; *II. 220*): "preced faint vS; following pL". But it nevertheless seems astonishing and worth remarking, that *H* has noticed the eF and often doubtful neby of *II. 219*, and yet should not have seen the relatively B Novæ. Can this peculiarity be owing to some variations of the objects in this region? *D'A* has here only 2 neb, and identifies *II. 219* with the first *Nova*.

GC.230 not with certainty seen. Atmospheric state v infavourable in consequence of strong aurora. *D'A* has: "Propterea *III.155* (=*GC.230*) in oculos non incurrit".

GC.249 barely seen in its place. Sky v much illuminated by aurora.
GC.250 has * 10^m nf.

I.108; III.250; III.251.

The coordinates for these neb 1860,0 according to *H:s* cat:

I.108 . . . . $1^h11^m56^s$; +2°31′ p. v B *
III.250, *III.251* . . . $1^h13^m\ 0^s$; +2°39′

From my obs the following places are obtained for the same epoch:

I.108 . . . $1^h11^m57^s$; +2°32′,8. Cl II; v B * f.
III.250 . . . $1^h12^m31^s$; +2°39′,3. Cl II.
III.251 . . . $1^h12^m53^s$; +2°39′,8. Cl I.

Referring *H:s* position *III. 250-251* to the second of the two, the identity of the objects can not be doubtful; it being of course easily imaginable, that *H* should have erroneously noted the last two neb, instead of the first two, as the faintest. *GC* consequently quite in error respecting these objects. Comp *Marth AN995* and *SN!*

I.108 inconsiderably Br than *III.250.*
III.250 irreg E nr in the decl-circle contains sev *..., yet has not the character of r.
III.251 estimated as one of the brightest of 3:d order.

h102; III156; III157; III158; Nova V; h113; h114; GC.302.

h102. The identification in *SN* certainly right, as no other nebs object is to be found nr: *GC* then has δ 5′ too S. Yet it remains to be remarked, that this neb is by no means eF and that h says nothing of the 2 B *... nr, which in all cases are v troublesome for the obs. The identification of *h102* with *III.156* evidently false! The three neb: *III.156*, *157*, *158*, according to my obs, are f *h102* nr 1 min and 3′ s.
III.156=GC.278 in the p part of a nr rectang Δ of *... 12^m. *GC* has AR 14^s too S.
III.157=GC.288 vF, immediately sp *h106* (=*III.158*); seen several times. The identification of *III.157* with *GC.278* evidently erroneous.
III.158=h106. With this identification the existing circumstances nr agree with *H*s description of *III.156, 157, 158:* "three form a rectangle Δ; in the legs eF, vS; at the rectangle vF, p L."; * a 11^m sf the neb.
Nova V nf *h106*: $\Delta\alpha=44^s$, $\Delta\delta=2'$; * 11^m12 np. Neb S, pB.
h113=GC.299; * 12^m np.
h114=GC.301. GC has AR some seconds too S.
GC.302 searched for 1867 Oct. 30, Nov 1 and 2, but not seen. Comp *SN!*

Rem. An eF neb just sf *h106* suspected. The neb *GC.288* not noticed by *D'A.*

Approx. Positions 1860,0 accord. to my observations.

h 102?	$1^h14^m\ 5^s$;	+32°47′,0	= *GC. 272*
III. 156	$1^h15^m\ 5^s$;	+32°43′,9	= *GC. 278*
III. 157	$1^h15^m19^s$;	+32°42′,4	= *GC. 288*
III. 158 (=h 106)	$1^h15^m21^s$;	+32°43′,4	= *GC. 289*
Nova V	$1^h16^m\ 5^s$;	+32°45′,7	= *Nova*
h 113	$1^h16^m47^s$;	+32°44′,3	= *GC. 299*
h 114	$1^h16^m54^s$;	+32°41′,7	= *GC. 301*

GC.274 has $*11^m$ np and $*11^m$ sf.
GC.293 Gr but not Br than the p neb.

h103 forms *Δ* with 2 *... s : 10^m11 p, 11^m f. Neb nova p 3^s à 4^s in the par?

Rem. As the state of the air was always v infavourable, nothing could be decided respecting the *R* neb. *SN.:* "Rossianæ neb. obscurissime quidem visæ, sed, nocte neutiquam favente, non satis tute."

h104 contains S *...; probably ⊕.
h107 not seen 1867 Nov 2, probably vvF; yet not searched for very carefully. Seen by *D'A.*

Rem. *GC.283—287* not seen. Comp *SN!* Half the field of the telescope s from *h104* seemed full with neby intermixed with pretty B *....

h117 indistinctly seen; r N.
GC.308 not e F; $*11^m$ np.

h118 Δ with 2 *... s : 10^n and 10^m11; rr and conspicuous ⊕. Astonishing, that the easy r of this neb is not remarked by other observers!
h120 not remarkably Fr than h118; contains S d* : 12^m np and 13^m sf nr in the same decl-circle. Place that of the * 12^m.

Rem. The *R* neb in the vicinity not seen and not searched for. The neb III. 171 also not seen; yet nothing with certainty decided respecting the existence of this object, as is can be easily overlooked am the great multitude of B *... in this region. *SN:* "Rossianas non commemoro, quum incerta sint loca".

GC.349 contains sev *...; place that of the brightest * 12^m in the s part of the neb.
Nova D'A contains at least one *; * 12^m f 2^s and 1′n.
h 131=the principal neb, has no essential C condens but contains sev F *... Place that of the brightest * = *a 11^m12.
h133 evidently S gr of *...; * 10^m11 sf.

h142 twice obs in v unfavourable circumstances. The neb has not character of r, yet contains some S *...; alm ◯, as the central condens inconsiderable; am the faintest of the 3d order.

II.253 am the faintest of the 4th order, indistinct and diffic; has not character of r, but contains probably sev condens knots — position that of the brightest point. The considerable E of the neb — sp to nf — only momentarily visible, as the sky v dull and murky. Neb ab 2′ lg and ⅓′ br; * 13^m f.

h147 am the faintest of the 4th order.

h149 *A* with 2 f *... 11m s and n; neb E np to sf.
h150 E probably sp to nf; ill seen.
h152 looks like a neby E nr in the par around sev S *...? ill seen; *a 11m sf and sev more *... f.

Rem. *h150* and *h152* nr of the same B and a trifle Br than *h149*.

II.228 Gr than *II.229*.
II.229 Br than the p; S d* (nebs?) sf. A nebs object also suspected bet the two neb?

II.221 not seen. *SN:* "Equidem nullus dubito, *h169* esse=*II.221*, quod et ante placebat omnibus."
h169 a trifle Br than *h175;* * 12m13 nf (h has np?); *9m10 sf. The *R* neb not noticed. *SN*: "Comites Rossianæ mihi non reperibiles."
h175 biN np to sf; both N perspicuously inv in neby; perhaps the neb contains sev F *...? Place that of the n N. The *R* neb not noticed. *SN:* "Juxta observata Rossiana præter hanc neb et *h169* tertia subest, quam nobis tamen usque adhuc non contigit videre."

GC.461 sp *h181*.
h181 probably contains sev S *...; * a 11m12 p 12s nr in the par.
GC.464 suspected nr in the place of *GC;* at all events eF; B * s. Not noticed in *SN!*

h194 E nr in the par? C Condens indistinct — perhaps sev knots? Neb not eF but v diffic in the murky air. Region v rich in stars.

h197 distinctly Br than *h194* and am the brightest of the 5th order; probably r, but this could not be decided in the dull air; *A* of * 11m and 2 *... 10m sf.

h208 E sp to nf; sev *... seen in it by glimpses. Air v bad: nothing seen of the *R* neb. *SN:* "Neb satelles, quæ auctore Rossio hanc ad meridiem comitatur, nobis non comparuit."
h210 an obvious ⊕, a trifle Br than *h208;* rM and sev more *... seen by glimpses; C condens v indistinct; *a 12m just sf. Nothing seen of *R* Nova. *SN*: "Comes hujus nebulæ Rossiana nostro telescopio nunquam in obtutum prodiit."

h217 irreg fig; * 11m12 p nr in the par.
h231 much of the same B as *h217* and *h234*; not Gr than 1/3'; * 10m np, *12m13 sf.
h233 bet 2 or more *... 13m; * 12m13 nf; vvF and tolerably diffic.
h234 E nr in the par; suspicion of r; * 11m sf.

Rem. The *R* neb in the vicinity not searched for, nor noticed. *SN:* "Neb 558 ac 559 Catal. Gener. spurias esse crediderim."

h226 Fr than *II.229*. Pos of axis of E by measure 1874 Jan. 14... 61°,7 (2).

h 242 evidently only the Br C part seen; E *A* with 2 *... 8^m9 and 9^m10.

h 260: * 10^m11 nf; strikingly Fr than *h 257*.
h 261 quite distinctly seen 1870 Jan. 22 nr in the same decl-circle with *h 260* and 1′ n; yet eF and not observable. Comp *SN!*

II. 619. Place that of the insignificant condens n in the neby; * 11^m n 1′; S neb nova nr in the same decl-circle 1′ s?

h 315 is irreg Rd; condens vF; *... 13^n or Fr seen by glimpses in the neby; detached neby immediately p.

Rem. The *R* diagr in *Phil Trans 1861* could not be verified, as state of air v unfavourable. Of the 5 *R* *... nr only α, β, ε obs: α has a F yet conspicuous nebs atmosphere, β and ε are purely stell.

h 355 is trip * inv in neby. Relativ positions of the *...: *a 11^m f, *b 11^m12 sp, *c 12^m np. Place that of the *a. Sev more eF stell points perceptible in the neby; * 13^m sp the neby. The discrepancies bet the descriptions of different observers respecting the stars in this neb are v remarcable. *H* has: "seems to have 1 or 2 *... in M or an irreg N"; *h:* "trip* in a neb."; *R:* "a multitude of *... and some unresolved neby."; *D'A* "quinque fixæ in nebula satis conspicua formæ ovalis oblongæ."

h 359 is no doubt ⊕; sev *... 13^m14 noticed; 2 *... nf nr in the same decl-circle: *a 10^m n, * 11^m12 s.

h 365 ◯; diam 10″ à 20″; light not equable; seems to r into sev sparkling knots. Place that of the brightest point in the p and more compact part of the neb.

h 368: 3 *... in the neby obs: *a 10^m p; *b 10^m11 nf; *c 12^m sf; sev stell points by glimpses nr a and b

Dunér Nova twice obs in v bad air and consequently v indistinctly seen; am the faintest neb of Cl. II; probably a S *gr.

h 434: * 11^m p in the par; * 12^m13 att at nf m.

h 447 = Rγ.
h 448 = Rβ; *A* with 2 *... 11^m nf and sf.
h 449 = Rα.

Rem. Nothing more here found nr.

GC. 1537; GC. 1538: S d* ab 3′ n; *12^m13 f ab 1′ n; * 11^m sf ab 45^s; * 10^m11 nf ab 46^s. Region v rich in *....

h 456 bet 2 *... 12^m, np and sf; through its flickering light and momentarily visible stell points character of ⊕. *R* has: "r".

h 457 has character of ⊕; many *... 11^m, 11^m12 and Fr around the neb. *R* says: "r, looks like a glob cl".

II.616 pretty B; has many *... nr: * 11^m12 sp, *a 11^m12 nf, * $9^m,4$ sp 21^s, *$9^m,5$ f 10^s and 3′ n. Seems not to have been reobserved since *H*^{s} time.

h481: * 10^m11 sp; neb has a conspicuous N 12^m, which seems momentarily to r. Neb generally has the character of r, but not sufficiently well seen for deciding. *R* has: "Centre r?" Comp also *SN!* A vvF neb nf *h481?*

h482 evident ⊕, rr; * 12^m13 sp (nebs?), which has 2 other *... 10^m11 and 11^m sp.

Rem. *SN* has: "Cumulus admodum coarctatus"; *R* says nothing of the r.

h495. Neby insignificant, E nr in the decl-circle np to sf; v indistinctly seen. N momentarily r.

h497 indubitably ⊕, distinctly though not constantly rr; obtuseang *A* with 2 *... 11^m n and sf. *R* says nothing of the r.

GC.1644, 1650=D'A Novæ noticed and observable, though not yet obs.

h500 al Br than *h501.*

II.259 considerably Br than *h507;* * 9^m10 nf; many more *... in the neighbourhood.

Rem. *GC* has "r", but the neb seems now to be entirely without the character of r. Neb not obs by *D'A* and, as far as I know, first in Upsala reobserved since *H*^{s} time. The new position not very well according with *H*^{s}.

h507 in a region rich in *...; poor *gr sf. As C, though F, is stell, the measures should be better accordant; perhaps more *... than one in C?

h526; GC.1705; h527.

h526: * 12^m13 nf, * 13^m sf, neb *A* with 2 *... 10^m.

GC.1705 vvS, nf att to *h526;* once the two seen distinctly separated, but no position taken. *GC* quite right; *R* erroneously identifies *GC.1705* with *h527.*

h527 eF but 1865 March 19 indubitably seen nr in the place of *GC,* yet too diffic for obs; probably ⊕. Neb of the same faintness immidiately nf? 2 *... 10^m11 n and sf. No more neby here seen. Comp *SN!*

h530 looks like * 10^m11 with F nebs atmosphere hardly Gr than ½′; d* 11^m p, * 12^m f, which *12^m fs. Sky v light.

Rem. *h* says "* with vF extensive neby"; *R:* "nucleus surrounded by v extensive neby"; *D'A* and *Rümker* respectively make the diam 60″ and 150″.

h532 E 4′ in 42° pos, ½′ br at N; planetary N 2″ situated in the Br n part of the neb, forms nr isosc *A* with 2 f *.... The neby nr extends to the n of these stars (11^m12), which however is not exactly on the central-line of the neby. The dist bet this * and N perhaps nr half the length of the s part of the neby.

Rem. The dimensions estimated by *h*, *D'A* and *Sf* inconsiderably Gr than those here indicated — but *H* has: 8′ lg and 3′ br; *R:* 10′ lg.

h550 a most indistinct and diffuse object, not to be exactly determined; hardly Br than Cl.III, yet sky v dull; neby E nr in the par sp to nf; C condens scarcely perceptible; a multitude of eF *... momentarily glimmering through the neby. S *Δ* of *... p.

Rem. In *SN* I find the following description according with my obs: "neb. ex iis est, quæ accuratam et perfectam loci determinationem non admittant, eam ex innumeris stellulis, admodum imbecillis, esse compositam, palam et apertum", and also "est sublustris classis sextæ." It is consequently v surprising, that *h* and *R* say nothing of the r. *R* has only: "an amorphous mass of neby, of uneven character." Even *H* has not seen the object resolved, though he has hypothetically "easily r."

h557 rr; * 10^m np and * 9^m10 (=BB.21° 1976) sf; r probably yet more conspicuous, if the v troublesome B *... did not exist.

Rem. *R* says nothing of the r, and, as far as I know, it has been before remarked only by *D'A:* "dissolvitur, et quidem nitide et aperte."

h565: * 12^m13 p ab 2′ s; r suspected, could not be decided on account of less favourable atmospherical circumstances; *h* has "r". Comp even *SN!*

h564 Gr and Br than *h593;* only seen in bright moonlight.

h566 rr; diam ab $^2/_3$′.

GC.1775 searched for in vain 1870 March 27 and 29; neby suspected nf *h566*, but nothing sure. Also not seen by *D'A*.

h567 rr; not Gr than $^1/_2$′; vvF and scarcely observable; d* (12^m; 12^m13) np.

Rem. According to the *Upsala*-obs both neb have many fine *... glimmering through the neby, and r is especially evident in *h566*. *H* has also "r" at *III.826=h567;* but *h*, *R* and *D'A* say nothing of r of these objects.

h568 is according to *GC* "cB", yet not found 1869 Jan. 18; not obs either by *R* or *DA*.

GC.1792=D'A Nova, similar to yet Fr than *h578;* evidently visible but not observable in moonlight.

h578: * 9^m10 (=BB.20° 2299) p 3′ n; * 10^m $1^1/_2$′ n from that *; nr in the par of the neb al d* f, the s component 11^m, possibly both *... inv in neby? Not to be decided in moonlight.

h584 a trifle Br than *h530;* diam perhaps 1′; * 11^m np.

II.57, 58 and *GC.1846* carefully but in vain searched for 1866 Apr. 5 and 6. The *R* neb suspected, but probably too F for the Upsala-refractor — not indicated in *SN*. The first two neb have not been reobserved since the time of *H* — are they spurious and identical with *h597* and *h598?*

h596 am the brightest of 4th order; N v indistinct seems to consist of sev S *.., but not to be decided, as the sky v dull; * 10^m f, and sev more *... nr.

I.56, *I.57* form the two nuclei of a continuous oval neby with 2 separate condens. The diagr in *Ph Tr 33* consequently gives no exact idea of this object; the diagr by *R* seems on the contrary to be a faithful image of the neb. Both N are nr on the axis of the neb: *I.56* nr central, pretty B; *I.57* nr the n edge, through its faintness scarcely observable. N *I.56*, E nr in the decl-circle, evidently r in 2 or 3 stellar points. State of the air only tolerable, and not more than 210 times magn. p. applicable.

h608; Nova; h609.

h608 F but conspicuous traces of r; ab $^2/_3'$; E in the par; * 10^m sf. Sky v serene.
Nova eF; f *h608* some seconds ab 2′ n; but not observable.
h609 hardly Fr than *h608!* Neb ab $^1/_2'$; E nr in the decl-circle; * $10^m,3$ np. The circumstance, that obs is scarcely troubled by the B * nr, prooves best, that even this neb is pretty B.

Rem. Are the objects in this region var? The two neb are evidently Br than for instance *II.116* and *h682*, and consequently much Br than is to be exspected according to the catalogues. The * *BB.* 10^h2019, f *h608* ab 40^s nr in the par, is on the contrary only 10^m, and consequently Fr than according to *BB.*

h622 C condens uncertain: once seen with well defined N, once rM.
h624 not much Fr than *h622*. Is there a vvF neb immediately s?
h627 is nowise eF, but obs diffic through the vicinity of *a 10^m p nr in the par. Astonishing, that this neb was not seen by *D'A*, and that *h* has not remarked the B *?

GC.1916=R.Nova vvF, v diffic for obs especially as the sky much illuminated from B northernlight.
h634 Fr and Sr than *h636;* * 12^m sf.
h636 in a trap of 4 *... 11^m.

h645 well seen; not particularly B and certainly not Cl.I; diam in the decl-circle nr $^1/_2'$; rr with magn. p. 210, sev distinct *... 12^m13 especially in the nf part; *a 10^m11 sf.

Rem. Other observers have not seen the neb resolved, and probably in consequence thereof have found the neb Br than I have? *D'A* for instance has: "Multum lucis, primæ manifesto classis etiamnunc.... Ad amplificationem 226 nebula non dissolvitur."

IV.79=M.82 E in pos 294° to 5′ à 6′, ab 1′ br; neb has a condens of ab $^1/_4'$ diam in the nf part, and the place refers to the C of it; *... glimmering in and around that condens spot.

h649=M.81 am the brightest of the 3d order; N planetary ab 2″; 2 *... 10^m11 sf, many more B *... nr.

h650 on a straight line bet 2 *... : *a 9^n np and *b 10^m11 sf; as the neby scarcely perceptible, although the sky pretty exquisite, neb looks nr like * 12^m.

h656 for the most part r into *... 13^n14, although state of the air v bad and moonlight. E in par? Place that of a Gr C * or N 12^m13.

Rem. *H* has: "cl of v comp S *..."; r even remarked by *h*, but not seen by *R* and *D'A*.

h659 equal to *h630* and inconsiderably Fr than that neb; diam ½′; 2 *... 12^m11 sp nr in a straight line through the neb.

h658 only seen in v unfavourable circumstances. Neb am the faintest af 3d order, probably C r.

h663 considerably E in the par to 3 à 4 times the breadth; *a 12^m sf.

h682 diam al more than 1′; has *... glimmering through the neby, of which a nr C * 12^m13; neb am the brightest of the 5th order; *a 10^m sp, *b 12^m13 sp.

GC.2054; GC.2055; GC.2057; h692; h693.

GC.2054=Rδ; sky always too light for exact obs. Neb probably S Rd ⊕, in which different C *... are by turns visible, as the measurments of decl, though diffic, ought otherwise to have agreed better; appears sometimes E in the par, which evidently is owing to an eF planetary object immediately sp — S neb?? That F object evidently identical with *D'A*: "* 20 magn. limbo proxima ad occas. versus" and "* 19^m magn. immed. præced. non nihil australior".

GC.2055=Rγ and *GC.2057*=the s branch of *h692* are both, according to *R*, ab of the same F. Searched for in vain in less favourable circumstances. Comp *SN!*

h692=Rα considerably E np to sf; pos (measured)=116°. Place that of the stell N, identical with the condens spot on the s side of the n branch in *R* diagr; * 13^m14 sp.

h693=Rβ has * 9^m ab 1′ n, which is followed by an other B *. Nebs object bet these 2 *...? Diam of the neb in the decl-circle measured=42″.

GC.2084=Winnecke Nova is dneb or biN: sp and nf. The sp N the Fr, and, owing to its dull and planetary appearance, seems not to be *; * 10^m11 immediately f ab 2′ à 3′ s.

Rem. *Winnecke* has considered the neb as a r *gr; *D'A* says: "Cumulus etc." According to my obs neb is simply biN, but not r. Sky only tolerably good.

h713 rectang *A* with 2 *... 12^m sf nr on the same par; * 9^m10 np.

h714 am the faintest of the 4th order; d* ($9^m,5$ and 10^m; dist ab 1') f ab $^1/_2$ min nr in the par.

Nova VI an insignificant object; p *h721* ab 68^s and 160″ s; *a 10^m f $0^s,2$ and 135″ n.
h721 similar to *h711* and inconsiderably Fr.

h724 ab 2′ lg and $^1/_2$′ br. Axe of E np to sf: pos(by measure)=112° (diffic). Sev knots or *... momentarily glimmering through in M? C condens consequently v indistinct. Sky not very favourable.

h737 not inconsiderably Br than *h739;* *a $8^m,5$ np.

h739 trap with 3 f *..., relative positions determined. Neb not Br than Cl.II. Why in *SN* the epithet "splendens" is employed, I can not conceive? I agree in the remarks of *D'A:* "qua ratione *milky* anno 1784, non intelligo", as the C part decidedly makes the impression of an unresolved ⊕. Neb 2′ or more E nr in the decl-circle; but the extreme part eF. *R:* "vF spiral with B centre; S * sf centre involved; two others following."

h751, 753 carefully but in vain searched for 1866 Apr. 8; nor seen 1863 Apr. 9; but the same year Apr. 17 I seem, according to the journal, to have seen at least a F trace of them, although sky v bad and the B neb *h748* looked vF. This obs quite certainly erroneous — the telescope probably was pointed to a false region? In *Ph. Tr. 61* there is nevertheless mention made of a "triple neb." Query wether the objects in question be var or do not exist?
h754 has evident traces of r, although seen in bad air.

h757: traces of C r or 2 C *...?
h758 is Sr but inconsiderably Fr than the p neb.
h761 not so vF, but only once obs in bad air, and then diffic.

h773: irreg C indistinctly seen — perhaps sev *...?
h775: *a 10^m sf.

II. 493 the brightest and best defined of the 3 in the vicinity.
II. 494 E nr in the par, indistinct and without C condens. Place that of *a 11^m12 in f end of the neby; * 11^m sf *a; * 13^m in p part of the neby.
I. 118 appears not to be capable of an exact determination.
Rem. Identification according to *SN.*

h784 inconsiderably Fr than *h500* and *501;* *, a little Br than the C of neb, nf.

h805 rN; sev S *...* also glimmering through other parts of the neby.
h813 E np to sf nr in the decl-circle; does not look like a r neb, yet *... visible in the neby.

Rem. The r of *h805* remarked by *h*, but not by *R* and *D'A*.

h810: sev *... 11^{m}12 à 12^m around the N, yet does not look like r neb; E np to sf.
h815 r; *D'A* says: "dissolvitur etc.", but *h* and *R* have not noticed the r.

h838 a curious circular planetary neb; comp *Ph. Tr. 50, 61!* Place that of the estimated geometr C; obs v diffic, as the appearance of the object v var and its light v flickering. Sky tolerable; illumination of the spiderlines deficient. * 12^m momentarily glimmering in p part; * 11^m nf, * 12^m sp. Diam of neb measured

in the decl-circle = 159″,6 (1)
— — par = 158″,7 (6)
} (v. diffic)

Time of passage = 18^s,8. Comp Ph. Tr. 33!

h847 am the brightest of the 3^d order; a little E np to sf; N indistinct; probably sev S *... in the direction of the E confluent with N. *H* has: "S nucl E par".

h854 inconsiderably Br than *h857;* evidently rM; * 11^m sp, * 12^m nf. Neb E np to sf; pos of E (by measure) = 169° (2)
h857 Br or Fr than *h854?* rM; E np to sf; pos of E (by measure) = 163° (2)

h861 E sp to nf.
h864 rM but indistinctly.

h871 suspected ⊕, as traces of r not wanting; * 11^{m}12 p in the par. Air pretty good, yet neb indistinct and diffic to form a clear idea of.

h891; h893; h894; h898.

h891 no doubt the brightest of the four.
h893 hardly Fr than *h894.*
h894 am the faintest of the 4th order, wholly the character of ⊕ with sev glimmering *...; indistinct and diffic to form an idea of; * 12^{m}13 att sp, * 10^{m}11 n; diam ab 1 ½′.
h898 Fr than the p neb, yet not eF and pretty good for obs; * 12^{m}13 nf.

Rem. *h* has "r" at *h894* and *h898; R* found *h894* the brightest of the 4, which agrees with *Sf*^{s} repeated observations — *D'A* seems dubious in *SN;* according to the *Upsala*-obs *h891* was quite certainly Br than *h894*; in *Ph Tr 33 h891* seems also to be rather the Br. Var?

h892 E np to sf, dimensions ab 1 ½′ and ¼′.

h897 E nr in the par.

h902 E nr in the par; indistinct but indubitable traces of C r; *H* has "r".

h933; h940; h939; D'A Nova.

h933 probably 2 *... 12^m13 inv in neby? * 11^m sf.

h940 incomparably the faintest of the 3 *Herschelian* neb; S *... (one Br) immediately f in the par.

h939 appears to be $^2/_3'$; perhaps a little E; its r v conspicuous with magn. p. 210. This neb the brightest of the 3, but even *h933* pretty B although vS.

D'A Nova nf *h939*, visible although the atmospheric state not v good; not obs. This neb is not inserted in *SN?*

Rem. *GC* has "r" at *h940* and *h939*.

h938; h944; h946.

h938 irreg Rd; C part E nr in the par; evident C r. 2 N in the same par? eF neb nova nf? * 9^m sf.

h944 Fr than *h938* and considerably Br than *h946*; dubious indications of r; * 9^m sf.

h946 has pretty distinct C condens; E sp to nf in ab pos = 45°.

Rem. According to h's terminology *h938* and *h944* are pF, *h946* vF; *h946* seems to be the greatest of the 3, and *h944* the smallest.

h943 only seen in moonlight, yet r undubitable; indications of Gr dimensions than 1'.

h945 E nr in par; appears to have the character of ⊕ — not to be decided, as sky v bad.

h1047 E nr in decl-circle, np to sf; dimensions ab 3' to 4' l and not quite $^1/_2'$ br; Nr?

h1049 rr; even the apparently compact N momentarily r into S *...; the C r here more conspicuous than for instance in *h636*. Neb no doubt am the brightest of the 4th order, but like all halfresolved objects continually v var in B.

h1050 E sp to nf, ab 20° with the par; dimensions ab 1 $^1/_2'$ and $^1/_4'$; Nr?

I.224 E sp to nf, ab 30° with the par; dimensions 1 $^1/_2'$ and $^1/_2'$; *A* with 2 bright *... s: 51°.1725 p and 51°.1726 f.

I.206 has the same character as the p neb, but is considerably Gr and Br. E sp to nf, ab 30° with the par; neb 1' br and at least 2' to 3' lg.

Rem. These two neb diffic for obs, and not to be sharply determined. No more neb heer seen.

h1088 E np to sf, and ab 1 $^1/_2'$ lg, but scarcely more than $^1/_6'$ br. Axe of E in measured pos 326°,8 (3) . . . . 1873 Febr. 18.

h1106 always ill seen, but r conspicuous; N E in the decl-circle. The unfavourable circumstances permitted no verification of the *R* description.

h1132 E np to sf, at least 13′ lg and ab $^2/_3$′ à 1′ br at N, situated nr 4′ from the n end of the neby. E by measure in

pos = 150°,1 (1) . . . . 1870 Apr. 21
— = 150,8 (4) . . . . 1871 Apr. 23

This pos refers to the generally rectilinear nebs radiations, but at the p part of s end of the neby the radiations seem to bend forward in direction of the diurnal motion. The C condens, even E in the axe of the neby, has a F C ∗, to which position refers. 1870 Apr. 21 N is stated to be evidently r? ∗ 11^m12 nf to N.

h1140 N in vF neby; C r evident.

Rem. *R* has: "v like a distant cl.".

h1146 v irreg and complicated object, quite without character of r. Neb has 3 knots α, β, γ; around the principal knot α (11^m) momentarily sparkling ∗... seen; β 12^m13 sf; γ, immediately p α in the par, it being diffic to separate these two knots, which probably form h^s dneb.

Rem. My description tolerably well according with *R*. The diagr in *Ph Tr 33* gives no true image of the object.

h1157: no trace of this neb seen. Comp *SN!*

h1147 E nr in the decl-circle, 1′ lg $^1/_3$′ br; N evidently r; ∗a 11^m np.

h1148 E nr in pos = 30°; 3′ à 4′ lg, $^1/_2$′ br; $^1/_5$′ n from the obs C appears momentarily an other Fr. Sky too light, but nevertheless indications of r at sev points.

h1153: size ab $^1/_2$′; a little E in the par; ∗ 12^m13 nf. Sev ∗... in M?
h1161 a little Br than *h1153*, but Fr than *h1222* and *h1147*; ∗ 11^m nf.

h1159 Br than *h1153*, but Fr than *h1161*; 2 ∗... 11^m n; sev ∗... in M? E nr in pos = 45°?

h1168 indubitably rM; perhaps not Rd.
h1185 considerably Br than the p; also rM.

Rem. *D'A* and *R* have not seen these neb r; *h* has *h1168* r, but *D'A* doubts the r. Of the 4 neb *h1168*, *1185*, *1186*, *1188*, the third is incomparably the brightest and *h1185* the greatest.

h1171 E np to sf in pos ab = 120°; neby hardly $^1/_3$′ br at the N, but at least 1′ lg; N round 1″; ∗a 12^m sf.

h1173 conspicuous ⊕; N indistinctly resolved in many sparkling ∗...; even around N the neby strongly mottled with evident traces of r, and some ∗... clearly seen. Is there a satellite neb immediately sf the nuclear part? Probably part of the *R* spiral, the object being one of R^s most renowned spiral neb.

GC.2840; h1176; h1177; h1180; GC.2849.

GC.2840 12^s p *h1176;* sev S *... nr. Only once ill seen, no position obtained.
h1176 momentarily E to s; brightest part Rd=1′ in the decl-circle (by measure); more than one * in M.
h1177 ab $^2/_3'$; sev *... in M?
h1180 r; sev star-points glimmering through the neby.
GC.2849 obs as nova 1864 Apr. 28; * 7^m immediately np makes the obs troublesome; neby E nr in the par.

h1178 and the other Neb in the vicinity.

The catalogue in Phil. Trans. 1833 has here for the epoch 1865,0

h1178 (Sweep 117) . . $12^h12^m,5$; +6°5′,3; 1st of 5
h1183 (— 238) . . $12^h13^m,1$; +6°5′,3; B; L; E.
h1187 (— 238) . . $12^h13^m,5$; +6°7′,7; vB; vL; Rd; 3 more seen.
h1189 (— 117) . . $12^h13^m,9$; +6°4′,3; vF; S.
h1190 (— 117) . . $12^h14^m,0$; +6°4′,7; vB; Rd; central of 4.
h1194 (— 117) . . $12^h14^m,4$; +6°8′,5; F; lE.

These positions notoriously nowise agree with the heavens and are hardly to be explained, without supposing errors in the given numbers. The conjecture of *Sf* seems then v happy; and I the more think this to be the case, as I have found a vvF neb, which premising certain conditions can be indentified with *h1190.*

If in the p list 0,8 minute be subtracted from the AR of *h1189, 1190, 1194,* and the descriptions of the two neb *h1189* and *h1190* be exchanged; the thus corrected positions will be, as follows:

h1178 (S 117) . . $12^h12^m,5$; +6°5′,3; 1st of 5
h1183 (S 238) . . $12^h13^m,1$; +6°5′,3; B; L; E.
h1189 (S 117) . . $12^h13^m,1$; +6°4′,3; vB; Rd.
h1190 (S 117) . . $12^h13^m,2$; +6°4′,7; vF
h1187 (S 238) . . $12^h13^m,5$; +6°7′,7; vB
h1194 (S 117) . . $12^h13^m,6$; +6°8,5; B; pL.

The corrections here employed being supposed right, it is evident, that *h1183* and *h1187* (S.238) are respectively identical with *h1189* and *h1194* (S.117); *h* has accordingly, under this supposition, not given positions for 2 B neb in this list, but has yet seen and intimated them ("trifolium"!), and they have first been determinated by *Sf* and *D'A.*

The places 1865,0 of all neb now existing in this region are then according to the Upsala-obs:

h1178 . . . . . $12^h12^m,5$; +6°7′,5; F
h1178a . . . . . $12^h12^m,9$; +6°2′,0; pB } seen by h, but no positions given.
h1178b . . . . . $12^h12^m,9$; +6°12′,8; pB }
h1183=(h1189) . $12^h13^m,0$; +6°5′,6; pB
h1190 . . . . . $12^h13^m,2$; +6°5′,6; vvF; vvS.
h1187(=h1194) . $12^h13^m,5$; +6°8′,1; B.

The 3 neb *h1178*a, *h1178*b and *h1187* make, as *Sf* remarks, v conspicuously a "trifolium", in M of which *h1183* is situated. That *h* in the general notice "the 1st of 5" (counted from *h1178*) should have overlooked the insignificant neb *h1190*, is easily conceivable.

h1178: * 13^m nf.
h1178a Sr than *h1183*, but of the same B.
h1178b not inconsiderably Br than *h1178*, but Fr than *h1183*.
h1183 distinctly rM; place that of the brightest * 12^m13 in M.
h1190 vvF; seen 1865 Apr. 24, 1866 March 15 and Apr. 12, and ought therefore to be beyond doubt, though sky was always tolerably bad. Neb is indicated as vS but not eF; yet no precise position obtained, as the air not sufficiently good.

Rem. Of these neb, *h1187* is the brightest, *h1183* al Fr than it yet Br than the two equal *h1178a* and *1178b; h1178* again is considerably Fr than these 2 neb, yet Br than the vS *h1190*.

h1186 vG and perhaps rN. Quite inexplicable, that my obs passages of this well def neb accord relatively ill!
III.300 is nf *h1188;* pretty conspicuous but not yet observed; * 12^m13 nr is troublesome.

Rem. *h1188* not more than half the size of *h1168*, but much the Br and nr equal to *h1185; h1186* incomparably the brightest of the 4: *h1168*, *h1185*, *h1186*, *h1188* — and *h1185* the greatest of them.

h1196: *a 10^m11 nr, np, which is v troublesome; otherwise object pretty B. Not obs by *D'A.*
h1202 has an irreg and probably not compact N; sev *... 12^n13 and Fr seen in the neby; the brightest sp N. As sky not v favourable, nothing more to be decided on the duplicity and form of the neb.
GC.2884. Not a trace of it discernible.

h1204 E np to sf, ab 3′ lg and $^1/_3$′ br; rM? * 10^m at n end of neby, faint * s from N.

h1201 remarkably Br than *h1178*, but considerably Fr than the other B neb in the region of this neb.
GC.2892 nr equal to and of the same brightness as *h1228*.

h1212, as seen, hardly Gr than $^1/_2$′.
h1221 remarkably Br than the p; E sp to nf in pos ab=45°; its stell C irreg — consisting of 2 *... ?

h1213 eF and diffic; the faintest and smallest of the 3; neb p 2 *... 11^m n and 11^m12 s. Atmospheric state tolerably infavourable. Is neb var?
h1215: diam $^1/_2$′; considerably Br than *h1213*, and I think it quite natural, that it should be seen in the *Mannheim*-refractor (*Sf*), although that seems inconceivable with respect to *h1213*. Comp *h* and *D'A! h1215* has * 11^m12 sp (the same noticed at *h1213).*
h1222: diam ab $^1/_2$′; *a 11^m12 sf.

h1228: diam ab $^1/_2{}'$; * 12^m np; * 9^m nr on the decl-circle of neb, 2′ à 3′ n; * *Sf* (124) sf.

h1232 rN 4″ à 5″; diam $^1/_2{}'$ à $^2/_3{}'$; bet 2 *... 12^m np and 11^m sf.
Rem. *III.97* not inserted in my working-list and not seen.

h1237; h1250: h1244; GC.2965; h1253; h1267.

h1237 bet 2^d and 3^d order of B; G N with evident traces of r; v ill seen, the sky being much too light.
h1250. Identification according to *Sf* and *D'A*. Neb has perhaps sev *... in M? Sky much too light.
h1244 ab $^1/_3{}'$ br in the decl-circle and ab 1′ lg in the par; Fr than *h1250;* C condens large and indistinct. Neb well seen 1865 Febr. 27; I then noticed no traces of r, which on an other occasion I thought that I perceived.
GC.2965 not seen; sky too light.
h1253 much resembles *h1237*, and is hardly Fr than this neb; N of *h1253* is even Br and distincter; no C r remarked — sky too light.
h1267 hardly Fr than *h1244;* at least momentarily seen E in pos ab=30°; *12^m np.
Rem. This region, so rich in neb, is not yet sufficiently examined.

h1239: * 9^m,5 sf.

h1242 am the faintest of the 2^d order; *a 9^m,7 sf and *b 12^m nf; neby at least 2′ and l E in direction of *b inv; neby also momentarily seems to extend to *a.
h1251 l E np to sf.

h1258 only once seen under v unfavourable circumstances. Neby much E np to sf; N indistinct, diam 2″ à 3″.

h1268; h1272; h1276.

h1268 once seen; v diffic, as sky tolerably light; Gr than the f neb *h1272;* * 13^m14 nf.
h1272 rather more diffic than the p neb, obs on same occasion; * 12^m np troublesome.
h1276: diam $^1/_2{}'$; B and easily obs.

h1274; h1275; h1287; h1290.

h1274: diam ab $^1/_3{}'$; equal to *h1148:* of the same B, but a little Sr.
h1275 Gr than *h1274.*
h1287 remarkably Fr than *h1290;* * 10^m nf.
h1290: diam ab $^2/_3{}'$ E sp to nf; a little Fr than *II.114* and *115*, but remarkably Br than *h1287;* N G and not stell.

h1286: diam did not appear Gr than $^1/_2{}'$; sky v light; N stell.

h1288; II.630; h1295.

h1288: diam ab $^2/_3'$; sev *... visible in N; a little Fr than *II.114* and *115*.
II.630: diam $^1/_2'$; E sp to nf; considerably F and diffic; neby milky with a dull N 4″ à 5″.
h1295 not much Fr than *h1288;* diam hardly $^1/_2'$; neby and the G N both E in the par; C r suspected but not seen. *GC* really has "r".

h1292; Struve Nova; h1293; h1294; h1305.

h1292: diam $^1/_3'$; am the brightest of the 4th order, and Br than *h1305*, which belongs to the normal neb of that order; 2 *... 11^m12 f n and s.
Struve Nova sp *h1294*: $\Delta\alpha = 16^s$, $\Delta\delta = 28''$; not eF, but * 12^m, just sp, makes it v diffic.
h1293 E nr in the decl-circle; N, G and dull, has a nr planetary appearance; Fr than *h1292* and *h1305*. F neb 2′ n 3^s f?
h1294 evidently ⊕; diam 3′ à 4′ at least, but extreme parts of the neby vF; brightest C part by measure = 80″; N irreg and resolved in sev B stell points, which together make the impression of * 10^n or 9^m10; the remaining C part also momentarily resolved into e fine *... mingled with neby; *a 11^m f exactly in the par.
h1305 pretty B, but *... 10^m11 sf and nf v troublesome.

Rem. Neby suspected at sev points of this not yet sufficiently explored region. *1871 in April*: it was remarked, that both the neb *h1292* and *h1293* were milky and of a blueish colour without character of r; *1865 in April*: *h2193* was on the contrary suspected to be ⊕.

II.114; II.115; II.116.

II.114. Neby vF and al Gr than 1′; N 2″, distinctly but not steadily r with magn p 320; makes an isosc *Δ* with 2 *... s, 11^m12 p and 11^m (double) f.
II.115 much of the same character as *II.114;* perhaps not quite so B but al Gr; N 3″ distinctly but not steadily r with magn p 320. As N of *II.114* was resolved into sparkling stell points, N of *II.115* divided into nebs *... or dull planetary knots.
II.116 am the brightest of the 5th order; diam $^1/_2'$ à $^2/_3'$; E sp to nf; the large N has no traces of r.

Rem. Respecting the r of these objects comp *SN!*

h1296; h1298; h1301.

h1296: diam ab $^1/_3'$; al Fr than *h1276*.
h1298 remarkably Br than *h1296;* by measure diam = 41″; evidently ⊕: sev stell points sparkling through the neby particularly in its n part.
h1301 the brightest of these 3 neb; ⊕ with many stell points sparkling through the neby around N, looks v indistinct; by measure diam = 103.″

h1307 remarkably Fr than *h1242;* diam ab $^2/_3'$; evident traces of C r.

h 1312 E np to sf; * 11^m12 in sf part.

h 1322 estimated much Fr and more diffic April 1871 than in nr equal circumstances Apr. 1865? F * p in the par and F *... f.

h 1329 E 2′ or more np to sf, and ab ½′ br; brightest part by measure in decl-circle =60″; N irreg and indistinct with evident traces of r; * 12^m s nr on the decl-circle of the N.

h 1343 E nr in direction of decl.

h 1349 E sp to nf in estimated pos = 60°; a trifle Fr than *h 1343*.

h 1348. G N r? Neby generally Rd and Sr than 1′, but momentarily I think it much extended particularly sf?

h 1345 E sp to nf; *a 11^m nf = star of comparison n:o 294.

h 1356; h 1358; h 1359.

h 1356 Fr than *h 1368;* not Sr than ½′; consisting of a v much condens C part, surrounded by Fr neby, both l E sp to nf; momentarily radiations even in other directions; obviously though only momentarily r M; *a 11^m12 np, *b 12^m sf. Neb v well seen 1869 April 12.

h 1358 al Sr and Fr than the f *h 1359*, with which it forms a dneb, the neby of the two neb momentarily running together; *h 1358* E sp to nf in pos = 70° à 80°; v fine *... visible in the neby, but, as the neby is milky with a blueish fosforescent light, its general appearance indicates no r.

h 1359 al Br, Gr and more distinct than the p neb; E sp to nf in estimated pos=30°.

Rem. These neb obs under v favourable circumstances. S d* 12^m13 (dist of components only few seconds of arc) p in the par; an other d* (dist of components=1′) f the neb: np component *a 11^m12, sf 12^m.

h 1357 E 12′ à 14′, np to sf; pos by measure:

1870 Apr. 19 . . . . 134°,5 (1)
1871 March 25 . . 137 ,3 (3)

Neby ½′ br at N, from which it is rapidly narrowing towards the ends; N also E in the axis of the neby, indistinct and not without indications of r; N surrounded by a pretty condens neby, beyond which the light rapidly decreases; *a 12^m13 nf the N.

Rem. I have seen that neb under v favourable circumstances: neb consists of two in form and size quite equal parallel branches, separated by a narrow dark canal; sp branch vB, the nf eF. The n branch certainly eF and only momentarily visible, but then always radiating to the same length as the s branch. N situated at the n edge of the sp branch and consequently just at the border of the dark canal. This canal not quite free from neby. The appearance nr agreeing with the diagr of *R.*

M.90: C part strongly condens and nr circular, diam $^1/_2'$ à $^2/_3'$; extreme parts of neby eF and not distinctly seen, although sky v perfect. E at least 6′ à 7′ sp to nf in estimated pos = 20°; greatest breadth at N not exceeding 1′. The C * looks like a real N: neb in its totality not to be estimated Fr than of the 3d order.
GC.3124 not looked for.

h1361 E np to sf in estimated pos = 135°; neby quite certainly 3′ lg and $^1/_3'$ br at N; N is G and E in the axis of the neby, its r evident.

h1368 vB N distinctly resolved.

h1378 pretty certain indications of C r; S *A* of *... sf.
h1383 Sr but not considerably Fr than the p neb; N has evident traces of r; *a11m np.

h1384: diam $^1/_2'$; am the brightest neb of the 4th order; r suspected but not ascertained; *a 10m11 f in the par.

h1386; h1402; h1405; h1408.

h1386 much of the same character as *h1368*, but r more conspicuous: sev distinct stell points both in the N and around it. Is the nf * 11m nebs?
h1402 conspicuously biN, or a dneb np and sf; components however probably nr in the same par, as, at the measure in decl, the micrometer-wire seems to occult both at once. Sky not sufficiently favourable. Place that of the seemingly brightest point and obs agreeing well; *a 10m np.

Rem. *h* describes neb as "F, R"; and it would consequently seem the more diffic to identify it with the neb here obs, which is B and evidently E, as *h* says nothing even of the duplicity, which nevertheless was very conspicuous and easily visible, although the atmospheric circumstances not v favourable. The duplicity is also remarked by *R*: "dneb; F neby connects them". On the other hand, as no other neb could be found nr, it is quite certain, that the object obs by me must be identical with *h1402*. This neb corresponds consequently to the 2: *GC.3171* and *GC.3172*. It is remarkable, that *D'A* 3 times of 5 has seen the neb Rd and even "circularis", and that the duplicity should have escaped his practised eye.
h1405 according to my observation am the brightest neb Cl. II, and should consequently be an easy object for the *Mannheim*-Refractor — yet it was hardly seen by *Sf?* Many *... twinkling through the neby, and r evident; C condens uncertain.
h1408 follows *h1405* s and so nr, that the neby of the 2 objects runs together; *h1408*, much of the same character as *h1368* and *h1386*, is yet much Br; N G and resolved.

h1399 bet 2 *... 12m sf, 11m np; diam no doubt Gr than 2′; N G and evidently r.

h1404 seen once in v unfavourable circumstances; E np to sf in estimated pos ab= 45°; perhaps 2′ lg, at N $^1/_2'$ br; M vB; N no doubt r in Sr *...; 2 *... np 11m12 and 10m11.

h 1414; h 1415. This dneb only once seen, sky much too light. The sp *h 1414* considerably E sp to nf; condens nr s end; neby milky and quite missing character of r (comp *SN!*). Neby of *h 1414* runs together with the S and Fr *h 1415*, which is nf and al oval. The diagr *Ph Tr 33* seems to represent nr right the relativ positions of the axes of E.

h 1417 hardly Gr than $^1/_3'$; perhaps a trifle Br than *h 1462;* both these neb Br than *h 1356.*

h 1419 al Gr than 1′ and inconsiderably E nr in decl; r indubious, even in twilight *... seen twinkling in the G N; *a 10^m11 sp.

h 1441 E sp to nf 4′ to 5′. Pos of axis of E by measure:
1874 April 8 . . . . 26°,1 (2).

h 1462; h 1466.

h 1462: diam perhaps $^2/_3'$, Gr than *h 1417;* probably ⊕, but r not constated; *a 10^m11 sp.
h 1466 few seconds br, but at least 3′ à 4′ lg; E sp to nf,
pos by measure = 30° (4) . . . 1869 Apr. 12.
S * by glimpses seen n from the stell N; traces of r around the N not quite missing; *a 9^nsp, *b 10^m11 s, *c 9^m10 sf.

Rem. Descript. *Ph. Tr. 61* has "R" — evidently a missprint for "r".

h 1486 once seen in tolerably favourable circumstances. Neby ab 4′ lg and 2′ br; E np to sf in pos ab = 100° à 110°. The N, some seconds in diam, r distinctly into sev e fine *...; along the n side of N a cavity or dark channel visible at first sight, although this feature of the neb before unknown to me.

Rem. The diagr *Ph. Tr. 33* pretty well represents the neb, although the relativ length al too G according to my obs. Description *Ph. Tr. 61* not quite clear: "dark and large spot at one side, around which is a close cl of well defined little * s". The "close cl of *..." is evidently identical with the resolved N, s from the dark spot — and "around" is possibly a missprint for "along". *SN:* "Solutio non improbabilis partim in ipso centro, partim in ea regione, quæ nucleum ad sept. insequitur". Astonishing, that *D'A* neither has distinctly seen the central resolvability, nor remarked the cavity; *h* has also: "resolvable but not resolved".

h 1498 E nr in the par: 2′ lg and $^1/_3'$ br; N in p part; * 11^m12 immediately np the neb.

h 1547 E sp to nf — ab 30° with the par. Obs diffic through thick fog!

h 1622; h 1623. This interesting couple of neb only once obs, and as sky then v bad, no comparison with the diagr of *h* or *R* possible; *h 1622* has *a 12^m sp.

h 1633 in vain searched for 1870 Apr. 20; circumstances unfavourable.

h1701; h1703; h1705.

h1701 E sp to nf nr in the decl-circle; considerably F; * 13^m sp, 2 *... 11^m nf.

h1703 irreg Rd, lE np to sf; diam ab 1′ or 2′; am the brightest of the 3^d order; C part Rd and vB with B C * and sev Sr *... glimmering around. Nebs rays, particularly in direction sp from M, momentarily seen. F *... nr. Neb obs 2 nights, but sky not v favourable.

h1705 am the faintest of the 4th order; sky v unfavourable and S *... nr troublesome. Neby seems to be ab ½′; eF *... momentarily glimmering through it.

Rem. *R* Novæ evidently identical with respectively *h1701* and *h1705* Comp *SN!*

h1778; h1779; GC.3848.

h1778 E np to sf; * 13^m n, d* p.

h1779: *a 11^m sf, * 12^m13 sp.

GC.3848 S and Rd; tolerably conspicuous even in less favourable circumstances; not yet determined.

h1874 quite of the same character as *h1132;* both neb am the faintest of the 3^d order; E np to sf,

pos by measure = 169°,3 . . . 1870 Apr. 21.

Neby at least 8′ lg, ½′ br at N; N is G and indistinct with traces of r; s part of the neby the Br; * 13^m in the neby ab 2′ s from N.

h1896 in the middle of a regular quadrangle of 4 *...; has indubitable traces of C r; v many *... in this region.

h1897 not observed.

II.542: nebs rays indicate E sp to nf? N is pretty constantly partially r and has a * at sp edge. Neb seen once in pretty favourable circumstances, but sky too light.

GC.4046: too F for observation.

II.541; III.511; GC.4043; GC.4044; h1901; h1902.

II.541 nr equal to *h1902;* G N distinctly r; * 10^m11 sp.

III.511 no doubt Cl.I; looks like a * out of focus; diam of the nebs atmosphere not exceeding ⅙′; * 12^m f in the par.

GC.4043 and *GC.4044* not seen.

h1901 am the brightest of the 3^d order; no conspicuous deviation from the Rd type; diam 1′ à 2′. Neb has a G irreg N with planetary C, whose diam 1″. Traces of r particularly in N; yet r here not found more conspicuous than in *II.541, h1902, II.542;* *a 11^m12 np; *b 12^m inv s from N. Astonishing, that *D'A* should have been the first to remark the conspicuous *b!

h1902 nr equal to *II.541,* both am the brightest of the 4th order and at least in fine air not considerably Fr than *III.511;* N of *h1902* stell; sev *... glimmering around it: r consequently indubitable.

II.751; II.752 seen in favourable circumstances, yet diffic although not eF; *II.751* the Br, diam not Gr than ¼′; diam of *II.752* ab 1′.

Rem. Position and description not well according with *GC.*

II.647 v conspicuous, but *a 9^m,4 sf v troublesome; F * bet *a and the neb.
h1961 al Br than *II.647;* E sp to nf; C condens uncertain, probably ⊕.

Rem. These neb erroneously indicated in *H*^{s} catalogue; *GC* agrees with the heavens, yet nothing remarked of the error in *Auwers'* cat. (*H*)?

II.753 has strongly condens M, nr to a N, in which sev *... twinkling; 2 of these, more steadily seen, make it look biN; Neb E nr in the decl-circle; sev *... in the neighbourhood: *a 11^m np, * 12^m sp, etc.

h1966 nr on the straight line bet 2 *... 12^n sp and nf.

h1967 forms trap with 3 *... n: * 10^n,5 p, 2 *... 11^m f — the first and second * on the same par, 3′ n from neb; diff of AR bet these 2 *... 27^s,5.

Rem. According to *h:* AR of the neb 48^s Gr. No other neb seen!

h1970

This interesting gaseous neb obs 6 times, but sky always too light. The characteristic appearance of the neb remarkably variable; I consequently here give the immediate descriptions from the different nights of observation.

1864 Aug. 4 and 13. Sky pretty serene; fine but unsteady images; magn p 210.

Neb is planetary, m pretty sharply cut off; neb however v much mottled or granulate, and its light having a peculiarly flickering appearance or rather a kind of rythmical variation, which makes neb look, as if it continually contracted and dilated. Al n from the geometr C is a rather dull nuclear condens momentarily visible, to which place refers. Neb vl E in the par;

diam. in decl-circle measured = 9″,5 (3)

Measures agree well.

1865 Aug. 11. State of the air rather favourable; images pretty fine and steady; magn p 320.

Neb this night v much reminds me of *h2000,* which was not at all the case last year; the usually so characteristic planetary appearance this night not v striking, as the light near the margine vF and edges v indistinct, but the C condens on the contrary v sudden to a conspicuous B N al n from geometr C. Light of neb steady and not granulated; strongly blueish.

Diam in decl-circle measured = 9″,5 (4).

Measures v diffic; the nr agreement with the last year only to be regarded as accidental.

1868 Aug. 11. Sky perfect; images v beautiful and steady; magn p 210, 320, 480.

Neb appears quite as an e compact ⊕; particularly in the n part of the neb sev discret stell points 12^m distinctly seen, remarkably B twinkling on the white-blueish

dull nebs ground. A vF haze appears all around the neb, radiating in the direction of the diurnal motion; its light also granulate as if consisting of e fine stars.

The peculiar contracting and dilating motion in the neby remarked 1864 this night again v conspicuous. The stronger magnifying powers show the neb inconsiderably distincter, but it is rather astonishing, that the neb looks so v nr equally B with the different eye-pieces.

Diam in decl-circle measured = 12″,6 (1).

1870 Juli 30. Sky pretty clear; images not v perfect; 320 magn p.

Neb al Br than * 9^m; planetary and of a strongly blueish light with no determinate C condens; edges (particularly the s) v diffuse. Neb has sev stell points twinkling in the neby mostly nr to the m and particularly at the n m. Neb vl E np to sf,

pos by measure = 100° (3).

Diam of the neb measured

in decl-circle = 10″,3 (4),

in par (passages) = 9″,6 (7).

1870 Aug. 6. Sky tolerably good; images v unsteady; magn p 320. Neb has much the same appearance as July 30; vl E np to sf,

pos by measure = 99°,5 (3).

Diam of the neb measured:

in decl-circle = 12″.7 (4)

in par (passages) = 11″,6 (7).

Rem. Neb has *a 9^m sf; *b 12^m np; *c 11^m12 sf; *12^m13 immediately nf *c.

h1973 in vain searched for 1866 Sept. 16; suspected in loco 1866 Oct. 5 — eeF?

M.92. A monographic study of this fine object is in progress.
Nova = **Sf.* 203, nf *M.92.* Does that * consist of e fine *... with a vS neby?

IV.37.

This gaseous neb observed 5 times; descriptions from the different evenings were, as follows.

1863 Oct. 1. Sky unfavourable; magn p 210.

Neb planetary and nr circular; strongly granulate with many glimmering stell points; edges well def. Position refers to the estimated geometr C.

1864 Oct. 14. Air tolerably good, moonlight; magn p 210 and 320.

Neb has many glimmering stell points particularly nearer the edges; no C condens. Position refers to the estimated geometr C.

Diam in decl-circle measured = 14″,2 (3).

1869 Feb. 23. Sky v bad, strong moonlight; magn p 320.

Neb planetary, without C condens, v much granulated with many glimmering stell points — particularly, when seen with low magnifying powers. Of H^s "B Rd centre" or $D'A^s$ C * 11^m no traces; on the contrary neb sev times seen *Fr in M* — annular? Neb al elliptic sp to nf,

pos by measure = 208°,6.

The m generally well defined, yet diffuse nebs rays at the ends and in direction of the longer axis.

Diam in decl-circle by measure = 18″,1 (5)
Diam in par (passages) = 16″,8 (10)

Rem. At the passages this night pos-circle evidently 10° in error; the obs $\Delta\alpha$ consequently corrected to this hypothesis.

1869 Febr. 26. Sky tolerably good, strong moonlight; magn p 160, 210, 320, 480, 640.

Neb this night long and scrupulously studied. D'A^s C * at length seen with 640 times magn p, and then even well seen with a magn p of only 320.

The * however only seen by glimpses, and thus much more diffic, than was to be supposed according to D'A^s description; * is 2″, has a dull planetary look, and through its white colour momentarily strongly contrasts with the blueish nebs ground.

Greatest condens of neb in all cases nearer the m, where many glimmering stell points — particularly n. I can thus the less get rid of the conviction, that neb really is annular, as in the moments, when D'A^s * is well seen, the nr surrounding part of the neby always appears considerably Fr than usually, although in the more condens part nr the m no var of B could be perceived. Neb this night much reminds me of *h 2050.*

Diam in par (passages) = 15″,3 (5).

1869 March 28. Sky not v favourable, strong moonlight; sev eye-pieces tried, not more than 320 times magn p to be employed.

Neb seen as before. *D'A^s* central * sometimes remarkably distinct; annular form hardly to be doubted — bet the central * and the n Br m a dark spot is for instance quite certain.

Rem. Neb was never seen in v favourable circumstances, and moon always troublesome. Neb has sev *... nr; * *Sf. 206* (9^m,2) np; *a 11^m nf; *b 12^m np.

II. 902 not eF; contains sev *..., but does not look as if r. The region rich in *...: * 12^m sp, 2 *... sf. Resulting AR of the neb deviates remarkably from *H^s* position.

h 2000 strongly blueish; has a B but not v well def N, E in decl — perhaps biN? Agreement bet different obs perfect. Neb compared with 4 neighbooring *...: *a 9^m nf, *b 10^{m}11 sf, *c 13^m nf, *d 12^m np.

II. 907 certainly not Br than Cl. III, although not eF; r suspected, looks like a ⊕; B *... nr troublesome. Resulting AR remarkably deviates from *H^s* position.

h 2023.

1865 Aug. 12. Sky tolerable; magn p 320.

Neb blueish and strongly granulate; n part Br. Pos of the axes of the outer limiting ellipsis by measure:

min. axe (np to sf) pos = 148°,9 (3),
maj. axe (sp to nf) pos = 57°,9 (3).

Diam of this ellipse measured in decl-circle = 58″,1 (5).

1865 Aug. 13. Sky tolerable; magn p 320.

The n part of the ring brightest; strongly granulate, with sev not vF * .. seen by momentary glimpses: 2 or 3 *... in the np part and two in the sf part of the ring distinctly seen; at the interior m of the ring 2 *... visible in the sp, and one in the sf part. *Hahns* * — al np the geometr C? — momentarily visible: dull and planetary. All these *... only so momentarily seen, that to adjust the micrometer-wire upon them was quite impossible. The nf part of the ring very diffuse with nebs radiations in the direction of the longer axis, which seem momentarily almost to destroy the annular form. The C vacuity not inconsiderably nebs. The diam of the outer ellipsis measured:

in decl-circle = 58″,8 (4).

Breadth of the ring measured = 11″,7.

1867 Aug. 6. Sky not v favourable; magn p 210, 320, 480.

Hahns C * distinctly seen without effort. The annulus strongly granulate; its nf end only momentarily well def.

1870 Aug. 8. Sky not particularly favourable and in all cases too light for that object, which is remarkably less B than for instance *h 1970;* magn p 320.

Twinkling stell points at the n outer m, which is best def. The diam of the outer ellipsis obtained through comparison of the edges with the f *a:

diam in decl-circle = 56″,5 (5),

diam in par (pass) = 59″7 (12).

Immediate measures give:

diam in decl-circle = 56″,1 (5).

Rem. *h 2023* never seen on sufficiently dark ground, wherefore of the sev F* ... in the vicinity only the well known *a f in par obs. The diff in decl, bet that * and the geometr C of the neb, was 1867 determined by means of a great number of measures in the following manner. The micrometer-thread was so placed, that its dist from the fixed thread was a little less than the diam in decl of the neb, and this thread-system was then symetrically adjusted on the annulus. If then c be the reading of the scale for the coincidence of the threads, and $\alpha - c =$ scale-dist bet the threads; the reading of the scale corresponding to the C of the annulus will be $\frac{1}{2}(c+\alpha)$. If then the moveable thread be adjusted upon *a, the refractor being left unmoved, and the reading of the scale be $= \beta$; the diff in decl sought will be

$$\beta - \frac{1}{2}(c+\alpha).$$

I had thought, that this manner of pointing the C of the neb by means of 2 threads would be much exacter than that of immediately adjusting a single thread upon it. This supposition was however not confirmed, as the probable error in one single measure in both cases is v nr the same = 0″,8. The results of the two methods of measuring meanwhile fully confirm each other, which gives an empirical *proof of the great exactitude, with which a sigle thread can be adjusted upon the geometr C of even a tolerably great discoid neb.*

GC.4473 (= *Hind Nova*) v diffic to obs, as the C condens none or v uncertain. Sev *... glimmering in the neby — r?

Rem. The apparent decl-diff bet the neb and comp* 1863 Aug. 20 was obtained = 96″,3; but, as an error in the reading of 1 whole part of the scale here is obvious, the above number is in the preceding tab augmented with 17″,23.

h 2037 has no C condens; its sf part brightest. Has neb a dark spot in its np part, or is the neb annular? Sev *... twinkling in the neby; * 13^m at nf edge. Diam in decl-circle by measure

1866 Sept. 1 . . . 82″.
1866 Sept. 15 . . . 99″.

Rem. I have since seen, that *R* here really has "annular."

h 2043 does not appear to be r, but sev *... inv: *a 12^m in nf part is the brightest, *b 12^m13 in sp part; the remaining inv *... not Br than 13^m, and only seen by glimpses. Immediately n from neb * Fr than 13^m.

h 2050 planetary with no C condens; well def Rd * in M; * 10^m11 sp.

Rem. *1865 Sept. 30:* neb some moments seen in verily perfect air, but obs soon interrupted by clouds. Evident traces of the annular form then seen, as the neby s from the C sev times momentarily disappeared. A curiously looking phenomenon!

Diam in decl-circle measured = 23″,3 (3).

h 2062 is an insignificant nebs object am the *... of the milky-way. Neb is not vF indeed, but can yet easily be overlooked (*R!*) am that multitude of *.... Neb has hardly any real C condens, but the M is however considerably Br than the extreme quite evanescent parts. S *... projected on the neby: place that of *a 12^m in p part. 2 *.., nr on the same decl-circle, p the neb immediately: * 11^m12 n, * 11^m s.

h 2072 annular neb, seen twice in tolerably unfavourable circumstances; yet the annular form quite evident and steadily seen; sev S *... by glimpses seen in the nebs ring. Position refers to the geometr C of the figure.

h 2075 in the p part of a quadrangle of 4 *... A, B, C, D. Neb in most cases has a nr planetary appearance; a stell C however momentarily glimmering through the neby, to which the place refers. 1866 Oct. 6 neb al while seen *in the most exquisite air* as a v fine ⊕, momentarily almost entirely resolved into e closely packed *... — elegant phenomenon! —; all *... of nr the same B, only two or three, just s from geometr C, al Br. Diam of neb in decl-circle by measure:

1866 Oct. 6 . . . 27″
1874 Aug. 24 . . . 38″ (4)
1874 Aug. 26 . . . 39″ (3)

Nova VII; h2081.

Nova VII, ½ min p *h2081*, first seen 1865 Sept 14. Neb is nr stell, its nebs atmosphere scarcely perceptible; yet looks quite differently from the surrounding *..., and has a peculiarly flickering light; forms an elongated *A* with 2 *... n : * $9^m,5$ p, * 10^m f.

Rem. Agreement of this object with I.103 in AR not bad, but difference in declconsiderable.

h2081 almost entirely r ⊕; *... v fine and gradually more densely crowded toward the geometr C, nr which the cl nevertheless momentarily seems to have a cavity or to divide into two parts, separated by a dark channel nr in direction of decl; C condens consequently v uncertain and sev *... of the same B twinkling in the same. The extreme *... of the gr extend to sp comp* n:o 378.

Rem. *R* has not obs this object v closely, and *D'A* says nothing of the dark spot nr M or C division of the cl.

h2086 trip* with a F nebs atmosphere. Place refers to the brightest *a 10^m11 in M; of the other *...: p 13^m14, f 13^m.

h2097 has a v uncertain C condens with sev S *... 13^m, by turns glimmering through the neby; r indubitable; place refers to M of the brightest part of neb. Ab 1′ s from neb 2 S *... 12^m13.

h2102 has *a $9^m,5$ immediately np.

III.209 not eF, looks like a ⊕, sev glimmering *... in the neby. Region rich in *...: a rectang *A* of *... f: * 10^m11 n, 2 *... 12^m s; these *... followed by an other Sr *A* of same form, which has its accute angle in opposite direction, *... 11^m12 à 12^m.

h2112 has a v fine * nf, *a $10^m,3$ f nr in the par of neb — this * not remarked by *h!*

h2121 has *a 11^m12 immediately sf; vF * bet *a and neb, and an other * sf *a; *b 11^m also sf neb, and Fr * p the *b.

h2132 a S ⊕, a trifle Br than *III.209;* r here distincter than in *III.209*; * 12^m13 at np m, ab so much n from geometr C, as a f * 10^m s from it. The * noticed at the n m has nothing peculiarly characteristic in its appearance, as sev *... inv in the neby are inconsiderably Fr than it. This object did not well bear a higher magn p than 160, although sky v fine.

h2135; *GC.4720.*

h2135 one of the faintest objects obs with the Upsala-refractor, sky however tolerably light from aurora. Measures not so v diffic! *... 13^m twinkling in the neby; 3 *... 12^m or 12^m13 sf nr in the par, * 12^m13 n, sev more *... nr.

GC.4720=D'A Nova f *h2135* nr in the par ab 2,4 min; neb bet 2 *... 12^m13, nr in the same decl-circle; * 10^m p. This neb is indeed eF, and its place consequently not to be determined with the Upsala-refractor, unless in a most glorious condition of the atmosphere.

h2139 has *a 12^m sp.

h2149 a conspicuous ⊕; 1867 Aug. 28 seen alm intirely r. The G N is sometimes planetary but momentarily r into *...; the remaining part of neb consists of e fine *..., which the most insignificant haze in the air is sufficient to exstinguish, or to intermix to a mere indistinct eF nebs mass. This circumstance easily makes the object to look v differently on different occasions, and consequently can be an apparent reason for suspecting a variability — probably not real? Neb has * 9^m,4 nf, *a 9^m,7 sf.

h2160: n from neb 3 *... forming a rather regular quadrangle with it: *a 11^m, *b 11^m12, *c 12^m13.

h2169 has the appearance of a ⊕, yet no traces of r seen; nr equal to *III. 209*, yet Br; * 19°4965 (6^m,3) sp is v troublesome; * 12^m just sp, * 10^m p in the par.

h2170: many *... distinctly glimmering through the neby; light v uncertain, as is commonly the case with theese S F ⊕; momentarily E sp to nf.

h2171 is not eF, and at any rate Br than *h2135;* * 12^m13 nf. Best seen with a magn p of 160.

Nova VIII; h2172; Nova IX; h2174; R Nova C; R Nova E; R Nova D

are all, with exception of the B *h2172*, eF, and only to be seen in v favourable circumstances.

Rem. Of *R* 5 Novæ the three C, E, D seen by me; *R* Nova B is, as already remarked in *SN*, identical with *h2174*. The 2 Novæ seen by me, not noticed in *Ph Tr 61.*

h2173 2′ à 3′ E exactly in direction of a sf * 10^m,5, which it however does not quite reach; neb ⅓′ br. Place determinable with the greatest exactitude.

h2175 E in the par to 1′, ⅓′ br; sev vF *... f each other in the axis of E; no certain condens.

h2183 has *a 10^m11 np.

h2184 has * 12^m nf.

Rem. Neb seen twice in unfavourable circumstances; the remaining F neb in that region not distinctly seen, but neby suspected in sev places.

h2189: M distinctly resolved into *... 12^m13; * 11^m nf, 2 *... 11^m and 12^m np.

Rem. vF neby suspected in sev places nr, but no time for verification.

h2199; h2202; h2202a; h2203.

h2199 has sev F *... centrally in direction of the par.

h2202 Fr than *h2203* but Br than *h2202a;* * 8^m, immediately f these neb, is v troublesome.

h2202a = D'A Nova not inserted in *GC.*

h2203 considerably Br than the two p.

II.249: * $10^m,5$ np v troublesome.

h2201: 2 *... 11^m and 11^m12 s; a rectang Δ of *... f the neb.

h2204 isosc Δ with 2 B *...: 8^o4984 (8^m) nf, *a $9^m,5$ sf

h2205 has 3 *... inv: *b 12^m13 nr C and looking like a N; *a 11^m12 and *c 13^m at the ends of neby, which is E in direction bet these two *... Place refers to *b.

h2214 with traces of C r; * 13^m at the m s from N, *a 12^m sp.
h2221 in vain sought for 1867 Nov. 29.

h2215 E sp to nf in pos ab = 45°; * 13^m f.
h2216 E np to sf in measured position = $99^o,5$; *a 11^m12 sf, which the neby reaches; sev S *... in the neby.

h2217 probably sev *... in M? *a 10^m sf.

h2218; h2219; D'A Nova; R Nova.

h2218 in a region rich in *...; 2 B *... p, F * sf.
h2219 (=GC.4912) E sp to nf in pos ab = 45°; *11^m sp.
D'A Nova (=GC.4913) nr in the same decl-circle with *h2219;* * 10^m np, *13^m sp. The B * v troublesome.
R Nova (=GC.4915) ab 1′ n from *h2219* al f, looks nr like *h2218;* not obs, but no doubt observable in favourable circumstances.

Rem. A poor * gr of pretty B *... follows the above neb ab $1\frac{1}{2}$ min; and the whole region f this *gr seems nebs: *a gr of S neb or a considerably extended neby with sev knots?* As yet the sky was not sufficiently dark, and neby vF and indistinct, no decision could be arrived at. This neby independently remarked in the autumns 1867 and 1869, as on the second occasion the elder notice was forgotten. Description and position do not at all agree with *III.181!*

h2221 searched for in vain 1867 Nov. 29; obs by *R*, but not seen by D'A.

h2222; h2223; III.238.

h2222 not eF, yet considerably Fr than *h2223.*
h2223 looks nr like *h2222,* although considerably Br; *a 10^m sf.
III.238 al Br and more easy than *h2222.* F * in the sp part of neby? A file of F *... nr in the same decl-circle f the neb.

GC.4934; D'A Nova; h2229; D'A Nova; h2230; h2231; h2233; GC.4942; GC.4943.

GC.4934 pretty B and distinct.
D'A Nova (= N:o 205 in AN. 1500) pB; Fr than *h2233*, but Br than *h2231.*
h2229 in vain searched for 1865 Oct. 14 and 16. Comp *SN!*
D'A Nova (= N:o 206 in AN. 1500) is not eF.
h2230, *h2233* nr equal and in brightness both of 3d order — the f perhaps a trifle the Br?
h2231 has *a 12m np.
GC.4942 not found 1865 Oct. 16.
GC.4943 al Br than *D'A 206 Nova;* *a 12m sp.

h2232 has traces of C r; neb n in an E *A* of * 11m12 p and 2 *... 11m f.

h2241.

observed six times, but the sky in most cases too light.

1865 July 22. Atmospheric state not unfavourable; even a magn p of 480 could be employed.

Neb is nr circular; m not v well defined; a dark spot in M or rather al s from the geometr C, to which position refers. I did not then know, that the neb had been indicated as annular by other obs; sev sparkling vv fine *... visible in neby.

Diam in decl-circle measured = 24″,2 (2).

Has the nf * 41°.4780 a F nebs atmosphere?

1867 Aug. 1. State of the air pretty favourable; magn p 210.

The cavity seems to be nr centrally situated, its diam ab 1/3 of the neb; many stell points sparkling through the neby of the ring. Neb l E sp to nf.

Diam in decl-circle measured = 24″,8 (4).

The nf * 41°.4780 seems also this evening surrounded by a F nebs atmosphere, which sometimes has nr the appearance of an interference-ring 6″ à 10″ diam, but is sometimes irreg and extended s in decl.; its N, of a dull white-yellow colour, is quite round and looks planetary, and is destitute of the characteristic sparkling appearance of the stars.

Rem. This * reminds me of other *ill defined weak-yellow* *..., which I have occasionally noticed. So for instance the * Sf. 207, the two 30°.3982 and 3987, etc. The last two *... I saw 1865 Sept. 24 v ill def, whereas on the same occasion a red * of the same B in the nearest vicinity (30°.3984) was most perfectly def. These *... deserve to be studied with the spectroscope!

1867 Aug. 4. Decl-diff bet geometr C of the neb and the comp* this night measured with two threads as above explained at *h2023.* The results quite agree with those, obtained by directly adjusting a single thread on the geometrical centre, and the two methods of measurment give even here nr the same exactitude.

1867 Aug. 25. Sky not v clear; magn p 210 and 320. Annular form this night not v distinct; but the partial r v conspicuous.

1869 Febr. 21. Sky not v clear, bright moonlight, and images v unsteady; magn p 210 and 320. Neb annular with partial r, much more conspicuous than in the annular neb of Lyra.

Diam in decl-circle measured = 25″,9.

1869 Aug. 3. Atmospheric state v favourable, but images unsteady; magn p 320 and 480. Neb annular with *... twinkling in the neby. Deviation of the ring from the circular form hardly perceptible.

Diam in decl-circle measured = 20″,4 (4)
Diam in the par (pass) = 21″,7 (7).

Rem. The S *... obs by Vogel in the nearest vicinity I designate with a, b, c.

*a — Neb. C = + 2ˢ,5; + 13″(13ᵐ)
*b — Neb. C = + 4ˢ,0; + 24″(12ᵐ13)
*c — Neb. C = + 8ˢ,0; −135″(12ᵐ13).

Of these *... I never succeeded in seeing *a! I certainly never saw the neb in v favourable circumstances, but it is yet v astonishing, that I never should have perceived a trace of the said *. Is there an error in the position, or is the * var? 1867 Aug. 25 for instance I sought v assiduously but in vain for this *; I then saw an eF *, which really forms d* (not v close) with the *b; this * ab a diam of the neb n from *b. The *a sought for also in vain 1869 Febr. 21 and Aug. 3 — magn p ab 500.

h 2242: * 11°5002 (9ᵐ,5) sp; sev *... glimmering around the C.
GC. 4967 much Fr but Gr than *h 2242;* *a 11ᵐ12 nf.

h 2257: *a 12ᵐ sp; * 16 Pisc. sf.
GC. 4994 not with certainty seen.

h 2264 looks like ⊕ with momentary C r; *a 11ᵐ sf.

h 2265 has a half resolved N of variable appearance; *a 10ᵐ sp, *b 11ᵐ12 sf.

h 2290; Nova XII; h 2300.

h 2290: *a 10ᵐ11 sp.

Nova XII in the same decl-circle with * 19°.5197 (7ᵐ,5), 1′½ n. Sev fine *... seen in the neby? The object in the autumn 1866 quite distinctly seen as a neb with stell C; in the autumn 1869 hardly visible!

h 2300 contains sev fine *..., but does not look r; v indistinct C condens, once a stell C obs; E sp to nf. Two dull *... 12ᵐ13 f the s end of the neby; d* or S neb immediately s from neb?

NOTE III.

A list is here given of the nights of observation together with their atmospherical conditions, which it is of great importance to know, since they in these more even than in other observations indicate the weight to be attributed to each determination. Moreover the coincidences of the spider-lines obtained for every evening are here given, in order to enable the reader, by a comparison of these with the definitive coincidences registered above, to estimate the accuracy of the last mentioned.

The different columns in the following table contain respectively:

Col. 1 and 2: The numerical succession and date of the nights of observation.

Col. 3: Position of the telescope.

Col. 4: Coincidences of the spider-lines. The numbers beside the coincidence-numbers indicate upon how many simple coincidences the coincidence given in the table depends.

Col. 5: Mean temperature in the Cupola during the observation.

Col. 6 and 7: Character of the atmospheric conditions during the observation. Here col. 6 gives, according to the known rule, a general graduation of favourableness of the atmosphere's condition, indicated by the numbers 1, 2, 3, 4.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1863		r			
1	Febr. 17	II	35,118 (2)	—	3	Images very unsteady. Windy.
2	Febr. 19	II	34,197 (4)	−1,5	3	Images unsteady. Windy.
3	Febr. 28	II	35,133 (3)	+1,5	2.3	Variable. Moonlight. Windy.
4	March 23	I	34,167 (2)	—	3	Unsteady and dull images. Aurora. Moon.
5	March 24	I	34,158 (3) 43,250 (2)	+1,8	3	Windy. Moon. Cirrhi.
6	March 25	II	—	—	3.4	Windy.
7	March 31	II	35,131 (2) 42,276 (4)	−2,5	2	Very strong moonlight.
8	April 9	I	34,150 (4)	+1,5	3	
9	April 12	I	35,281 (4)	+2,0	2.3	Images very unsteady.
10	April 13	I	35,296 (2) 35,311 (1)	+2,0	2	Calm; but images tremulous.
11	April 14	I	34,231 (2)	+3,0	2	Perfectly calm, but images very tremulous.
12	April 15	I	35,167 (2)	+4,8	2	Calm; images tremulous and not very well defined. Very strong aurora.
13	April 17	I	34,217 (2)	+5,0	2.3	Windy. Images tolerably steady and well defined but dull. Aurora.
14	April 19	I	34,223 (2)	+5,3	3	Cirrhi; windy; images dull and unsteady.
15	April 22	I	34,228 (1)	—	2.3	Cloudy. Strong gale — chronometer difficult to hear. Moon. Observation interrupted by clouds.
16	April 24	I	27,458 (2)	+1,2	1.2	Atmospheric state variable: for the most part perfectly clear, but images extremely tremulous. Moon.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1863		r	°		
17	April 30	I	36,429 (1)	+ 2,5	2	Air very damp; perfectly calm and images pretty steady. Strong moonlight. Observation soon interrupted by fog.
18	May 1	I	44,546 (1)	+ 4,8	2	Air very dry; strong wind and moon. Images tolerably steady. Illumination deficient.
19	July 28	II	24,979 (1)*	+17,2	3	Twilight and moon; windy. Images extremely disturbed.
20	Aug. 2	II	32,389 (2)*	+ 9,0	2.3	Strong moon and twilight; perfectly calm; images nevertheless very unsteady, indistinct and variable.
21	Aug. 7	II	32,392 (1)*	+12,0	1.2	Moon.
22	Aug. 9	II	32,393 (1)*	+16,1	2.3	Strong wind. Images very tremulous and ill defined.
23	Aug. 14	II	33,193 (2)*	+11,3	1.2	Slight wind. Images very unsteady.
24	Aug. 17	I	24,969 (2)	+14,5	2.3	Air perfectly calm and very damp; variable. Observation interrupted by fog.
25	Aug. 20	I	24,970 (1)	+13,0	2	Air calm and damp.
26	Aug. 25	I	24,973 (2)	+11,5	2	Tolerably windy; images nevertheless steady. Bright moon.
27	Sept. 2	I	32,405 (3)*	+12,0	2	Variable, cloudy, feeble wind. Strong moon. Images not very well defined.
28	Sept. 15	I	—	+ 8,8	2	
29	Sept. 18	I	32,392 (2)	+ 7,5	1.2	Atmosphere very variable.
30	Sept. 21	II	33,195 (1)*	+ 8,6	2	Quiet and steady images; soon clouding.
31	Sept. 29	I	24,975 (1) 24,169 (1)	+ 7,0	1.2	Strong gale, images very tremulous but measures satisfactory. Strong moonlight.
32	Sept. 30	I	24,975 (2)	+ 8,5	2.3	Windy and images very tremulous. Strong moonlight.
33	Oct. 1	I	24,973 (2)	+ 8,1	2.3	Calm, but images unsteady; very damp; strong moon. Soon fog and clouds.
34	Oct. 21	I	24,985 (1)*	+ 3,0	2	Strong gale and excessively tremulous images. Strong moonlight.
35	Oct. 24	II	24,983 (3)*	− 0,5	1.2	Steady images. Strong moon.
36	Nov. 14	I	24,973 (2)	0,0	2.3	Very strong aurora; images dull.
	1864					
37	Jan. 24	I	24,186 (4)	− 1,5	2.3	Cirrhi and wind; images however pretty good and steady.
38	Jan. 29	I	24,185 (1)	− 6,6	3.4	Observations very difficult.
39	Febr. 19	I	24,190 (3)*	− 7,0	2	Slight wind; strong moon.
40	March 13	I	32,397 (2)	0,0	2	Images unsteady. Moon. Interrupted by clouds.
41	March 16	I	24,977 (2)	− 8,5	1.2	Moon. Images a little unsteady.
42	Apr. 7	I	24,187 (2)	− 4,7	2.3	Calm. Images considerably tremulous.
43	Apr. 8	I	32,411 (2)*	− 1,0	2	Wind. Images very tremulous.
44	Apr. 10	I	24,989 (1)*	+ 3,5	3.4	Too bad for continued observation.
45	Apr. 12	I	32,410 (2)*	+ 3,0	2	Unsteady images. Moon.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1864		r	°		
46	Apr. 13	I	33,202 (2)*	+ 2,1	2	Moon.
47	Apr. 14	I	33,186 (2) 24,977 (1)	+ 0,3	1.2	Moon. Steady images.
48	Apr. 15	I	24,978 (2)	+ 2,4	1.2	Moon. Pretty steady images.
49	Apr. 19	I	24,186 (1)	+ 4,3	2	Calm. Images unsteady. Moon very strong.
50	Apr. 26	I	24,186 (1)	+ 4,6	2	Calm. Images very tremulous. Twilight and moon at the end of observation.
51	Apr. 28	I	33,186 (2)	− 0,6	1.2	Images pretty steady but not fine. Twilight at the end of observation.
52	Apr. 30	I	33,186 (2)	− 1,0	3	Images steady but diffused.
53	May 1	I	24,188 (1)	− 0,9	2.3	Soon clouding. Steady images.
54	May 2	I	24,978 (2) 33,185 (1)	− 2,0	1.2	Images unsteady. Twilight.
55	May 4	I	24,189 (1) 33,186 (1)	− 1,5	2	Twilight.
56	May 5	I	24,979 (1)	− 1,1	1	Night very fine; but twilight too strong.
57	Aug. 4	I	38,177 (1)* 38,396 (2)*	+14,0	1.2	
58	Aug. 13	I	38,393 (1)	+12,2	1.2	Fine but unsteady images.
59	Aug. 14	I	30,169 (1)	+13,3	2	Aurora and moon.
60	Aug. 30	I	30,934 (2)	+ 8,4	1.2	Air damp; images extremely tremulous.
61	Aug. 31	I	30,176 (1)* 38,379 (2)	+10,7	1.2	
62	Sept. 14	I	30,934 (2) 30,174 (1)	+ 8,9	1	Images perfect and steady. Very damp. Strong moonlight.
63	Sept. 29	I	—	—	2	Interrupted by clouds.
64	Oct. 4	I	38,385 (1)*	+ 4,5	2	Variable. Interrupted by clouds.
65	Oct. 5	I	30,177 (1)* 38,386 (1)*	+ 5,5	1 2	Calm; very damp; images unsteady.
66	Oct. 6	I	30,950 (1)* 30,178 (1)*	+ 6,1	1!	Atmosphere for a while very exquisite! I never saw the like of it — sky quite dark and air excessively transparent.
67	Oct. 7	I	30,948 (1)* 30,180 (1)*	+ 4,0	1.2	Images not steady.
68	Oct. 14	I	30,949 (1)* 30,179 (1)	− 1,1	1.2	Aurora; clouds.
69	Oct. 28	I	30,941 (2) 30,175 (2)	− 2,3	2.3	Aurora; very variable.
70	Oct. 31	I	30,943 (1) 30,176 (1)	− 5,5	2.3	Variable; wind. Images tolerably unsteady.
71	Nov. 25	I	30,172 (1)	− 3,4	2.3	Clouds; strong gale; excessively tremulous images. Chronometer difficult to hear.
72	Dec. 22	I	30,172 (1)	− 8,5	3	Aurora; images very dull, diffused and tremulous.
73	Dec. 23	I	30,172 (2)	− 4,3	3	Aurora; cirrhi; strong gale. Images excessively tremulous and dull.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1864		r	o		
74	Dec. 24	I	30,172 (1)	− 7,8	2.3	Strong aurora. Images excessively tremulous and diffused.
75	Dec. 27	II	30,187 (1)*	− 2,8	2	Aurora; strong gale. Excessively tremulous and dull images. Hoarfrost on the objectglass
76	Dec. 28	II	30,952 (1)* 30,184 (1)* 38,403 (1)*	− 0,2	2.3	Very strong gale. Excessively tremulous images.
	1865					
77	Jan. 1	II	38,407 (2)*	−11,5	2.3	Images diffused and dull. Illumination deficient.
78	Jan. 5	II	30,182 (2)* 38,399 (1)*	− 2,5	2.3	Windy; moon. Images diffused and extremely tremulous.
79	Jan. 6	II	30,189 (1)*	− 5,5	2	Windy; strong moon. Images unsteady.
80	Jan. 7	II	30,954 (1)*	− 5,7	2.3	Windy; strong moon. Bad and tremulous images.
81	Jan. 11	II	30,155 (1)	+ 1,1	2.3	Strong gale and moon. Images very unsteady and diffused.
82	Jan. 31	II	30,920 (1)	−13,8	2	Images unsteady.
83	Febr. 13	II	30,927 (1)	−13,5	2	Very unsteady and diffused images.
84	Febr. 14	II	30,926 (1)	− 5,0	3.4	
85	Febr. 27	II	30,932 (1)* 30,922 (1) 38,374 (1)*	− 4,5	1.2	Fine and steady images.
86	Febr. 28	I	38,376 (1)*	− 4,5	2.3	Variable.
87	March 17	I	30,935 (1)*	−10,2	2	Images unsteady.
88	March 18	I	30,932 (1)* 39,150 (1)*	−12,5	2	Images unsteady.
89	March 19	I	30,936 (1)*	− 10,2	2	Images pretty fine and not unsteady.
90	March 27	I	30,939 (1)*	− 10,8	3	Excessively tremulous and diffused images. Deficient illumination.
91	March 30	I	30,936 (1)* 30,923 (1) 39,152 (1)* 39,142 (1)	− 4,6	2	Aurora. Pretty steady and fine images.
92	March 31	I	30,920 (1)	− 1,8	2.3	Strong gale; moon. Excessively tremulous and diffused images.
93	April 1	I	30,923 (1)	—	3	Strong gale; moon; soon clouding. Excessively tremulous and diffused images.
94	April 2	I	30,922 (1)	—	3	Strong moon; soon clouding. Images very tremulous.
95	April 3	I	30,933 (1) 39,138 (1)	—	2.3	Strong moon. Images pretty and not so very unsteady.
96	April 15	I	39,134 (1)	+ 5,5	3	Slight wind; soon clouding. Images very unsteady.
97	April 16	I	30,931 (1) 39,134 (1)	+ 4,0	2	First twilight; then a splendid aurora and sky very light. Fine and pretty steady images. Illumination deficient.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1865		r			
98	Apr. 19	I	30,943 (1)* 30,930 (2) 39,154 (1)*	+ 2°,5	2	Sky very light. Pretty steady images.
99	Apr. 20	I	30,946 (1)* 30,937 (1) 39,154 (1)* 39,143 (1)	+ 3,3	2	Windy; images unsteady.
100	Apr. 21	I	39,142 (2)	+ 8,7	2	Sky light; images unsteady.
101	Apr. 22	I	30,927 (1)	—	2	Sky not very dark; images tolerably steady.
102	Apr. 23	I	30,951 (2)*	+ 2,8	2	Sky very light; images pretty steady.
103	Apr. 24	I	30,932 (2) 39,139 (2)	+ 1,9	2	Images pretty fine and steady.
104	July 19	I	—	+18,3	2	Cirrhi; sky very light.
105	July 22	I.II	—	—	2	Sky very light; images pretty steady.
106	July 23	I	—	—	2.3	Sky light; images not bad but dull.
107	Aug. 11	I	26,433 (2) 34,630 (2)	+14,2	1.2	Very damp. Sky very light.
108	Aug. 12	I	26,446 (1)* 34,647 (2)*	+14,0	1.2	Moon; sky very light; damp. Images very steady.
109	Aug. 30	I	—	+ 8,3	2.3	Variable. Images unsteady.
110	Sept. 7	I	26,440 (1)	+15,0	2	Strong moonlight; windy, and very tremulous images. Illumination deficient.
111	Sept. 12	I	26,448 (2)*	+ 8,0	2	Strong moon. Images extremely tremulous.
112	Sept. 13	I	26,455 (1)*	—	3	Very strong gale. Images bad and excessively tremulous.
113	Sept. 14	I	26,446 (3)*	+ 9,4	2	Strong gale; images very unsteady.
114	Sept. 19	I	26,450 (1)* 34,648 (1)*	+ 8,5	3	
115	Sept. 20	I	26,438 (1)	—	1.2	Very strong aurora.
116	Sept. 21	I	26,437 (1)	+10,5	1.2	Aurora; soon clouding.
117	Sept. 24	I	26,439 (1)	+11,9	2.3	Strong gale. Images diffused and extremely tremulous.
118	Sept. 26	I	26,437 (1)	+11,8	2.3	Variable. Images very unsteady.
119	Sept. 28	I	26,440 (1)	+ 6,0	2.3	
120	Sept. 29	I	26,433 (2)	+ 6,5	1.2	Strong gale and very tremulous images.
121	Sept. 30	I	26,431 (1)	+ 7,3	1.2	Images pretty steady; soon clouding.
122	Oct. 4	I	26,433 (1)	+ 6,8	2.3	Strong moon.
123	Oct. 10	I	26,445 (1) 34,640 (1)	− 0,5	1.2	Moon. Images pretty steady.
124	Oct. 14	I	26,431 (2)	− 1,1	2	Aurora; windy. Illumination deficient.
125	Oct. 16	I	26,436 (1)	—	3	Strong aurora; windy. Images diffused and unsteady.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1865		r	°		
126	Oct. 17	II	26,456 (2)* 34,667 (1)*	− 3,0	2.3	Variable. Dull images.
127	Oct. 26	II	26,433 (2)	− 2,3	2	Strong aurora.
128	Nov. 10	II	26,433 (3) 34,632 (2)	− 2,0	2.3	Variable. Images tremulous and diffused.
129	Nov. 15	II	—	+ 0,5	3	Hoarfrost on the object-glass.
130	Nov. 25	II	26,425 (1)	+ 4,9	3.4	
131	Nov. 27	II	—	+ 2,6	3	
132	Dec. 5	II	26,423 (1)	− 3,5	2	
133	Dec. 6	I	26,431 (1)	− 6,0	2	Strong moon. Images well defined and steady but dull.
134	Dec. 10	II	26,430 (1) 44,110 (1)	+ 4,2	2.3	Strong gale; variable. Ima ges tremulous and diffused.
135	Dec. 12	II	34,646 (1)*	− 1,3	3	Soon clouding.
136	Dec. 18	II	34,641 (1)*	+ 3,6	3	Wind. Images excessively tremulous and dull.
137	Dec. 19	II	—	—	2	Fine images; soon clouding.
	1866					
138	Jan. 12	II	26,435 (2) 34,631 (1)	− 6,8	3	Windy. Images excessively tremulous and diffused.
139	March 9	II	26,446 (1)* 26,438 (2)	− 8,0	3	Aurora. Images dull.
140	March 10	II	26,432 (1) 34,627 (1)	− 6,5	2.3	Aurora and windy.
141	March 15	II	26,433 (1) 34,630 (2)	− 9,8	2	Fine but very unsteady images.
142	March 16	II	26,433 (2)	− 5,3	2.3	Dull images.
143	March 24	I	34,623 (1)	− 9,8	2.3	Images unsteady and not very fine.
144	Apr. 5	I	26,422 (2) 34,622 (1)	+ 1,0	3	Sky variable; soon fog.
145	Apr. 6	I	26,424 (2)	+ 1,1	2.3	Sky not very transparent.
146	Apr. 7	I	26,428 (1)* 26,422 (1) 34,617 (1) 44,104 (1)	+ 2,5	2.3	Aurora. Images pretty well defined but dull.
147	Apr. 8	I	26,424 (1) 34,624 (1)	+ 3,3	3	Aurora.
148	Apr. 10	I	26,433 (1)* 26,424 (1)	+ 6,7	2	Images unsteady; soon clouding.
149	Apr. 11	I	26,424 (1)* 26,420 (1) 34,639 (1)*	+ 4,7	2	Aurora; but sky relatively dark. Images a little unsteady.
150	Apr. 12	I	26,425 (2) 34,618 (1)	+ 4,7	2	

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1866		r	°		
151	Apr. 13	I	26,419 (1) 34,620 (1)	+ 6,2	2.3	Strong gale. Images diffused and excessively tremulous.
152	Apr. 20	I	26,421 (1)	− 3,4	2	Twilight; moon; aurora. Images somewhat dull.
153	Aug. 19	II	28,956 (1)	—	3.4	Very bad.
154	Sept. 1	I	29,044 (1)	+12,5	3	Soon clouding. Deficient illumination.
155	Sept. 15	I	29.042 (2)	+10,7	2	Strong gale; illumination flickering and unsteady; chronometer difficult to hear. Images excessively tremulous.
156	Sept. 16	I	29,045 (1)	+11,3	2	Soon clouding.
157	Oct. 2	II	29,055 (1)*	+10,8		Extremely variable; soon clouding.
158	Oct. 3	II	29,052 (1) 37,241 (1)	+ 9,6	2.3	Very damp; object-glass covered with moisture.
159	Oct. 4	I	29,055 (1)* 37,248 (1)*	+ 8,3	2	Sky very light from strong aurora; very damp. Images pretty fine and tolerably steady.
160	Oct. 5	I	—	+ 8,5	3	Aurora; soon fog. Images dull and indistinct.
161	Oct. 8	I	37,247 (1)*	+ 8,0	3	Sky soon quite covered.
162	Oct. 11	I	29,056 (1)*	+ 5,7	3	Observation difficult. Sky very light from strong aurora; images excessively tremulous, diffused and dull.
163	Oct. 15	I	29,060 (1)*	+ 2,2	1.2	Sky not very dark.
164	Oct. 16	I	29,063 (1)*	+ 1,5	1.2	Images pretty steady.
165	Oct. 17	I	29,060 (1)*	+ 1,0	1	Images fine and steady; sky however very light.
166	Nov. 10	I	30,200 (1)	− 0,7	2	Pretty fine and not very unsteady images.
	1867					
167	Jan. 5	I	30,179 (2)	−14,3	2	Slight wind. Images unsteady but pretty well defined.
168	Febr. 4	I	30,175 (2)	− 0,8	2.3	Strong gale.
169	Febr. 8	I	30,177 (2)	− 4,0	2.3	Very splendid aurora; strong fog.
170	Febr. 23	I	30,176 (2)	−10,0	1.2	Moon. Images fine and steady.
171	March 1	I	30,176 (1)	− 8,5	2	Slight aurora. Images somewhat dull but steady and beautiful.
172	March 9	I	30,177 (1) 38,358 (1)	− 7,1	2	Windy.
173	Aug. 1	II	28,014 (3)*	+11,4	1.2	Sky very light. Fine and pretty steady images.
174	Aug. 4	II	27.386 (3)*	+12,8	2.3	Observation through clouds. Images not bad.
175	Aug. 5	II	27,338 (3)*	+10,0	1.2	Sky however much too light.
176	Aug. 6	I	27,328 (4)	+13,9	2.3	Observation through clouds. Sky very light.
177	Aug. 7	I	27,330 (4)	+11,4	2.3	
178	Aug. 25	II	27,223 (3) 35,429 (1)* 35,419 (1)	+13,0	2	Sky light.
179	Aug. 26	II	27,223 (1)	+16,7		Aurora; sky first very fine, soon clouding.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1867					
180	Aug. 28	II	27,225 (2) 35,432 (2)* 35,424 (2)	+11°,5	2	Aurora. Images very unsteady.
181	Aug. 30	II	27,224 (2) 27,227 (2)* 35,418 (2)	+13,6	2	Aurora. Images unsteady.
182	Sept. 3	II	27,225 (2) 35,425 (1)	+ 9,6	2.3	Images not bad, steady; clouding.
183	Sept. 4	II	27,224 (1) 35,421 (1)	+11,0	2.3	Sky very light from aurora; images not bad.
184	Sept. 5	II	27,222 (1) 27,230 (1)* 35,435 (1)*	+ 9,2	1.2	Air very damp; fog; sky tolerably light. Images pretty fine and steady.
185	Sept. 19	I	44,893 (1)	+11,5	2.3	Cirrhi; moon; windy.
186	Sept. 25	II	27,217 (2) 44,900 (2)*	+ 2,9	2.3	Strong aurora.
187	Sept. 26	II	—	+ 5,9	3.4	Aurora; windy; soon clouding.
188	Sept. 29	II	27,227 (2) 35,429 (2)	+ 5,8	2.3	Aurora; windy. Images dull, diffused and tremulous.
189	Oct. 3	II	27,220 (2)	+ 4,7		Extremely variable; soon clouding.
190	Oct. 21	II	27,229 (2)* 35,436 (2)	+ 7,6		Extremely variable; soon clouding.
191	Oct. 22	II	27,220 (2)	+ 8,2		Clouds; variable; soon quite cloudy.
192	Oct. 25	II	27,232 (2)* 27,224 (2)	+ 7,4	1.2	Sky light from aurora.
193	Oct. 30	II	27,222 (2)	+ 4,1	2	Aurora; variable; soon clouding.
194	Nov. 1	I	27,241 (2)* 27,228 (2)	+ 1,7	1.2	Aurora.
195	Nov. 2	I	27,226 (1) 35,437 (2)* 35,426 (2)	+ 0,8		Aurora; windy. Extremely variable.
196	Nov. 27	I	27,251 (1)* 27,236 (1) 35,475 (1)*	− 4,4	2.3	Sky very light from strong aurora. Images unsteady and diffused.
197	Nov. 29	I	27,268 (1)* 35,467 (1)*	− 7,3	2	Slight aurora; soon clouding.
	1868					
198	Jan. 16	II	27,233 (2)	− 1,5		Aurora; very variable.
199	Febr. 16	II	27,251 (1)* 35,448 (1)* 35,436 (1)	− 3,1		Observation difficult and troublesome. Atmospheric state very variable; clouds — soon quite cloudy. Images extremely tremulous.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1868		r	o		
200	Febr. 18	II	35,453 (2)* 35,429 (2)	− 0,6	1.2	Slight wind.
201	Febr. 24	II	27,227 (1)	+ 2,0	1.2	Strong gale; soon clouding.
202	Febr. 25	II	27,246 (1)* 27,236 (1)	− 0,7	2	Aurora; windy — variable.
203	Febr. 27	II	27,238 (1)	+ 3,3	3.4	Moon; violent gale — soon clouding.
204	Febr. 28	II	27,230 (1)	− 2,0	3	Moon; hoarfrost — soon clouding.
205	March 11	II	27,235 (1)	− 1,5	1.2	Images remarkably steady — but soon fog and clouding.
206	March 16	II	27,237 (1)	− 0,9	2	Strong gale; soon clouding.
207	March 28	II	27,235 (2) 35,431 (1)	0,0	1	Feeble moonlight.
208	March 29	II	35,437 (1)*	+ 5,0	3	Cirrhi; clouding.
209	Apr. 8	II	27,364 (2)	+ 2,7	1.2	Strong moon and clouds.
210	Apr. 9	II	27,369 (3)	− 0,7	1.2	Twilight; strong moonlight. Images pretty fine.
211	Apr. 10	II	27,369 (1)	0,0	3.4	Strong aurora and cirrhi — observation soon interrupted by clouds.
212	Apr. 14	I	27,366 (2)	+ 2,0	1.2	Twilight; strong aurora.
213	Apr. 15	I	27,367 (1)	+ 3,7	1	Twilight — soon abruptly clouding.
214	Apr. 27	I	27,371 (1)	+ 5,5	3.4	Through clouds.
215	May 9	I	27,368 (2)	+ 6,7	2.3	Strong twilight and moon.
216	Aug. 11	I	34,963 (4)	+18,5	1	Atmosphere very damp. Images beautiful and steady.
217	Sept. 9	II	34,967 (1)	+11,8	2.3	Cirrhi.
218	Sept. 13	II	34,965 (1)	+ 8,1	1.2	Very damp.
219	Sept. 20	II	34,961 (1)	+ 9,3	2	Sky light from aurora — later: strong fog.
220	Sept. 21	II	34,966 (1)	+ 7,2	2.3	Sky very light from aurora; very damp. Images feeble and indistinct.
221	Sept. 22	II	34,965 (2)	+ 6,1	2	Strong aurora; soon suddenly clouding.
222	Oct. 9	II	—	+ 7,8	3.4	
223	Oct. 12	II	34,966 (2)	+ 7,5	3	Cirrhi; very damp — soon fog and quite cloudy.
224	Oct. 17	II	—	+ 7,7	2.3	Strong gale. Sky very light and images dull.
225	Nov. 5	I	—	+ 1,2	1.2	Images unsteady.
226	Nov. 7	I	34,971 (2)	− 1,9	3	Fog — soon clouding.
227	Nov. 10	I	—	− 0,5	1.2	
228	Nov. 11	I	34,973 (2)	+ 2,5	1	Very damp — soon suddenly clouding.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1868					
229	Nov. 12	I	—	− 0,3	1.2	Sky very light.
230	Nov. 20	I	34,973 (3)	− 7,5	2.3	Aurora; slight wind. Images unsteady.
231	Dec. 9	I	34,966 (2)	− 7,5	2.3	Sky very light; slight wind. Illumination deficient.
232	Dec. 12	I	—	− 2,9	2	Observation very troublesome through gale and deficient illumination.
	1869					
233	Jan. 7	I	34,975 (2)	− 2,5	3.4	Sky very light from strong aurora; soon suddenly covered. Images very dull.
234	Jan. 10	I	34,978 (3)	− 0,6	2.3	Sky very light from aurora. Object-glass dimmed.
235	Jan. 18	II	34,973 (2)	− 5,3	2	Moon; cirrhi.
236	Febr. 2	II	—	0,0	2	Slight aurora; windy. Images dull and indistinct.
237	Febr. 3	II	34,975 (2)	0,0	2.3	Sky illuminated by a splendid aurora; windy.
238	Febr. 21	I	34,989 (2)	− 7,5	2	Strong moonlight. Images unsteady.
239	Febr. 23	II	34,975 (3)	− 6,6	2.3	Very strong moonlight; soon clouding. Images very tremulous.
240	Febr. 26	II	34,973 (3)	+ 1,0	2	Very strong moon; soon clouding.
241	March 1	II	34,977 (1)	− 7,1	3	Atmosphere misty. Images very dull.
242	March 3	II	34,973 (1)	− 5,0	3.4	Strong aurora; cirrhi; hoarfrost.
243	March 4	II	—	− 6,5	2.3	Aurora. Hoarfrost.
244	March 5	II	—	− 7,2	1.2	Aurora.
245	March 6	II	34,981 (3)	− 7,5	1.2	Slight aurora.
246	March 7	II	34,974 (3)	− 6,6	1.2	Sky very light from aurora. Hoarfrost.
247	March 13	II	—	− 5,1	2.3	Sky very light from aurora. Observations difficult.
248	March 14	II	34,975 (3)	− 2,0	3	Soon quite covered.
249	March 28	II	34,972 (2)	− 1,5	2	Strong moon; cirrhi. Images unsteady.
250	March 30	II	34,981 (3)	+ 3,0	1.2	Atmosphere very dry; windy. Images unsteady.
251	March 31	II	—	+ 4,3	2	Aurora. Images dull and tremulous.
252	Apr. 8	II	—	+ 4,3	2	Strong aurora — soon clouding.
253	Apr. 12	II.I	34,982 (3)	− 1,2	1	Slight aurora — splendid atmosphere.
254	Apr. 21	I	34,975 (3)	+ 7,2	1.2	Strong moon; sometimes feeble clouds.
255	Apr. 22	I	34,972 (2)	+ 7,5	3	Strong moon — soon quite covered.
256	Aug. 3	II	—	+ 16,0	1.2	Images unsteady.
257	Aug. 13	I	—	+ 13,5	1.2	Sky tolerably light; very damp.
258	Aug. 27	I	34,973 (3)	+ 13,1	2	Moon tolerably strong; clouds.
259	Aug. 29	I	—	+ 9,2	1	Moon tolerably strong. Images unsteady.
260	Aug. 30	I	34,982 (4)	+ 7,1	1	Tolerably strong moon; atmosphere dry.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1869		r	°		
261	Aug. 31	I	34,975 (2)	+ 7,4	1	Moon; sometimes slight wind. Images unsteady.
262	Sept. 2	I	—	+ 5,0	2.3	Aurora and clouds. Very variable.
263	Sept. 3	I	—	+ 5,2	2	Aurora and clouds. Variable.
264	Sept. 4	I	34,984 (3)	+ 7,8	2.3	Strong aurora and cirrhi; sky very light; very damp.
265	Sept. 5	I	34,976 (2)	+10,2	2.3	Very strong aurora and cirrhi; sky very light; damp.
266	Sept. 9	I	34,975 (3)	+15,5	3.4	Clouding.
267	Sept. 10	I	34,975 (7)	+15,9	3	Strong aurora; sky very light; misty. Images very dull and indistinct.
268	Sept. 30	I	34,972 (2)	+ 5,5	2.3	Pretty fine and steady images, but sky too light.
269	Oct. 2	I	—	+ 4,4	1	Slight aurora; damp; observation interrupted by clouds.
270	Oct. 3	I	34,974 (2)	+ 3,6	1.2	Sky very light from aurora.
271	Oct. 10	I	—	+11,0	2	Observation hurriedly made through clouds.
272	Nov. 1	I	34,980 (4)	+ 0,2	1	Sky tolerably light. Images unsteady.
273	Nov. 5	I	—	− 4,9	3.4	Through clouds.
274	Nov. 6	I	34,979 (4)	− 5,9	2	Aurora. Sky very light.
275	Nov. 27	I	34,980 (4)	− 0,5	2	Clouding.
276	Nov. 28	II	—	− 4,5	1.2	Sky light.
	1870					
277	Jan. 19	I	34,981 (2)	− 8,8	3	Strong moon; windy. Images diffused and extremely tremulous.
278	Jan. 22	I	—	− 6,9	1.2	Sky very light from aurora. Images steady and fine.
279	Jan. 23	I	34,981 (2)	− 3,7	3	Clouding.
280	Febr. 1	I	—	—		Observation difficult — sky illuminated by a splendid aurora.
281	Febr. 17	I	34,983 (2)	− 1,9	2	Strong moon. Images extremely tremulous.
282	Febr. 19	I	—	− 5,5	2	Dull and extremely tremulous images.
283	Febr. 25	I	—	− 9,6	2	Windy. Unsteady images.
284	March 5	II	34,980 (10)	+ 0,5	1.2	Images a little dull but exquisitely steady — soon clouding.
285	March 7	II	34,980 (4)	+ 1,0	2	Soon clouding.
286	March 8	II	—	− 4,6	2	Moon; strong aurora. Images dull, yet momentarily pretty fine.
287	March 9	I	—	− 5,9	2	Very strong aurora and moon: tolerably strong gale — soon clouding.
288	March 11	II	34,978 (6)	− 8,5	1.2	Strong moon; feeble cirrhi.
289	March 26	II	34,977 (6)	− 4,0	2.3	Strong aurora; damp; hoarfrost; very variable. Observation interrupted by clouds.
290	March 27	I	—	− 2,4	3.4	Aurora and cirrhi.
291	March 28	I	34,981 (6)	− 1,0	3	Extremely bright aurora — soon clouding.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1870		r	°		
292	March 29	I	34,981 (6)	− 2,0	1.2	Slight aurora; hoarfrost.
293	March 30	I	34,980 (6)	− 1,9	2	Sky illuminated by a very strong aurora.
294	April 1	I	34,983 (6)	+ 1,8	1.2	No visible trace of aurora, sky however very light Stars strongly scintillating.
295	April 3	I	34,983 (10)	+ 0,5	1.2	Slight aurora. Atmospheric state on the whole not unfavourable: images however a little dull and very unsteady.
296	April 18	I	34,982 (4)	+ 6,6	2.3	Twilight; aurora. Unsteady and dull images.
297	April 19	I	34,982 (15)	+ 7,0	1.2	Twilight and aurora. Images pretty fine, though unsteady.
298	April 20	I	34,980 (6)	+ 9,9	1.2	Twilight; moon; aurora.
299	April 21	I	34,981 (6)	+11,0	1.2	Twilight; strong aurora. Images extremely unsteady.
300	April 24	I	34,981 (4)	+ 5,6	2	Twilight; slight aurora, but sky relatively dark; windy. Images extremely tremulous.
301	April 27	I	—	+ 4,2	2	Twilight; windy; no traces of aurora — variable.
302	April 29	I	34,980 (4)	+ 3,0	1.2	Strong twilight; very damp. Observation interrupted by fog.
303	April 30	I	34,982 (4)	+ 3,9	1.2	Steady and fine images, but sky too light from twilight.
304	May 2	I	—	+ 5,4	1.2	Strong twilight; windy; clouding.
305	July 30	I	35,034 (4)	+16,3	1	Images a little diffused and unsteady.
306	Aug. 6	I	35,221 (3)	+17,4	1.2	Damp, though tolerably windy. Images unsteady.
307	Aug. 8	I	34,998 (12)	+15,5	2	Very damp; clouds here and there.
	1871					
308	Jan. 24	I	34,992 (6)	− 9,4	3	Windy.
309	March 9	II	34,992 (4)	+ 2,8	2	Strong gale. Observations ill according.
310	March 17	II	34,989 (4)	− 1,6	1.2	Sky very light from aurora.
311	March 23	II	34,992 (4)	+ 4,3	2.3	Aurora; cirrhi; clouding.
312	March 24	II	34,997 (4)	+ 3,5	2.3	Sky very light from aurora; misty; windy.
313	March 25	II	34,995 (4)	+ 4,5	2.3	No traces of aurora, sky however very light. Tolerably strong gale and atmosphere very dry. Images excessively tremulous and diffused.
314	March 26	II	34,994 (4)	+ 4,5	1.2	Images singularly unsteady. Soon clouding.
315	April 7	I	34,996 (4)	+ 2,6	3	Observation through clouds.
316	April 8	I	34,995 (4)	− 0,7	2	Between cirrhi sky exquisite.
317	Apr. 13	I	34,993 (4)	+ 0,6	2	Sky very light from strong aurora; windy. Observation partially disturbed by passing clouds.
318	Apr. 14	I	34,996 (4)	− 1,0	2	Splendid aurora; misty.
319	Apr. 18	I	34,996 (4)	− 1,4	2.3	Sky extremely light from splendid aurora.
320	Apr. 19	I	34,995 (4)	+ 2,4	3	Clouding.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1871		r	o		
321	Apr. 22	I	34,995 (4)	− 1,4	2	Atmosphere very misty; observation soon interrupted.
322	Apr. 23	I	34,993 (4)	− 1,6	2	Strong moon and twilight; atmosphere a little misty.
323	Apr. 24	I	34,994 (4)	+ 0,3		Feeble aurora and moon; very variable. Only bright objects observable.
324	Apr. 27	I	34,985 (4)	+ 3,3	1.2	Strong moon and slight aurora. Images extremely unsteady.
325	Apr. 28	I	34,982 (4)	+ 3,1	2	Twilight; strong moon. Images very unsteady.
326	Dec. 26	I	34,992 (4)	+ 1,0	3	Very bright moonlight.
	1872					
327	Jan. 1	I	34,993 (4)	+ 3,0	3	Strong gale; clouding.
328	Febr. 2	I	35,000 (4)	+ 0,7	1	Suddenly clouding.
329	Febr. 16	I	34,998 (4)	− 4,6	2	Moon; unsteady images.
330	Apr. 25	I	34,994 (4)	+ 4,8	1.2	Strong moon; fog; extremely damp.
331	Apr. 26	I	34,991 (4)	+ 9,0	3	At first twilight; then misty and clouding.
332	Apr. 28	I	34,993 (4)	+ 7,9	3	Misty and very damp.
333	Oct. 1	II	34,990 (4)	+ 6,3	3	Misty. Observation difficult.
334	Oct. 25	II	34,989 (4)	+ 7,2	2.3	Damp; atmosphere soon too misty for observation.
335	Nov. 1	II	34,987 (4)	+ 7,1	2.3	Strong gale; misty.
336	Nov. 3	II	34,991 (4)	+ 7,0	3	Clouding.
337	Nov. 8	II	34,988 (4)	+ 5,3	2.3	Moon; misty; very damp.
338	Nov. 28	II	34,991 (4)	+ 2,3	2	Sky very light.
339	Dec. 12	II	34,992 (4)	− 4,3	2.3	Strong moon. Clouds here and there.
340	Dec. 18	II	34,992 (4)	−12,0	3	Strong moon.
	1873					
341	Jan. 16	I	34,982 (4)	+ 1,5	2.3	Moon; clouds; windy.
342	Jan. 28	II	34,988 (4)	− 2,6	2	Images somewhat dull and unsteady.
343	Febr. 4	II	34,991 (4)	− 2,4	2	Strong moon. Images extremely tremulous. Observation interrupted by fog.
344	Febr. 16	II	34,990 (4)	− 0,4	2.3	Images dull and extremely tremulous; soon clouding.
345	Febr. 18	II	34,991 (4)	+ 1,3	2.3	Clouds; soon quite covered. Images extremely unsteady.
346	Febr. 19	II	34,991 (4)	+ 2,3	1.2	Soon misty. Images very unsteady.
347	Febr. 24	II	34,989 (4)	− 7,8	2	Slight wind and aurora.
348	Febr. 25	II	34,989 (4)	− 3,4	2.3	Cirrhi; observation soon interrupted.
349	March 18	II	34,989 (4)	− 1,9	2.3	
350	March 20	II	34,988 (4)	− 3,7	2	
351	March 26	II	34,988 (4)	+ 3,8	2	

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1873					
352	March 27	II	34,989 (4)	+ 3,1	2	
353	March 28	II	34,986 (4)	+ 5,7	2.3	Aurora; sky very light.
354	March 29	II	34,990 (4)	+ 5,2	2.3	Aurora; sky very light.
355	March 30	II	34,987 (4)	+ 6,0	1.2	
356	Apr. 2	II	34,989 (4)	+ 7,3	2	Moon and strong aurora; clouds here and there.
357	Apr. 15	II	34,990 (4)	+ 1,7	2	Dull images.
358	Apr. 16	II	34 986 (4)	+ 2,0	2	
359	Aug. 24	I	34,986 (4)	+16,1	1.2	
360	Aug. 26	I	34,993 (4)	+16,2	1	Slight aurora.
361	Sept. 22	I	34,993 (4)	+ 8,2	1.2	Images unsteady.
362	Sept. 23	I	34,993 (4)	+ 7,7	1	Sky tolerably light, but images fine and steady.
363	Sept. 24	I	34,988 (4)	+10,4	1.2	Images perfect — fine and steady; but sky light from aurora.
364	Sept. 26	II	34,987 (4)	+11,5	2.3	Damp and variable; soon clouding.
365	Sept. 30	II	34,990 (4)	+ 6,4	1	Strong gale and slight aurora.
366	Oct. 12	II	34,989 (4)	+ 8,2	2.3	Feeble moon and sky misty. Dull images.
367	Nov. 24	I	34,996 (4)	− 0,7	2	Slight aurora; sky very light and variable.
368	Nov. 25	I	34,997 (4)	− 2,4	3	Aurora; sky misty.
369	Dec. 6	I	34,998 (4)	− 4,6	3	Moon. Observation through clouds soon interrupted.
370	Dec. 9	II	34,996 (4)	+ 3,2	3	Observation soon interrupted by clouds.
371	Dec. 10	II	34,995 (4)	+ 3,2	2	Suddenly clouding.
372	Dec. 11	II	34,993 (4)	+ 2,1	1.2	Violent gale. Observation soon impossible.
373	Dec. 13	II	34,992 (4)	+ 2,2	Var.	Sky at first dull and misty, then very fine.
374	Dec. 15	II	34,992 (4)	+ 1,0	2.3	Soon clouding.
375	Dec. 17	II	34,992 (4)	− 2,0	1.2	Strong gale.
376	Dec. 20	II	34,993 (4)	+ 0,5	Var.	Sky not favourable; soon clouding.
377	Dec. 21	II	—	− 1,5	1.2	Soon suddenly clouding.
378	Dec. 22	II	34,995 (4)	+ 2,0	2	Strong gale; soon clouding.
379	Dec. 23	II	34,994 (4)	0,0	2	Soon clouding.
380	Dec. 26	II	34,995 (4)	− 0,5	2	Strong moonlight.
381	Dec. 31	II	34,996 (4)	− 2,0	2	Strong moon and windy. Images very unsteady. Illumination deficient.
	1874					
382	Jan. 13	I	34,992 (4)	+ 0,7	2.3	Unfavourable. Soon clouding.
383	Jan. 14	I	34,993 (4)	− 3,1	1.2	Sky very light.

Night	Date	Refract.	Coincidence of Microm.	Temp. C.		Remarks on the images and state of the atmosphere
	1874		r	o		
384	Jan. 24	II	34,994 (4)	− 1,5	1.2	Strong moonlight.
385	Jan. 25	II	34,994 (4)	− 1,8	2.3	Clouds; observation soon interrupted.
386	Febr. 3	I	34,995 (4)	0,0	2.3	Moon; misty; windy.
387	Febr. 6	I	34,995 (4)	− 0,5	2	Soon clouding.
388	Febr. 7	I	34,995 (4)	− 8,5	2	Sky apparently quite pure but very light.
389	Febr. 8	I	34,994 (4)	− 5,8	2	Clouds; strong gale and illumination unsteady.
390	Febr. 12	I	34,992 (4)	− 0,4	2	Strong gale; soon clouding.
391	Febr. 23	II	34,992 (4)	− 0,4	1.2	At first tolerably strong moonlight; then soon clouding. Images perfectly steady.
392	March 2	II	34,992 (4)	− 0,5	2.3	Strong moonlight; soon clouding.
393	March 12	II	34,991 (4)	− 6,3	2	Images steady, but sky not quite pure.
394	March 14	II	34,993 (4)	− 1,8	2.3	Images unsteady and dull.
395	March 15	II	34,996 (4)	− 4,6	1.2	
396	March 21	II	34,989 (4)	− 2,5	1.2	Observation interrupted by fog.
397	March 22	II	34,991 (4)	− 1,5	1.2	Tolerably strong gale.
398	March 27	II	34,992 (4)	+ 0,5	2.3	Moon.
399	Apr. 5	II	34,991 (4)	+ 5,0	2	Images steady and pretty fine but dull.
400	Apr. 8	II	34,992 (4)	+ 3,5	1.2	Slight wind.
401	Apr. 13	II	34,991 (4)	+ 0,5	2	Strong aurora.
402	Apr. 14	II	34,993 (4)	+ 0,8	1	Images fine, steady and clear; sky however tolerably light.
403	Apr. 18	II	34,993 (4)	+ 1,8	2	Images tremulous and diffuse — slight wind.
404	Apr. 24	I	34,993 (6)	+ 5,7	1.2	Images unsteady. Strong moonlight.

As during the observations the opening of the cupola was always turned from the wind, it may be confidently assumed, that the position of the instrument was never perceptably affected by that cause.

RESEARCHES

ON

THE SALTS OF SELENIOUS ACID.

BY

L. F. NILSON.

[Presented to the Royal Society of Upsala, The 31 Octobre 1874.]

UPSALA,
PRINTED BY ED. BERLING.
1875.

In the year 1817 BERZELIUS [1]) published his discovery of selenium. In discussing its chemical properties, he also gave a report of the only compound of oxygen obtained by him of the new element, viz. selenious acid (or, as he then called it, selenic acid) and of its salts with most bases then known. In doing so he, for the most part, does not go beyond experiments of reaction, only rarely stating the quantitative composition of a very few salts, with the view of ascertaining the basicity of the new acid. For this purpose he makes use of the selenites and diselenites of sodium and barium and the selenites of lead and silver. He also proved the existence of tetraselenites of the alkaline metals and diselenites of a good deal of other ones; in a word, he set forth the general characteristic of the basicity of selenious acid, for which chemistry is still at the present time indebted almost to him alone. I think we may make this assertion, without fear of mistake, though MUSPRATT [2]), many years later, has made the salts of selenious acid the object of his researches. For as far as his statements are not a mere repetition of what BERZELIUS had published thirty years before — and this is often the case — the results of his work, owing to deficient and erroneous methods, appear little calcutated to inspire confidence, and it will be evident from the following pages, that most of the formulæ he furnishes and thinks himself authorized to assume for a considerable number of salts, are erroneous.

No author, as far as I am aware, except the two above mentioned, has made the matter in question the exclusive object of any research. There occur, however, some scattered reports on a few salts of selenious

[1]) Afhandlingar i Fys., Kemi och Miner. VI. s. 42—144.

[2]) The Quart. Journ. of the Chem. Soc. of London 1849 p. 52.

acid. Thus WÖHLER [1]), a couple of years before MUSPRATT published his paper, obtained a tetraselenite of zinc, of which the latter seems to be ignorant; in 1853 KÖHLER [2]) gave some information of the selenites of mercury, and KUHLMANN [3]) described in 1864 some of the selenites of thallium. Finally, in the latest time, ATTERBERG [4]), in a treatise on glucinum-salts, made mention of three basic salts of this metal; CLEVE [5]), in the course of his researches on the earth-metals, has also examined salts of selenious acid with yttrium, lanthanum, didymium and thorium; HÖGLUND [6]) has contributed to the knowledge of erbium in this respect, and JOLIN [7]) to the knowledge of the selenites of cerium.

As the statements of these authors will be more particularly spoken of in the following pages, it may be superfluous to give here a more detailed account of them.

As MUSPRATT'S treatise, as already mentioned, cannot pretend to supply the rather scanty knowledge of the salts of selenious acid which we owe to BERZELIUS, and as the other authors here mentioned have only treated of particular salts, I undertook the investigation of the compounds of this acid, of which, accordingly, an account will now be given.

My original intention was indeed to apply myself to the study of its double salts, supposing that these ought to afford as interesting a matter for an examination, as the known double salts of sulphurous acid, already treated of by several authors. But even a superficial review of the literature on this subject suggesting that there is but little known about the selenites in general, I thought it useful to give some previous attention to the simple salts of the acid. The results of my investigations are stated below in more than one hundred, analytically determined compounds, whereas the double salts will be treated of in another paper, which I hope soon be able to publish. Before I enter upon the peculiar subject of my treatise, it will be convenient previously to give an account of the manner in which pure selenious acid has been obtained.

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[1]) Ann. d. Chemie u. Pharm. LXIII S. 279. (1847).
[2]) Ann. d. Ph. u. Ch. LXXXIX S. 146.
[3]) Bull. de la Soc. chim. de Paris [2] II p. 330.
[4]) K. Sv. Vet. Akad:s handl. 1873 N:o 5.
[5]) Bih. t. Sv. Vet. Akad:s handl. I N:o 8, II N:o 6, 7. 8.
[6]) Bih. t. Sv. Vet. Akad:s handl. I N:o 8.
[7]) Bull. de la Soc. chim. [2] XXI p. 533.

PREPARATION OF SELENIOUS ACID FROM A SELENIFEROUS DEPOSIT.

The material for the preparation of the acid was a deposit from the sulphuric acid manufactory at Falun. For it I am indebted to the kindness of the Director M:r A. Bohman, who obligingly placed large quantities at my disposal.

The deposit was a slippery mass of a strong red colour, moistened throughout with sulphuric acid and, for the most part, composed of sulphate of lead, sulphur and selenium [about 2.5 per cent.] Besides, I have found in it smaller quantities of iron, copper, tin, antimony, arsenic, bismuth, calcium, magnesium, aluminium and phosphoric acid. Washed and dried, it formed a powder of grey colour with a reddish tint.

Berzelius, as is known, discovered selenium in the residue from the sulphuric acid manufactory of Gripsholm, which consumed sulphur from Falun. His material was consequently taken from the same source, as that here employed. The question in what minerals selenium is to be found in the neighbourhood of Falun, and what is the reason of its occurring in the leaden chambers, is no more settled now, than at the time of Berzelius' first researches. He found it, in minerals from Falun, only as a subordinate ingredient, for instance in iron pyrites and galena and was but able to advance the hypothesis that there, as at Skrikerum, it ought to be possible to find selenium as an essential ingredient of a mineral. The only discovery that can be said to indicate the existence of anything of that kind at Falun, has been made by Mr Tholander[1]) and appears, according to an analysis, to be composed of bismuth with bismuth-glance and galena, containing about 1 per cent of selenium.

For treating such a seleniferous deposit, as the above-mentioned, the following methods have been suggested and adopted.

Berzelius[2]) treated it with nitro-muriatic acid, precipitated the solution with hydrosulphuric acid, dissolved once more the precipitate in aqua regia, expelled the excess by heat, acted upon the residue with alkali, heated first alone, then with sal-ammoniac, and at last selenium sublimated, free from impurities previously present in it, of iron, zinc, copper, tin, arsenic, mercury and lead. A precisely similar proceeding is indicated by Liebe[3]) in 1860, only with the difference that the heating with sal-ammo-

[1]) Geol. fören. i Stockholm förh. Bd. II N:o 3.

[2]) Afhandlingar i Fys., Kemi och Miner. VI s. 50.

[3]) Arch. Pharm. S. 150. [1860].

niac is not carried so far as to cause sublimation of the selenium, which remains when the mass is extracted with water.

Wöhler [1]) employed two different methods. The seleniferous deposit was mixed with nitrate and carbonate of potassium and deflagrated: it was then uninterruptedly boiled with hydrochloric acid, in order to convert the selenic acid into selenious acid, which then was reduced with sulphurous acid. The other method consisted in igniting the seleniferous deposit with black flux, and extracting by means of boiling water the selenide of potassium thus formed. This solution, when exposed to the air, oxidates and yields the selenium.

Another procedure is suggested by Böttger [2]) and recommended by Otto [3]). It depends upon the circumstance that the selenium which exists uncombined in the above-mentioned deposit, is dissolved by boiling with sulphite of sodium, forming selenio-hyposulphite, from which selenium is precipitated by hydrochloric acid.

As regards the three methods first mentioned, they seemed to me to require operations so troublesome, that I was inclined, without hesitation, to give the preference to Böttger's method. In order to try its applicability, 2 kilogrs. of the deposit, completely washed and dried, were boiled with a concentrated solution of 425 grs. crystallized sulphite of sodium, in accordance with the given direction, until the mass was quite black. The filtrate being treated with hydrochloric acid, not even a trace of selenium was precipitated, but only a few flakes of sulphur with its peculiar yellowish colour. The method was consequently unavailable. And the cause is easily intelligible. It lies namely in the presence of the sulphate of lead, which is transposed with the seleniohyposulphite, first really formed, so that sulphate of sodium, selenide of lead and sulphurie acid are formed:

$$Na^2.O^2.SSeO + Pb.O^2.SO^2 + H^2O = Na^2.O^2.SO^2 + Pb.O^2.SSeO + H^2O$$
$$\overbrace{Pb\,Se} + \overbrace{H^2.O^2.SO^2}.$$

That such a transposition really takes place, was confirmed, in the first place, by the circumstance that, although the mass, by washing, had been wholly purified from free sulphuric acid, yet sulphurous acid, during the boiling, was continually disengaged, owing to the action of the free sulphuric acid thus produced on an excess of sulphite of sodium; and, mo-

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[1]) Berzelius Lehrb. d. Chemie 2 Bd. S. 198. Annal. d. Chemie u. Pharm. XCI S. 122 (1843.)

[2]) Journ. f. prakt. Chemie XCIV S. 439.

[3]) Lehrb. d. Chemie 4:te Aufl. I S. 634.

reover, by the fact that selenio-hyposulphite was really first formed, because the solution that was obtained by boiling the mass with sulphite of sodium only for a few moments, on adding hydrochloric acid, afforded a large precipitate of pink selenium, whereas its quantity, on further boiling, gradually diminished and finally quite disappeared. Thus for instance, a small quantity of the deposit, treated for a minute in this manner, gave a solution which with hydrochloric acid separated only sulphur.

This method is consequently quite unfit for producing selenium from the seleniferous residue of the leaden chambers, whenever the latter contains any compound which, like sulphate of lead, exercises a disturbing influence on the operation, because it is impossible to prevent the almost instantaneous decomposition of the selenio-hyposulphite of sodium into insoluble selenide and soluble sulphate.

Already in the year 1840 WIGGERS [1]) proved that selenium, like sulphur, may be dissolved in boiling cyanide of potassium, which was later confirmed by WÖHLER [2]); and ROSE [3]) has found that, from a mixture of sulphur and selenium, it chiefly extracts the latter. This property of a solution of cyanide of potassium and the relation of the thus formed selenocyanide of potassium to hydro-chloric acid, when the liberated selenocyanic acid resolves itself into hydro-cyanic acid and free selenium, has, according to OTTO [4]) been made use of to extract selenium from "Flugstaub". To this statement, however, he adjoins the remark that cyanide of potassium is by no means preferable to the cheaper sulphite of sodium.

In the case of the seleniferous residue from Falun, this method proved very simple and profitable. It most decidedly has the advantage of BÖTTGER'S method and every other hitherto known. Even for obtaining selenium from "Flugstaub", the mode of proceeding proposed by BÖTTGER seems to me inapplicable, or at least less profitable, for as this latter substance contains, among other things, also sulphate of copper it would probably be impossible to prevent a decomposition of the selenio-hyposulphit of natrium formed, by which at least some part of the selenium that is contained in the Flugstaub, remains in the residue as insoluble selenide of copper.

In order to obtain selenium from the seleniferous deposit, I have found it best to proceed in the following manner.

A quantity of ordinary cyanide of potassium is dissolved in warm water to tolerably strong concentration, and then allowed to act upon the

[1]) Ann. d. Chemie u. Pharm. XXIX S. 319.
[2]) Ann. d. Chemie u. Pharm. CIX S. 125.
[3]) Ann. d. Ph. und Ch. CXIII S. 624.
[4]) Lehrb. d. Chemi 4:te Aufl. I. S. 634.

seleniferous mass, when this, by means of washing has been freed from sulphuric acid. This will best take place in large porcelain-bowls, heated by steam.

The cyanide of potassium very soon exerts a dissolving action upon the uncombined selenium and sulphur of the mass, yielding selenio- and sulphocyanate of potassium, the latter only in small quantities, whilst the sulphate of lead is little or not at all attacked by it; in all cases, cyanide of lead is insoluble even in cyanide of potassium, so that the lead is in no danger of being dissolved. The red colour of the mass disappears in a few minutes; and when it has become quite grey without a tint of red, the whole amount of selenium is dissolved, of which I convinced myself by testing the residue for selenium. In fact, by observing this discolouration, one may avoid employing too large a quantity of cyanide of potassium, which would only be unnecessarily lost. The undissolved residue is now allowed to deposit itself and the solution separated by means of decantation and reiterated washing with warm water, as long as any remarkable quantity of selenium is to be observed in the washing-water, or, in other words, until, on adding hydrochloric acid and boiling, only a very slight opalescence in redish tint is discernable. If the solution thus obtained, after being filtered, is precipitated with hydrochloric acid, the whole amount of selenium contained in it is yielded in cherry-coloured flakes, under a brisk evolution of hydrocyanic acid; the precipitation ought therefore to be undertaken in the open air. The sulphur remains in the solution as sulphocyanic acid. The selenium is now obtained as a red voluminous precipitate which, when heated under water, cakes together in black, considerably smaller masses. Selenium obtained in this manner is still impure with small quantities of iron and copper, in form, no doubt, of ferrocyanide of copper. The cause of the presence of these compounds is undoubtedly to be looked for in the action of cyanide of potassium on the compounds of the above-meantioned metals existing in the deposit. Double salts of the cyanide of potassium and the cyanides of the two other metals enter into the solution, wherefrom, on adding hydrocloric acid, ferrocyanide of copper, being insoluble in this acid, is precipitated [1]) and thus mixes with selenium precipitated at the

[1]) Experiments, made by me, have pointed out that these solutions do not affect each other, before hydrochloric acid is added, when ferrocyanide of copper instantly subsides. Information on this subject being, to the best of my information, still wanting, it may be convenient here to observe, that ferrocyanide of potassium may be employed for the detection of copper in the double cyanures, merely by adding hydrocloric acid at the same time.

same time. The above mentioned are the only metals to be observed; the others are no doubt contained in the deposit in such compounds as are not attacked by cyanide of potassium.

In order to separate the selenium from the above-mentioned admixtures, I made an attempt to dissolve it in caustic soda, by which operation, as is known, selenide and selenite of sodium are formed. From this mixture, all selenium was precipitated as an extremely fine powder, hydroselenic and selenious acid mutually decomposing each other. Selenium treated in this way yielded, after oxidation with nitric acid, a solution green-coloured by copper; that copper could not, however, be precipitated with excess of soda. The strongly alkaline solution assumed an intense green colour and resembled a concentrated solution of nickel. On adding hydrosulphuric acid, it was discoloured and a green precipitate obtained, that blackened on boiling and, when submitted to the blow-pipe, gave the reactions of copper.

The dissolution in caustic soda having led to the above-mentioned result, it became obvious that the method of purifying, that I intended by the above proceeding, to avoid because it seemed to be attended with too much difficulty, viz. that of transforming selenium into selenious acid and sublimating the latter, must be resorted to. For this purpose selenium was dissolved in nitric acid and the solution evaporated to dryness in the water-bath; the selenious acid was then converted into anhydride, which was obtained in cakes of fine, satiny prisms, slightly green-coloured by copper. If the evaporation takes place too rapidly and with too great an excess of nitric acid, not insignificant quantities of selenious acid are also carried away with the fumes. The anhydride thus obtained was then laid in a tolerably wide tube of hard-melted glass, divided in the middle by an inserted stopple of asbestos, with or without a contraction of the glass at that point, and drawn out at both ends, so that the one part, wherein the impure acid was placed, might, by means of a wide caoutchouc-tube, be connected with an apparatus for drying and filtering through cotton-wool the current of air that was to be sucked through the tube during the operation; this was necessary, partly in order that the anhydride might be brought to sublimate into the other part, with which an aspirator, applied for the purpose, was connected, and partly that the sublimate might not obstruct the glass-tube. In order not to lose any selenious acid, which, in case of a rapid current of air, might easily pass into the aspirator, it will be convenient to place, between this and the sublimation tube, a Liebig's bulb-apparatus, filled with water. The heating is undertaken on a combustion-furnace; and by regulating the heat and the rapidity of the current of air,

there is no difficulty, after a little practice, in passing the anhydride into the other part of the tube, where it is obtained, now quite pure, in colourless, acicular, long crystals or, if the quantity is larger, in solid, firm crystal-cakes. The impurities remain as a melted, green mass, which, besides acid, contains the oxides of iron and copper. The pure anhydride being dissolved in water — which is most easily done by immersing that part of the tube into a somewhat wider tube with water — the same tube may be employed several times. Above all, it is indispensable in these sublimations to keep up a uniform current of air through the tube; selenious acid will, otherwise, obstruct it and cause its bursting, together with the inconveniences consequent thereupon. If it be neglected to filter the air through cotton, a little part of the anhydride is reduced by organic dust-particles, and the sublimate coloured red by selenium. When large quantities of acid are employed, it easily happens that the acid, in an impure state, retains a little nitric acid, owing to imperfect evaporation; it has therefore been found very convenient, before beginning the sublimation, slightly to warm the whole tube, whereby nitric acid is driven off and passes into the water of the bulb-apparatus.

SALTS OF SELENIOUS ACID.

The salts here to be treated of have been obtained in the following manner. The method, that seems to have been employed by Muspratt to procure a neutral alkali-salt, viz. that of treating the carbonate with selenious acid, as long as carbonic acid is given off, being found attended with serious difficulties, and the reaction of the neutral salts on litmus being moreover no criterion of the saturation of the acid since the neutral salt reacts as if alkaline, neutral salts of alkali were prepared as follows. To a given quantity of carbonate was added the requisite quantity of selenious acid, weighed in form of anhydride, such as it was obtained by the process of sublimation just described. The strongly sour reacting di- and tetraselenites of the alkaline metals were procured in the same manner. With such a mode of preparation, it was easy to remove the difficulty of which Muspratt complains.

Among the neutral salts of selenious acid only a very few are soluble in water, viz. those of the monatomic metals. The others are only with difficulty or not at all to be dissolved; they are easily obtained from selenite of sodium, by means of double decomposition with some other soluble salt, forming thereby a very often crystalline precipitate.

From them, acid salts have been prepared, in most cases by treating with the necessary quantity of selenious acid for forming di- and tetraselenites; in this manner, many metals, besides di-, also give tetraselenites; some, as magnesium, glucinum, cobalt, instead of tetraselenites, afford triselenites, some others have yielded sesquiselenites, for instance ammonium, aluminium, cadmium, didymium, formerly unknown stages of saturation of selenious acid. Moreover, acid salts have been obtained also in other stages of saturation; this is especially the case with those which form sesquioxides, of which group $^4/_3$-selenites seem to be particularly characteristic; more seldom, they give $^5/_3$-selenites. That a metal does not give any acid salts may be looked upon as a very rare exception. BERZELIUS' supposition that all acid salts would be soluble in water, is not supported by experimental evidence; on the contrary, it is not unusual that insoluble acid salts occur, as well of the heavy metals, as of the earth-metals. His statement that selenious acid shows very little tendency to form basic salts, must also be submitted to some correction. It happens namely very often that, on mixing neutral solutions, basic salts are precipitated; this is the case with glucinum, aluminium, chromium, iron, indium, uranium, cerium, lanthanum, didymium etc. Such selenites as are but sparingly, or not at all soluble in water, for the most part, readily dissolve in hydrochloric acid; some few, however make an exception from this rule; this is especially the case with selenite of zirconium, which can be dissolved, only when heated for a very long time with the warm acid. Sometimes the salt, on dissolving in hydrochloric acid, yields free selenium. That this is the case with the ferrous and mercurous selenites, has been already some time known; the salts of thallium present the same phaenomena, which depends on an oxidation, at the expense of the selenious acid, into ferric, mercuric etc. salts that are dissolved. For the same reason, a stannous selenite cannot exist.

After these general remarks, and before entering upon the description of the different compounds, I will first give an account of the analytical methods applied, in order that I may not be obliged in every particular case to state the mode of determining the composition.

ANALYTICAL METHODS.

BERZELIUS, in his analyses, has not determined selenious acid otherwise than as a loss. On accounting for the analyses of the selenites of sodium, he mentions an unsuccessful attempt to precipitate selenium from

their solutions, previously acidulated with hydrochloric acid. But the method was judged unfit; for, when the sulphide of selenium seemed to be completely deposited, there was still, on evaporating, obtained a precipitate of the same compound; and, on further evaporation to dryness, the residue, when heated, smelled strongly of selenium, and the crucible was attacked by it.

Muspratt has, concerning this matter: "The determination of selenious acid is very difficult; fusion with nitrate of potash is the first step to convert the acid into selenic acid, which latter can then be precipitated by nitrate of baryta;" he also points out some precautions that must be observed on heating the selenate of barium previously to the weighing, but he only once in his analyses applies this method of determination. In other cases, a water estimation, of the details of which, except in a single instance, (diselenite of sodium), we are left wholly ignorant, is most often considered to be sufficient; more rarely is the quantity of the base determined, but so much the oftener a formula assumed, without any kind of analytical basis.

Rose [1]), in a detailed memoir on the quantitative determination of selenium, also mentions the methods of determining selenious acid. His researches proved, in the first place, with regard to its precipitating and weighing as selenate of barium, that this method of proceeding suffers from two great faults, rendering it completely unavailable. Selenate of barium is namely soluble in water and still more so in acids; it shares, moreover, with the corresponding sulphate the property of precipitating, along with it, other salts present in the solution, which, though soluble by themselves, cannot be washed out from the precipitate with boiling water. If a good result happens to be obtained, the only cause is, that the two faults compensate each other.

The method of precipitating selenious acid with hydrosulphuric acid, stated to be unavailable by Berzelius, is considered by Rose as leading to the complete precipitation of the acid, in form of selenium mixed with sulphur — $Se + S^2$ —; the quantity of selenium is calculated from the weight of the precipitate dried at 100°. Should there be any reason to suppose that the precipitate contains excess of sulphur — which easily takes place on precipitating with hydrosulphuric acid — he recommends treatment with oxidating agents and to reduce the selenious acid then formed with sulphurous acid. It seems to me, however, to be a shorter way to treat the precipitate, obtained by means of hydro-

[1]) Ann. d. Ph. u. Ch. CXIII S. 472.

sulphuric acid, with cyanide of potassium and acidulate the solution with hydrochloric acid; selenium is then, as ROSE has found, completely precipitated. In analyzing the salts of mercury, I employed both methods at the same time, and with complete success.

The best method for the quantitative determination of selenious acid was found by ROSE to be reduction with sulphurous acid; it is then necessary to add hydrochloric acid. This is the way I have chosen for the analysis of most of the selenites here described. The salts, soluble as well as insoluble, have been acidulated with about 10 c. c. concentrated hydrochloric acid, by means of which even the latter are brought into a soluble state; then a solution of sulphurous acid, saturated at the ordinary temperature, has been added, and the covered vessel, holding only 100 c. c., placed to digest at about 60° for nearly 12 hours, when it was usually impossible to remark any smell of sulphurous acid and the reduction of selenium was completed. If the reduction be accelerated by means of stronger heating, small quantities of selenious acid are carried off with the speedily disappearing sulphurous acid gas, as tetrachloride of selenium, by which a loss of selenium, often amounting to several percent, may be caused. By proceeding in the manner just mentioned, the loss, on the contrary, is very slight or none, as the numerous selenium determinations, communicated in the following pages, will show. OTTO's [1]) statement that selenium thus becomes impure with heavy metals, if any such occur in the solution, has not been confirmed in a single instance. Selenium was obtained in shining, black, caked masses at the bottom of the glass; only a small quantity had been deposited on the sides as a yellowish-red, brilliant covering, which was easily removed with a feather. When washed by means of decantation with boiling water, it was collected on a weighed filter, dried at 100° and weighed. The bases, after evaporation of the filtrate and expulsion of the excess of sulphuric acid, have in most cases been determined as waterfree sulphates, but, when this was impossible, as oxides, usually by heating the sulphates to whiteness. Sometimes, as in the cases of the salts of barium, strontium, calcium etc., peculiar methods have been found necessary, which will be accounted for in their proper places. Where the contrary is not particularly remarked, all the selenites have been prepared for analysis by previous pressing between folds of bibulous paper.

[1]) Lehrb. d. Chemie 4:te Aufl. I S. 634—635.

SELENITES OF POTASSIUM.

1. Neutral: $K^2O^2.SeO + H^2O$.

This salt was prepared from carbonate and selenious acid, weighed off in corresponding molecular quantities, and the solution continually boiled, in order to remove all dicarbonate possibly formed. It was then allowed to evaporate over sulphuric acid at the ordinary temperature, the syrupy mass becoming filled with laminar, four-sided, probably rhombic prisms, with the ends obliquely truncated on both sides. The salt is so deliquescent that the crystals could not be collected and freed, by pressing, from mother-liquor. In order to obtain the salt in a fit state for analysis, it was therefore dried over sulphuric acid to constant weight.

Analyses:

1) 0.5992 gr. salt gave 0.2115 gr. selenium or 0.2972 gr. selenious acid and 0.4715 gr. sulphate of potassium, containing 0.255 gr. potash.
2) 0.9133 gr. salt gave 0.3239 gr. selenium or 0.4551 gr. selenious acid and 0.7108 gr. sulphate of potassium, containing 0.3844 gr. potash.

Centesimally represented:

	experiment			theory	
	1.	2.			
Potash	42.56	42.09	K^2O	94.2	42.20
Selenious acid . . .	49.60	49.83	SeO^2	111.0	49.73
Water (loss)	7.84	8.08	H^2O	18.0	8.07
	100.00	100.00		223.2	100.00

Berzelius found this salt insoluble in alcohol; when alcohol is added to the aqueous solution, the selenite that has but little tendency to crystallize, separates as an oily layer under the alcoholic solution. Like the other neutral selenites of the alkaline metals, it is persistent in a fused state, if protected from the action of reducing gases; otherwise, it is partially reduced to selenide of potassium. Muspratt communicates an analysis of selenite of potassium showing an amount of 45.02 pr. ct. potash, without mentioning, how the analyzed compound has been previously treated. This amount of potash nearly agrees with the formula for waterfree salt, which requires 45.91 pr. ct.; it is also possible that he had before him such a salt, obtained by previous heating.

2. **Diselenite:** $KH.O^2.SeO$.

This salt was obtained by treating 1 mol. carbonate with 2 mol. anhydride of selenious acid, and crystallized from this solution, after strong concentration over sulphuric acid, as laminar, four-sided, probably rhombic prisms, the ends of which were obliquely truncated on both sides. The salt is very deliquescent on exposure to the air. On account of this, it was, before the analysis, desiccated between folds of bibulous paper and dried over vitriol oil.

Analyses:

1) 0.9765 gr. salt gave 0.4563 gr. selenium or 0.6411 gr. selenious acid and 0.5068 gr. sulphate of potassium, containing 0.2741 gr. potash.
2) 0.9285 gr. salt gave 0.434 gr. selenium or 0.6098 gr. selenious acid and 0.4795 gr. sulphate of potassium, containing 0.2593 gr. potash.
3) 0.7845 gr. salt, when heated for several days at 100°, lost 0.042 gr. of its weight.

Centesimally represented:

	experiment				theory	
	1.	2.	3.			
Potash	28.07	27.93	—	$\frac{1}{2}K^2O$	47.1	28.19
Selenious acid . .	65.65	65.68	—	SeO^2	111.0	66.42
Water	6.28	6.39	5.35	$\frac{1}{2}H^2O$	9.0	5.39
	100.00	100.00			167.1	100.00

Consequently the salt at 100° completely, though very slowly, loses its hydrogen in form of water, and becomes an anhydrous salt without fusing; at a higher temperature, half the selenious acid passing off, it is converted to a neutral salt.

MUSPRATT communicates a potash determination on diselenite of potassium, closely agreeing with the formula above.

3. **Tetraselenite:** $KH.O^2.SeO + H^2.O^2.SeO$,

crystallizes from a solution, containing 1 mol. carbonate to 4 mol. selenious acid, in form of glass-shining prisms, permanent on exposure to the air and generally exhibiting a great number of combination-faces. There is seldom observed a rhombic prism, combined only with predominant macrodiagonal end-faces and pyramid-faces of the same order, but generally also

with a predominant basal endface, owing to which the crystals assume the form of slightly oblique rhombic laminæ, the edges truncated by the pyramid-faces and the obtuse angles by the endfaces. Several other combinations were also to be observed. The salt is very soluble.

Analyses:

1) 0.8858 gr. salt gave 0.4708 gr. selenium or 0.6615 gr. selenious acid, and 0.2573 gr. sulphate of potassium or 0.1392 gr. potash.
2) 0.8798 gr. salt gave 0.4665 gr. selenium or 0.6555 gr. selenious acid, and 0.2578 gr. sulphate of potassium or 0.1394 gr. potash.
3) 0.8022 gr. salt, when kept over sulphuric acid for three days, only lost 0.0014 gr. of its weight, but, on heating at 100° during the same time, parted with 0.0435 gr.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Potash	15.71	15.85	—	$^1/_2K^2O$	47.1	15.91
Selenious acid. . .	74.68	74.50	—	$2SeO^2$	222.0	74.97
Water (loss) . . .	9.61	9.65	5.42	$^1/_2H^2O$	27.0	9.12
	100.00	100.00			296.1	100.00

At 100° the salt gives off 5.42 pr ct. water; this forming upwards two thirds the whole amount, it seems probable that the product thus obtained, being viscid and fused in heat, glazed and slightly red-coloured by reduced selenium in cold, is an anhydrous salt of the formula:

$$KH.O^2.SeO + SeO^2.$$

In all probability, more water cannot be expelled at this temperature, the loss during the last 40 hours not exceeding 0.0005 gr. It is very remarkable that neither BERZELIUS nor MUSPRATT has obtained this salt crystallized, though it undoubtedly most easily among all the salts of potassium and without difficulty affords large crystals. Both of them also declare the salt to be highly deliquescent, although it is completely permanent on exposure to the air. It is evident that their method of preparation has not given a pure product.

SELENITES OF RUBIDIUM

have hitherto not been investigated. As was to be expected, they exhibit a complete analogy to the salts of potassium.

1. Neutral: $Rb^2.O^2.SeO + H^2O$.

This salt, with the greatest difficulty, separates from a syrupy solution, prepared from carbonate and selenious acid in corresponding molecules, and is obtained in microscopical crystals of a prismatic appearance, united in groups and with the ends ensiformly pointed; most of them are there furnished with an angle directed inwards. The salt is so extremely deliquescent that, even under the receiver of the air-pump, it was very difficult to obtain it in a fit state for analysis. It was desiccated, as well as possible, between folds of bibulous paper; the analysis indicated that by this treatment and whilst weighed, it had absorbed a not inconsiderable quantity of water.

Analyses:

1) 0.936 gr: salt gave 0.221 gr. selenium or 0.3105 gr. selenious acid and 0.7665 gr. sulphate of rubidium or 0.5367 gr. rubidia.
2) 0.4175 gr. salt gave 0.098 gr. selenium or 0.1377 gr. selenious acid and 0.337 gr. sulphate of rubidium or 0.2359 gr. rubidia.

Centesimally represented:

	found			calculated	
	1.	2.			
Rubidia	57.34	56.50	Rb^2O	186.8	59.15
Selenious acid . .	33.18	32.98	SeO^2	111.0	35.15
Water (loss) . . .	9.48	10.52	H^2O	18.0	5.70
	100.00	100.00		315.8	100.00

The less close correspondence between found and calculated values is a consequence of the extreme deliquescence of the salt; this, however, does not render its composition in the least doubtful.

2. Diselenite: $RbH.O^2.SeO$.

When the solution has evaporated down to a syrupy mass, this salt is obtained in oblique, four-sided prisms with basal end-faces; they have

commonly their acute combination-edges between the prism-faces and the basal end-faces truncated and, besides, exhibit several other combinations. The salt is very deliquescent, but becomes opake in contact with dry air.

Analyses:

1) 0.465 gr. salt gave 0.1673 gr. selenium or 0.2351 gr. selenious acid and 0.2843 gr. sulphate of rubidium or 0.199 gr. rubidia.
2) 0.4215 gr. salt gave 0.152 gr. selenium or 0.2136 gr. selenious acid and 0.257 gr. sulphate of rubidium or 0.18 gr. rubidia.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Rubidia	42.80	42.70	$\frac{1}{2}Rb^2O$	93.4	43.79
Selenious acid . . .	50.56	50.68	SeO^2	111.0	52.01
Water (loss)	6.64	6.62	$\frac{1}{2}H^2O$	9.0	4.20
	100.00	100.00		213.4	100.00

The analysis has given too low values of alkali and acid, owing to the hygroscopic tendency of the salt; the composition here assumed is not, however, subject to the least doubt, as agreeing with that of the corresponding salt of potassium.

3. Tetraselenite: $RbH.O^2.SeO + H^2.O^2.SeO$.

A solution of this salt was prepared by decomposing 1 mol. carbonate of rubidium with 4 mol. selenious anhydride. Being evaporated over sulphuric acid until it had the consistency of a thick syrup, it yielded a salt in long prisms, united to a radiate crystal-cake, in which the form of the individual crystals was not to be ascertained. It is permanent on exposure to the air. Like the acid alkaline salts in general, it easily receives a reddish tint from partially reduced selenium.

Analyses:

1) 0.616 gr. salt gave 0.283 gr. selenium or 0.3976 gr. selenious acid, the rubidium estimation was lost.
2) 0.52 gr. salt gave 0.239 gr. selenium or 0.3358 gr. selenious acid and 0.198 gr. sulphate of rubidium or 0.1387 gr. rubidia.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Rubidia	—	26.68	$\frac{1}{2}Rb^2O$.	93.4	27.28
Selenious acid . .	64.55	64.58	$2SeO^2$	222.0	64.83
Water (loss) . . .	—	8.74	$1\frac{1}{2}H^2O$	27.0	7.89
		100.00		342.4	100.00

SELENITES OF CÆSIUM

have also hitherto not been investigated; they show a complete analogy to the selenites already treated of, except in the proportion of water contained in the neutral salt.

1. Neutral: $Cs^2O^2.SeO + 3H^2O$.

The salt, prepared from carbonate of cæsium with a necessary quantity of selenious acid, crystallized from a very concentrated solution and formed a radiate mass of which the individual crystals seemed to be long, oblique, four-sided prisms, with obliquely truncated ends. It is so deliquescent that the crystals, within a few moments, liquefy in air. After desiccating between bibulous paper, it was consequently, for the following analyses. dried over sulphuric acid under the receiver of the air-pump.

Analyses:

1) 1.11 gr. salt gave 0.194 gr. selenium or 0.2726 gr. selenious acid and 0.903 gr. sulphate of cæsium or 0.7034 gr. oxide of cæsium.
2) 0.9175 gr. salt gave 0.1595 gr. selenium or 0.2241 gr. selenious acid and 0.749 gr. sulphate of cæsium or 0.5835 gr. oxide of cæsium.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Oxide of cæsium . . .	63.37	63.60	Cs^2O	282	63.09
Selenious acid	24.56	24.42	SeO^2	111	24.83
Water (loss)	12.07	11.98	$3H^2O$	54	12.08
	100.00	100.00		447	100.00

2. **Diselenite:** $CsH.O^2.SeO$.

A solution, obtained by treating 1 mol. carbonate of cæsium with 2 mol. selenious anhydride, having assumed the consistency of syrup, afforded the diselenite in small, oblique, four-sided tablets, often six-sided the acute angles being truncated, rarely forming long prisms. The salt is deliquescent on exposure to the air, but not in so high a degree as the neutral salt. It was prepared for analysis by pressing and subsequent drying over sulphuric acid under the receiver of the airpump.

Analyses:

1) 0.679 gr. salt gave 0.2005 gr. selenium or 0.2817 gr. selenious acid and 0.4665 gr. sulphate of cæsium, corresponding to 0.3634 gr. oxide of cæsium.
2) 0.6945 gr. salt gave 0.206 gr. selenium or 0.2895 gr. selenious acid and 0.4765 gr. sulphate of cæsium, corresponding to 0.3712 gr. oxide of cæsium.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Oxide of cæsium . . .	53.52	53.45	$\frac{1}{2}Cs^2O$	141	54.02
Selenious acid	41.49	41.68	SeO^2	111	42.53
Water (loss)	4.99	4.87	$\frac{1}{2}H^2O$	9	3.45
	100.00	100.00		261	100.00

3. **Tetraselenite:** $CsH.O^2.SeO + H^2.O^2.SeO$.

This salt, deposited easily from a solution, containing 1 mol. oxide of cæsium to 4 mol. selenious anhydride, in large, four-sided, laminar crystals, remains unaltered on exposure to the air.

Analyses:

1) 0.5 gr. salt gave 0.199 gr. selenium or 0.2797 gr. selenious acid and 0.2288 gr. sulphate of cæsium, containing 0.1782 gr. oxide of cæsium.
2) 0.5385 gr. salt gave 0.217 gr. selenium or 0.3049 gr. selenious acid and 0.248 gr. sulphate of cæsium, containing 0.1932 gr. oxide of cæsium.

Calculated on 100 parts:

	found			theory	
	1.	2.			
Oxide of cæsium . .	35.64	35.88	$^1/_2Cs^2O$	141	36.15
Selenious acid	55.94	56.66	$2SeO^2$	222	56.92
Water (loss)	8.42	7.46	$1^1/_2H^2O$	27	6.93
	100.00	100.00		390	100.00

SELENITES OF AMMONIUM.

1. **Neutral:** $Am^2.O^2.SeO + H^2O$.

If the strongest caustic ammonia be added to anhydride of selenious acid, a lively combination takes place under effervescence and evolving of great heat. Particles of selenium are then sometimes separated, probably owing to the strength of the heat and the decomposition between ammonia and selenious acid, which, as BERZELIUS has shown, cannot consist together at a higher temperature. A solution thus prepared and still containing ammonia in great excess, when allowed to evaporate to strong concentration over sulphuric acid, affords a salt in well developed, large crystals; this is not, however, a neutral salt, but a sesquiselenite, as will be soon shown. BERZELIUS' supposition that a neutral salt is obtained in this manner, does not therefore hold good; it seems also to have been pronounced without any previous analyses.

On the contrary, if selenious anhydride be dissolved in alcohol, in which it is very soluble, and ammonia-gas passed into the solution, this is considerably heated and, when nearly saturated with gas, affords the neutral salt so speedily that the gas-delivery-tube is very liable to be obstructed by it. The salt is thus obtained in small, acicular, milk-white crystals, united in bundles, and, for reasons easily intelligible from what follows, must be speedily desiccated and weighed in a closely covered vessel, in order to obtain it in a state fit for analysis. Ammonia-determinations on this and following compounds are made by heating the salt with soda-lye, passing the gas into hydrochloric acid, evaporating the solution at 100° and weighing the remaining sal-ammoniac.

Analyses:

1) 0.4678 gr. salt gave 0.2663 gr. sal-ammoniac or 0.1295 gr. oxide of ammonium.
2) 0.8428 gr. salt gave 0.472 gr. sal-ammoniac or 0.2296 gr. oxide of ammonium.
3) 0.794 gr. salt gave 0.3625 gr. selenium or 0.5093 gr. selenious acid.
4) 0.4415 gr. salt gave 0.199 gr. selenium or 0.2796 gr. selenious anhydride.

Calculated on 100 parts:

	1.	2.	3.	4.
Oxide of amonium	27.69	27.24	—	—
Selenious acid . . :	—	—	64.15	63.34

numbers very closely corresponding to the formula above, which requires:

Am^2O	52	28.73
SeO^2	111	61.32
H^2O	18	9.95
	181	100.00

Though the salt was prepared for analysis with the greatest speed and care, yet the result exhibits a less close correspondence with the calculated values, the compound very speedily giving off considerable quantities of ammonia, of which more here after.

Muspratt, who has obtained a salt of ammonium in the way here mentioned, found it to contain 32.49 per cent oxide of ammonium. Considering, however, the great tendency of this salt to give off ammonia, it seems rather unaccountable, how he has been able to obtain this high percentage of oxide, not only amounting to, but even, on using the atomic weights here employed, exceeding by 0.6 per cent what is required by the formula for a waterfree salt $Am^2.O^2.SeO$.

When the neutral salt now described was left in a vessel, covered with a glass-plate, under the mother-liquor in which it had been formed, the milk-white, fine needles were changed into comparatively much larger, short, thick, transparent, colourless crystals, the form of which it was impossible more closely to determine. Like the foregoing salt, it smelled of ammonia and soon liquefied in contact with the air. In spite of its changed appearance, it proved to be completely unaltered in composition.

Analyses:

1) 0.5085 gr. salt gave 0.22 gr. selenium or 0.3091 gr. selenious acid.

2) 0.806 gr. salt gave 0.354 gr. selenium or 0.4974 gr. selenious acid.
3) 0.74 gr. salt gave 0.4315 gr. sal-ammoniac or 0.2099 gr. oxide of ammonium.
4) 0.7262 gr. salt gave 0.413 gr. sal-ammoniac or 0.2009 gr. oxide of ammonium.

Calculated on 100 parts:

	1.	2.	3.	4.
Oxide of ammonium	—	—	28.36	27.67
Selenious acid	60.78	61.73	—	—

numbers closely agreeing with the formula above.

If, on the contrary, this salt, in one or the other form, be taken up from the mother-liquor and desiccated by pressing, it gives off ammonia and smells strongly of it. On this account, it suffers a considerable loss of weight, when it is kept in contact with the air, and then speedily deliquesces. If such a liquid, obtained by spontaneous deliquescence, when no longer emitting any smell of ammonia, be allowed to crystallize over sulphuric acid, it affords the residue of the neutral salt in form of monoclinic, four-sided prisms with basal end-faces, a crystalline form, characteristic for the diselenite mentioned below. It was also proved by the analysis of the product, that this salt had been formed, for

1) 0.531 gr. salt gave 0.2892 gr. selenium or 0.4063 gr. selenious acid or, in 100 parts, 76.52 (required 76.03).
2) 1.268 gr. salt gave 0.465 gr. sal-ammoniac or 0.2262 gr. oxide of ammonium or 17.84 pr. ct. (required 17.81).

2. **Sesquiselenite:** $2Am^2.O^2.SeO + H^2.O^2.SeO$.

This salt was obtained, in the manner already accounted for, in very well developed (monoclinic?) prisms with basal end-faces and hemidoma. The crystals often reach a considerable length, 30 m. m. and more. They are transparent, shining and deliquescent on exposure to the air.

Analyses:

1) 0.5155 gr. salt gave 0.2491 gr. sal-ammoniac or 0.1211 gr. oxide of ammonium and 0.262 gr. selenium or 0.3681 gr. selenious acid.
2) 0.5983 gr. salt gave 0.2851 gr. sal-ammoniac or 0.1387 gr. oxide of ammonium and 0.308 gr. selenium or 0.4328 gr. selonious acid.

Represented in 100 parts:

	found 1.	found 2.		calculated	
Oxide of ammonium.	23.50	23.18	$2Am^2O$	104	22.86
Selenious acid	71.41	72.34	$3Se^2O$	333	73.18
Water (loss)	5.09	4.48	H^2O	18	3.96
	100.00	100.00		455	100.00

Over sulphuric acid, the salt diminishes in weight and becomes milk-white; the loss, in one experiment, was 15 pr. ct., or the whole amount of water and half the amount of oxide of ammonium, so that the residue was possibly an anhydrous salt of the formula $Am^2.O^2.SeO + 2SeO^2$; I am, however, able to furnish no other quantitative determination, to confirm the truth of this supposition.

3. **Diselenite:** $AmH.O^2.SeO$.

Was obtained from a syrupy solution, prepared by acting upon the neutral salt with an appropriate quantity of anhydride of selenious acid, and crystallized in four-sided, probably monoclinic prisms with basal end-faces; the acute edges between these and the prism-faces were very often truncated. They are generally united into larger aggregates and deliquesce speedily on exposure to the air.

Analyses:

1) 0.5694 gr. salt gave 0.213 gr. sal-ammoniac or 0.1035 gr. oxide of ammonium and 0.3082 gr. selenium or 0.4331 gr. selenious anhydride.
2) 1.1434 gr. salt gave 0.4156 gr. sal-ammoniac or 0.202 gr. oxide of ammonium.

Centesimally represented:

	found 1.	found 2.		calculated	
Oxide of ammonium . .	18.18	17.67	$\frac{1}{2}Am^2O$	26	17.81
Selenious acid	76.07	—	SeO^2	111	76.03
Water (loss)	5.77	—	$\frac{1}{2}H^2O$	9	6.16
	100.00			146	100.00

It has already been remarked that this salt is prepared from the neutral selenite, by means of spontaneous decomposition in contact with the air.

The analogy between the selenites, now analyzed, and the corresponding carbonates is obvious. As one of the several products of decomposition, a sesquiselenite appears here, corresponding to the sesquicarbonate of ammonium, frequently met with in commerce.

4. Tetraselenite: $AmH.O^2.SeO + H^2.O^2.SeO$.

From a solution, obtained by treating the neutral selenite of ammonium with precisely three molecules of selenious anhydride or 184 p. ct., the tetraselenite crystallized over sulphuric acid under the receiver of the airpump, only with great difficulty, however, the crystals being so deliquescent, that it is hardly possible to ascertain their form under the microscope, before they have liquefied at the edges. The salt forms large, long, four-sided prisms, with oblique end-faces. It was analyzed after long keeping over sulphuric acid, when there remained a white, opake salt-mass.

Analyses:

1) 0.822 gr. salt gave 0.1415 gr. sal-ammoniac or 0.0688 gr. oxide of ammonium.
2) 0.7555 gr. salt gave 0.128 gr. sal-ammoniac or 0.0622 gr. oxide of ammonium.
3) 0.5465 gr. salt gave 0.3415 gr. selenium or 0.4798 gr. selenious anhydride.
4) 0.556 gr. salt gave 0.347 gr. selenium or 0.4875 gr. selenious anhydride.

In 100 parts:

	1.	2.	3.	4.
Oxide of ammonium	8.37	8.23	—	—
Selenious acid	—	—	87.80	87.69

These values, it is true, do not agree with the formula above, $AmH.O^2.SeO + H^2.O^2.SeO$, which requires

$\frac{1}{2}Am^2O$	26	9.45
$2SeO^2$	222	80.73
$1\frac{1}{2}H^2O$	27	9.82
	275	100.00;

but it is hardly doubtful that the obtained, highly deliquescent salt has this composition, in analogy to all tetraselenites of the monatomic metals. The

analyses seem to prove that ammonium and selenious acid have very little tendency to combine, the tetraselenite not being sufficiently permanent to retain its ammonia, when kept for a long time over sulphuric acid. I have not had the opportunity of ascertaining whether it be possible to obtain a final product of free selenious acid in this manner, the ammonia totally disappearing.

SELENITES OF THALLIUM.

Kuhlmann has examined the neutral salt and the diselenite of this metal, without, however, adjoining any numbers. What here follows will, for the chief part, confirm his statements.

1. Neutral: $Tl^2.O^2.SeO$.

This salt was prepared by saturating the diselenite described below with an appropriate quantity of carbonate of thallium. The solution, as well at the ordinary temperature as when warmed, yielded the salt in extremely thin, yellow, shining, oblique, four-sided lamels, with one or, more generally, all the summits cut off; they probably belong to the rhombic system. The salt is so little soluble in cold water, that a somewhat dilute solution in warm water, on cooling easily deposits part of it [1]).

Analyses:

1) 0.487 gr. salt gave 0.067 gr. selenium or 0.0941 gr. selenious acid and 0.4585 gr. sulphate of thallium or 0.3858 gr. thallious oxide.
2) 0.539 gr. salt gave 0.0745 gr. selenium or 0.1047 gr. selenious acid and 0.5065 gr. sulphate of thallium or 0.4261 gr. thallious oxide.

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[1]) I avail myself of the opportunity of correcting a mistake that has been inserted in Otto's Lehrbuch der Chemie, 2:ter Th. S. 1388, where it is stated that Werther (Journ. f. prakt. Ch. XCI S. 385, XCII S. 128, 351) has examined a thallium-zinc-selenite, isomorphous and composed analogically to the corresponding sulphate. Werther has never stated to have operated with such a salt, but the statement will be correct, if selenite be replaced by selenate, or if it be placed under the rubric "selensaures Thalliumoxidul". To the double salts of thallium I shall have occasion to return in connexion with the other compounds of this kind.

Centesimally represented:

	found 1.	found 2.		calculated	
Thallious oxide	79.22	79.06	Tl^2O	424	79.25
Selenious acid	19.32	19.43	SeO^2	111	20.75
	98.27	98.49		535	100.00

2. **Diselenite:** $TlH.O^2.SeO$.

This salt was obtained by means of double decomposition of sulphate of thallium with the anhydrous salt of barium, $Ba.O^2.SeO + SeO^2$, which shall be hereafter described: it separates readily from an aqueous solution in large, yellow, wax-shining, four-sided, possibly quadratic prisms, combined with pyramid of another order, terminal face and other faces. It dissolves very easily even in cold water.

Analyses:

1) 0.562 gr. salt gave 0.1332 gr. selenium or 0.1872 gr. selenious acid and 0.425 gr. sulphate or 0.3575 gr. thallious oxide.
2) 0.5025 gr. salt gave 0.1174 gr. selenium or 0.165 gr. selenious acid and 0.38 gr. sulphate of thallium or 0.3197 gr. thallious oxide.

Centesimally represented:

	found 1.	found 2.		calculated	
Thallious oxide . .	63.61	63.62	$\frac{1}{2}Tl^2O$	212	63.86
Selenious acid . . .	33.31	32.84	SeO^2	111	33.43
Water (loss)	3.08	3.54	$\frac{1}{2}H^2O$	9	2.71
	100.00	100.00		332	100.00

3. **Tetraselenite:** $TlH.O^2.SeO + H^2.O^2.SeO$.

By treating 1 mol. of the next preceding salt with 1 mol. selenious anhydride, a solution was obtained which, when it had assumed such a consistency that it was scarcely liquid, afforded this salt in very large, long, probably monoclinic prisms, with predominant orthodiagonal end-faces and the ends obliquely truncated on both sides. The compound is very soluble in water and permanent in contact with the air.

Analyses:

1) 0.7395 gr. salt gave 0.2525 gr. selenium or 0.3548 gr. selenious acid and 0.4155 gr. sulphate of thallium or 0.3494 gr. thallious oxide.
2) 0.598 gr. salt gave 0.2043 gr. selenium or 0.287 gr. selenious acid and 0.3215 gr. sulphate of thallium or 0.2705 gr. thallious oxide.

Calculated on 100 parts:

	experiment		theory		
	1.	2.			
Thallious oxide . .	47.26	45.24	$\frac{1}{2}Tl^2O$	212	45.99
Selenious acid . .	47.98	48.00	$2SeO^2$	222	48.16
Water (loss) . . .	4.76	6.76	$1\frac{1}{2}H^2O$	27	5.85
	100.00	100.00		461	100.00

When this salt, for the analysis, was treated with hydrochloric acid, pink selenium was thrown down, thallious oxide being oxidated at the expense of the selenious acid. A similar process takes place, when ferrous and hydrargyrous selenites are dissolved in hydrochloric acid.

SELENITES OF SODIUM.

1. Neutral: $Na^2.O^2.SeO + 5H^2O$.

When a solution containing this salt, prepared quite in the same manner, as the corresponding salt of potassium, by evaporation at the ordinary temperature, has assumed the consistency of syrup, it begins to afford the salt, either in microscopic needles united in bundles, strongly contracted in the middle, or, more rarely, in very large, four-sided prisms, with the ends on both sides obliquely cut off. The crystals are persistent in contact with the air and extremely soluble in water. In dry air, they part with their crystallization-water.

Analyses:

1) 0.689 gr. salt gave 0.207 gr. selenium or 0.2909 gr. selenious acid and 0.3708 gr. sulphate of sodium or 0.1619 gr. soda.
2) 0.641 gr. salt gave 0.1915 gr. selenium or 0.269 gr. selenious acid and 0.344 gr. sulphate of sodium or 0.1502 gr. soda.

3) 0.4783 gr. salt gave off 0.1628 gr. water, when kept over sulphuric acid.
4) 0.95 gr. of the salt that was obtained crystallized in large, four-sided prisms, lost 0.324 gr. of its weight, when kept over sulphuric acid.

Calculated on 100 parts:

	1.	2.	3.	4.
Soda	23.50	23.43	—	—
Selenious acid . .	42.22	41.97	—	—
Water	—	—	34.02	34.10,

the formula requiring:

Na^2O	62	23.58
SeO^2	111	42.20
$5H^2O$	90	34.22
	263	100.00

In waterfree state, this salt is obtained well crystallized in small, milk-white, four-sided, quadratic prisms with basal end-faces, if the crystallization takes place at a higher temperature 60°—100°.

0.453 gr. of this salt did not diminish in the weight, either in dry air, or when heated to 100°, or when fused.

By heating this salt, such as it was obtained after fusion, with sal-ammoniac and weighing the remaining chloride of sodium, Berzelius determined its amount of soda to be 35.50 pr. ct.; the calculated quantity is 35.84 pr. ct.

2. Neutral: $Na^2.O^2.SeO$.

Muspratt says that he has not been able to obtain neutral selenite of sodium crystallized with water from its aqueous solution; may be he has overlooked that this salt, when kept in dry air, loses the 5 mol. of crystallization-water that it always contains, when allowed to crystallize at the ordinary temperature, from a neutral solution. The material, on which he undertook a soda-determination, was dried over sulphuric acid and then contained 35.50 pr. ct. soda. It may also be very possible that it had not deposited from a completely neutral, but from an acid solution, and, in this case, waterfree selenite of sodium is obtained, as I had occasion to learn in an attempt to ascertain, whether there existed any sesquiselenite of sodium. A solution, prepared for this purpose from 2 mol. of carbonate and

3 mol. of selenious anhydride, afforded, after evaporation to a syrupy mass, a salt in the crystalline form of waterfree selenite of sodium.

Analysis:

0.6 gr. salt gave 0.2715 gr. selenium or 0.3815 gr. selenious acid and 0.459 gr. sulphate of sodium or 0.2004 gr. soda.

In 100 parts:

	experiment		theory	
Soda	33.42	Na^2O	62	35.84
Selenious acid	63.58	SeO^2	111	64.16
	97.00		174	100.00

The compound thus obtained was consequently a waterfree selenite which, to judge from the analysis, must have been in some measure contaminated with diselenite; in fact, this salt began afterwards to crystallize in large quantities. For this reason, it seems as if no sesquiselenite could be formed.

3. **Diselenite:** $NaH.O^2.SeO$.

Prepared as the corresponding selenite of potassium, this salt crystallizes from its solution, when considerably concentrated, in radially arranged groups of probably monoclinic prisms, combined with basal end-face, pyramid of another order, hemi-pyramid- and hemi-dome-faces. The crystals have a cuneate appearance and a strong tendency to form radiate aggregates. The salt is permanent, both in ordinary and dry air and also at 100°, at which temperature it does not fuse. In water, it is very soluble.

Analyses:

1) 1.2343 gr. salt gave 0.644 gr. selenium or 0.9049 gr. selenious acid and 0.5803 gr. sulphate of sodium or 0.2533 gr. soda.
2) 1.217 gr. salt gave 0.635 gr. selenium or 0.8923 gr. selenious acid and 0.5718 gr. sulphate of sodium or 0.2497 gr. soda.
3) 1.5314 gr. salt showed undiminished weight, when kept over sulphuric acid and, likewise, on heating to 100°.

Represented in 100 parts:

	found			calculated	
	1.	2.			
Soda	20.52	20.52	$^1/_2Na^2O$	31	20.53
Selenious acid . .	73.31	73.32	SeO^2	111	73.51
Water (loss). . . .	6.17	6.16	$^1/_2H^2O$	9	5.96
	100.00	100.00		151	100.00

BERZELIUS employed this salt to prove the existence of diselenites. For this purpose, it was carefully melted; but, in spite of all precautions, he was not able to prevent a small quantity of selenious acid being carried off with the water. On this account, the soda determination of the fused salt, made in the same manner as that of the neutral salt, proved to be a little too high, or 22.17 p. ct. instead of the calculated 21.83 p. ct.

For reasons unknown to me, MUSPRATT seems to have arrived at a contrary result; he says namely: "the biselenite does not suffer decomposition except at a very high temperature". Under this empression, and as "it parts on a sand-bath only with its water of crystallization", he heated the salt in this manner without determining the temperature even approximately, then finds a loss of weight of 10.41 p. ct., assumes this to be expelled water of crystallization and finds this number casually agreeing with a formula $Na^2O.SeO^2 + H^2O.SeO^2 + 2H^2O$, which is assumed to express the real composition of the salt, though no other ingredient has been determined. I need scarcely remark that BERZELIUS' observation, regarding the phaenomena presented by the salt on heating, is true and that nearly half the loss of weight, obtained by MUSPRATT, consisted in selenious acid that had sublimated.

4. Tetraselenite: $NaH.O^2.SeO + H^2.O^2.SeO$,

A solution, obtained by treating 1 mol. carbonate of sodium with 4 mol. selenious acid, afforded this salt readily in very large, glass-shining, hard, probably monoclinic prisms in combination with ortho- and klino-diagonal end-faces, of which the predominant one gave a laminar form to the crystals, and with terminal face, hemi-pyramid, dome etc. The salt has a tendency to effloresce and is permanent both in ordinary and dry air.

Analyses:

1) 0.571 gr: salt gave 0.3193 gr. selenium or 0.4586 gr. selenious acid and 0.143 gr. sulphate of sodium or 0.0624 gr. soda.

2) 0.832 gr. salt gave 0.4712 gr. selenium or 0.6621 gr. selenious acid and 0.2103 gr. sulphate of sodium or 0.0918 gr. soda.
3) 0.5358 gr. salt, being kept over sulphuric acid, exhibited no change of weight; at 100°, it fused down to a tough, viscid mass and then, after long heating, lost 0.035 gr.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Soda	10.93	11.04	—	$\frac{1}{2}Na^2O$	31	11.07
Selenious acid. . .	78.57	79.58	—	$2SeO^2$	222	79.28
Water (loss) . . .	10.50	9.38	6.53	$1\frac{1}{2}H^2O$	27	9.65
	100.00	100.00			280	100.00

At 100°, the salt lost $\frac{2}{3}$ of its hydrogen, one half very speedily, the other much more slowly. It seems probable that here, as was also the case with the tetraselenite of potassium, an anhydrous salt of the composition $NaH.O^2.SeO + SeO^2$ remains as a mass, viscid when warm, glassy when cold, and, owing to reducing gases, slightly redcoloured by traces of separated selenium.

The foregoing analyses show that this salt contains no water of crystallization. MUSPRATT is of another opinion, on the strength of two soda determinations — 11.56 and 12.41 pr. ct. — which in his numbers, have been erroneously set down as amount of water. In other respects, these determinations agree better with the composition of the waterfree salt, than with the formula $Na^2O.SeO^2 + 3H^2O.SeO^2 + H^2O$, assumed by him. The material he employed for the analysis is so much more surely identical with the salt described above, as it was previously dried over sulphuric acid.

As magnesium, glucinum and cobalt have given salts in a stage of saturation between diselenite and tetraselenite, I made an experiment with sodium that proved that no triselenite exists of this metal. A solution, obtained by treating one mol. carbonate of sodium with three mol. selenious anhydride, yielded namely, on spontaneous evaporation, a salt in oblique, four-sided prisms, with ortho- and klino-diagonal end-faces and several combination-faces at the ends. After recrystallization from water,

0.5168 gr. salt, fumed with sulphuric acid, gave 0.129 gr. sulphate of sodium or 0.0563 gr. soda; in 100 parts: 10.90, a value that proves the compound to be a tetraselenite.

As the mother-liquor afterwards separated diselenite with its peculiar crystalline form, it is evident that there exists no triselenite of sodium, nor, consequently, of the other alkaline metals.

SELENITES OF LITHIUM.

To the best of my information, no compounds of this metal with selenious acid have been hitherto examined. They exhibit a composition analogous to that of the other alkaline metals and have also been obtained in a similar manner.

1. Neutral: $Li^2O^2.SeO + H^2O$.

A solution containing this salt, when allowed to evaporate at the ordinary temperature, hardly yields distinct crystals, but readily, a white, butter-like mass. If, on the contrary, the evaporation takes place at a higher temperature, a salt is obtained in microscopical, white, satiny, extremely fine needles which are very slowly dissolved in cold, more easily in warm water. In all probability, there exists a salt, containing more water and being, both in cold and in warmth, much more soluble than this, which, just on account of its being less soluble, is deposited on heating the solution.

Analyses:

1) 0.4963 gr. salt gave 0.2467 gr. selenium or 0.3466 gr. selenious acid, and 0.3473 gr. sulphate of lithium or 0.0947 gr. lithia.
2) 0.4919 gr. salt gave 0.2436 gr. selenium or 0.3423 gr. selenious acid, and 0.347 gr. sulphate of lithium or 0.0946 gr. lithia.
3) 0.184 gr. salt, when kept over sulphuric acid, did not diminish in the weight, but lost 0.01 gr. at 100°.

Calculated on 100 parts:

	experiment				theory	
	1.	2.	3.			
Lithia	19.08	19.23	—	Li^2O	30	18.87
Selenious acid . .	69.82	69.58	—	SeO^2	111	69.81
Water (loss). . . .	11.10	11.19	5.44	H^2O	18	11.32
	100.00	100.00			159	100.00

The salt, being completely permanent on exposure to the air, parts with half its water of crystallization at 100°.

2. **Diselenite:** $LiH.O^2.SeO$.

This compound crystallizes from a strongly concentrated solution in laminar, probably rhombic prisms with predominant brachydiagonal end-faces and dome. The crystalline form reminds very much of that of arragonite, and agrees with it even in the circumstance, that twin-formation seems to be rather common. The salt is permanent both in ordinary and dry air and dissolves readily in cold water.

Analyses:

1) 0.6965 gr. salt gave 0.4075 gr. selenium or 0.5725 gr. selenious acid and 0.2778 gr. sulphate of lithium or 0.0758 gr. lithia.
2) 0.689 gr. salt gave 0.4032 gr. selenium or 0.5665 gr. selenious acid and 0.273 gr. sulphate of lithium or 0.0745 gr. lithia.
3) 0.3737 gr. salt, when kept over snlphuric acid, presented no change of weight; but lost 0.0257 gr., when heated to 100°.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Lithia	10.88	10.81	—	$\frac{1}{2}Li^2O$	15	11.11
Selenious acid . . .	82.21	82.21	—	SeO^2	111	82.22
Water (loss)	6.91	6.98	6.88	$\frac{1}{2}H^2O$	9	6.67
	100.00	100.00			135	100.00

Consequently, the compound, at 100°, changes into an anhydrous salt $Li^2.O^2.SeO. + SeO^2$, under loss of water, and without fusing at this temperature; the change, however, takes place but slowly.

3. **Tetraselenite:** $LiH.O^2.SeO + H^2.O^2.SeO$.

As is the case with the corresponding selenites of the two common alkaline metals, this salt also crystallizes more easily, than both the neutral salt and the diselenite. The crystals are long, four-sided prisms, with obliquely truncated ends. They are permanent even in dry air, but fuse at 100° and give off water.

Analyses:

1) 0.7948 gr. salt gave 0.4719 gr. selenium or 0.6631 gr. selenious acid and 0.1633 gr, sulphate of lithium or 0.0445 gr. lithia.
2) 0.8005 gr. salt gave 0.476 gr. selenium or 0.6688 gr. selenious acid and 0.165 gr. sulphate of lithium or 0.045 gr. lithia.
3) 0.6385 gr. salt did not diminish in weight by being kept in dry air; at 100°, it speedily lost 0.023 gr. of its weight, and then gradually diminished, until the loss amounted to 0.037 gr.

Centesimally represented:

	found			calculated		
	1.	2.	3.			
Lithia	5.60	5.62	—	$\frac{1}{2}Li^2O$	15	5.68
Selenious acid . . .	83.43	83.53	—	$2SeO^2$	222	84.09
Water (loss)	10.97	10.85	5.79	$1\frac{1}{2}H^2O$	27	10.23
	100.00	100.00			264	100.00

The ascertained loss of weight at 100° makes a little more than half the quantity of water that could have been parted with. Of the 5.79 pr. ct., 3.60 pr. ct. went off very speedily, the rest rather slowly. It seems more than probable that, also from this salt, as is the case with the tetraselenites of potassium and sodium, $\frac{2}{3}$ of the hydrogen may be expelled at 100°; the melted state of the salt, however, makes it considerably more difficult to expel the water.

SELENITES OF SILVER.

Neutral: $Ag^2.O^2.SeO$.

This is the only selenite of this metal; as being extremely difficult to be dissolved, even in boiling water, it is easily obtained by double decomposition or, since it is also dissoluble in dilute nitric acid, by treating nitrate of silver with free selenious acid. It is also formed by direct combination between oxide of silver and an excess of selenious acid, which cannot dissolve it. The salt crystallizes in white, fine, needles and undergoes no change on exposure to sun-light. After strong heating, pure, me-

tallic silver remains. BERZELIUS from whom these statements are for the most part taken, found by analysis that the salt had the above composition.

In accordance with THOMSEN'S [1]) project, selenite of silver has been employed to procure selenic acid, and this method, depending upon the action of moist bromine on this salt, makes it now an easy task to obtain this acid in a pure state which, by the methods formerly known, was attended with much difficulty.

With this salt, the description of the selenites of the monatomic metals is at an end. They show a striking analogy, one to another, as to constitution and properties. All, except silver, have afforded, besides neutral salts, also diselenites and tetraselenites; the latter are all composed according to the formulæ:

$$RH.O^2.SeO \text{ and}$$
$$RH.O^2.SeO + H^2.O^2.SeO.$$

The compounds are easily soluble in water, except selenite of silver, which dissolves but to an inconsiderable amount; the neutral salts of thallium and lithium are rather difficult to be dissolved, as compared with the others, that most often deliquesce. As usual, there is to be observed a closer agreement of properties in the salts, if the metals in question be arranged in two groups, with potassium and sodium as types. Thus, for instance, all neutral salts of the potassium-group [2]) are composed according to the formula

$$R^2.O^2.SeO + H^2O.$$

and all deliquesce, on exposure to the air; this is also the case with the diselenites. The diselenites as well as the neutral salts of the sodium-group are on the contrary permanent in the air.

Now follows to pass to the compounds of the diatomic metals with selenious acid.

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[1]) Tidskrift for Physik og Chemie Bd 8 s. 286.

[2]) Except cæsiumselenite with 3 mol. water.

SELENITES OF CALCIUM.

1. Neutral: $3Ca.O^2.SeO + 4H^2O$.

By mixing solutions of chloride of calcium and selenite of sodium, a white precipitate is obtained in extremely small, microscopic prisms, united in asteriated groups.

Analyses:

1) 0.4995 gr. salt, dissolved in the smallest possible quantity of hydrochloric acid, gave, on adding sulphuric acid and alcohol, 0.3488 gr. sulphate of calcium or 0.1436 gr. lime, and from the filtrate was precipitated 0.2 gr. selenium or 0.281 gr. selenious acid.
2) 0.5243 gr. salt, treated in the same manner, gave 0.3655 gr. sulphate of calcium or 0.1505 gr. lime and 0.208 gr. selenium or 0.2923 gr. selenious acid.
3) 0.345 gr. salt, kept over sulphuric acid, parted with 0.0125 gr. water.

Centesimally represented:

	experiment 1.	2.		theory		
Lime	28.75	28.70	—	3CaO	168	29.32
Selenious acid . .	56.26	55.75	—	$3SeO^2$	333	58.12
Water (loss) . . .	14.99	15.55	3.62	$4H^2O$	72	12.56
	100.00	100.00			573	100.00

The salt, which is brilliant white and satiny, dissolves in dry air and then loses $^1/_4$ of its water, according to 3). BERZELIUS has remarked that this salt, when heated in glass-tubes, eats through the glass and runs out in a fused state; a most remarkable effect, produced also by the salts of magnesium and manganese.

2. Diselenite: $Ca.O^2.SeO. + H^2.O^2.SeO. + H^2O$.

A solution, obtained by the action of 1 mol. of lime on 2 mol. of anhydride of selenious acid, on evaporating over sulphuric acid, afforded very easily the salt in beautiful monoclinic prisms with basal end-faces pyramid and dome-faces. It is pretty soluble in water and persistent in the air.

Analyses:

1) 0.5327 gr. salt when analyzed in the same manner as in the preceding case gave, 0.2246 gr. sulphate of calcium = 0.0925 gr. lime and 0.2672 gr. selenium or 0.376 gr. selenious acid.
2) 0.58 gr. salt gave likewise 0.2465 gr. sulphate of calcium = 0.1015 gr. lime and 0.2892 gr. selenium or 0.4063 gr. selenious acid.

Centesimally represented:

	experiment			theory	
	1.	2.			
Lime	17.36	17.50	CaO	56	17.83
Selenious acid	70.59	70.06	$2SeO^2$	222	70.70
Water (loss)	12.05	12.44	$2H^2O$	36	11.47
	100.00	100.00		314	100.00

3. **Diselenite:** $2[Ca.O^2.SeO + SeO^2] + H^2O$.

When to 1 molecule of the preceding salt 2 molecules of selenious acid were added — by which it was intended to ascertain whether any tetraselenite were formed — the solution, allowed slowly to evaporate, at about 60°, in a low vessel covered with a glass-plate, afforded a salt in very large, short crystals in form of irregularly six-sided lamels; possibly they are formed by a rhombic prism with macrodiogonal dome- and end-faces. The salt, permanent in the air, is very soluble in water.

Analyses:

1) 0.522 gr. salt, treated in the above manner, gave 0.2405 gr. sulphate of calcium = 0.099 gr. lime and 0.2847 gr. selenium or 0.4 gr. selenious acid.
2) 0.5712 gr. salt gave 0.2645 gr. sulphate of calcium = 0.1089 gr. lime and 0.3136 gr. selenium or 0.4406 gr. selenious acid.
3) 0.5765 gr. salt, after fuming with sulphuric acid and heating, gave 0.281 gr. sulphate of calcium or 0.1157 gr. lime.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Lime	18.97	19.06	20.07	2CaO	112	19.51
Selenious acid . . .	76.63	77.14	—	$4SeO^2$	444	77.35
Water (loss)	4.40	3.80	—	H^2O	18	3.14
	100.00	100.00			574	100.00

As the anhydrous salt now mentioned crystallizes from a solution, which contains the constituents in such proportion that tetraselenite of calcium could be formed, free selenious acid remaining, it seems as if there existed no tetraselenite of this metal.

SELENITES OF STRONTIUM.

1. **Neutral:** $2Sr.O^2.SeO + 7H^2O$.

This salt is precipitated in form of an indistinctly crystalline powder, when a solution of chloride of strontium is treated with selenite of sodium. Permanent in ordinary air, it parts with its water on being kept over sulphuric acid; when heated it fuses to a hard, white, porcellaneous mass, without changing, if protected from the action of reducing gases.

Analyses:

1) 0.4403 gr. salt, fumed with sulphuric acid, gave 0.291 gr. sulphate of strontium or 0.1641 gr. strontia.
2) 0.4867 gr. salt, treated in a similar way, gave 0.3222 gr. sulphate of strontium or 0.182 gr. strontia.
3) 0.4945 gr. salt, gently heated over free fire, parted with 0.1105 gr. water; on subsequent heating in an atmosphere of oxygen, the loss of weight increased to 0.1115 gr.
4) 0.5367 gr. salt, heated in an atmosphere of oxygen parted with 0.12 gr. water.
5) 0.4095 gr. salt lost over sulphuric acid 0.0925 gr.
6) 0.5175 gr. salt, dissolved in a small quantity of hydrochloric acid and treated with sulphuric acid and alcohol, gave, from the filtrate of the sulphate of strontium, 0.148 gr. selenium or 0.2079 gr. selenious acid.

Calculated on 100 parts:

	experiment						theory		
	1.	2.	3.	4.	5.	6.			
Strontia	37.27	37.40	—	—	—	—	2SrO	207	37.29
Selenious acid	—	—	—	—	—	40.18	$2SeO^2$	222	40.00
Water	—	—	22.55	22.36	22.59	—	$7H^2O$	126	22.71
								555	100.00

2. Diselenite: $Sr.O^2.SeO + H^2.O^2.SeO$.

A solution, prepared from 1 mol. carbonate of strontium and 2 mol. selenious acid, afforded on evaporation at the ordinary temperature no crystallizing salt. On evaporating slowly however at 50°—60° the salt crystallized in pretty large — the length 5 mm. and more — shining, transparent, monoclinic prisms with basal end-faces, in the same form, consequently, as the diselenite of calcium.

Analyses:

1) 0.515 gr. salt, treated in the same manner as the salts of calcium, gave 0.2693 gr. sulphate of strontium = 0.1519 gr. strontia and 0.24 gr. selenium or 0.3372 gr. selenious acid.
2) 0.5103 gr. salt gave likewise 0.2653 gr. sulphate of strontium = 0.1497 gr. strontia and 0.2357 gr. selenium or 0.331 gr. selenious acid.
3) 0.2903 gr. salt. fumed with sulphuric acid, gave 0.1523 gr. sulphate of strontium or 0.0865 gr. strontia.

Centesimally represented:

	experiment				theory	
	1.	2.	3.			
Strontia	29.50	29.35	29.80	SrO	103.5	30.13
Selenious acid . . .	65.48	64.85	—	$2SeO^2$	222.0	64.63
Water (loss)	5.02	5.80	—	H^2O	18.0	5.24
	100.00	100.00			343.5	100.00

This salt is permanent on exposure to the air and very easily dissolves both in cold and warm water.

To ascertain the behaviour of the diselenite of strontium, when treated with an excess of acid, an experiment was also made, by which the question whether there existed any tetraselenite of strontium, was answered in the negative. The solution evaporating at 50—60°, the diselenite crystallized unaltered in its characteristic form, and two specimens, fumed with sulphuric acid, exhibited the folloving results:

0.569 gr. salt gave 0.2873 gr. sulphate of strontium = 0.162 gr. strontia or 28.47 pr. ct.

0.7705 gr. salt gave 0.3935 gr. sulphate = 0.222 gr. strontia or 28.81 pr. ct.

values very nearly agreeing with the per-centage of strontia calculated by the formula, assumed above.

SELENITES OF BARIUM.

1. Neutral: $BaO^2.SeO + H^2O$.

A solution of nitrate of barium in a few moments grows turbid, on adding diselenite of sodium, and, within a short time, deposits a white, voluminous precipitate of satiny crystals. Placed under the microscope it exhibits a very beautiful crystallization of small prisms, asteriatedly grouped and obliquely truncated at the ends. The mother-liquor deposited, on evaporating, the diselenite, mentioned below.

Analyses:

1) 0.327 gr. salt, dissolved in hydrochloric acid, gave 0.2658 gr. sulphate of barium or 0.1745 gr. baryta, and out of the filtrate form this 0.0884 gr. selenium or 0.1242 gr. selenious acid.
2) 0.632 gr. salt gave likewise 0.5013 gr. sulphate of barium = 0.3292 gr. baryta and 0.179 gr. selenium or 0.2515 gr. selenious acid.

Centesimally represented:

	found			calculated		
	1.	2.				
Baryta	53.36	52.09	BaO	153	54.25	
Selenious acid	38.00	39.79	Se^2O	111	39.37	
Water (loss)	8.64	8.12	H^2O	18	6.38	
	100.00	100.00		282	100.00	

This salt, which is permanent in the air, holds consequently 1 mol. crystallization water. BERZELIUS found it to be waterfree. It is but slightly soluble in water. When heated, the waterfree salt is persistent, when protected from reducing gases; otherwise it is partially reduced to selenide of barium. PETTERSSON [1]), who has proved that selenite of barium is able to endure even the heat in Schlösing's furnace without decomposition, provided that the crucible containing it be suspended in another crucible a little larger, and also found that selenate of barium, under such circumstance is converted into selenite of barium, has employed such a mode of procedure in order to convert selenite of barium into selenite, for the quantitative determination of selenic acid.

[1]) Zeitschr. für analyt. Ch. 1873 S. 287.

2. **Diselenite:** $Ba.O^2.SeO + SeO$.

By dissolving 1 mol. carbonate of barium in 2 mol. boiling selenious acid this salt was obtained. The solution evaporating at 40—50°, it was deposited in satiny, small, four-sided prisms, truncated in the ends. The crystals are commonly arranged in aggregates, the individuals radiating from a common point. The salt is but slightly soluble in cold, more so in warm water.

Analyses:

1) 0.5253 gr. salt gave 0.3228 gr. sulphate of barium from the solution, acidulated with hydrochloric acid, or 0.212 gr. baryta, and from the filtrate 0.218 gr. selenium or 0.3063 gr. selenious acid.
2) 0.686 gr. salt gave 0.4235 gr. sulphate of barium = 0.2781 gr. baryta and 0.2846 gr. selenium or 0.3999 gr. selenious acid.
3) 0.31 gr. salt, obtained by the treatment of the diselenite with 2 mol. of selenious acid, gave, when fumed with sulphuric acid, 0.19 gr. sulphate of barium or 0.1248 gr. baryta.

In 100 parts:

	found			calculated		
	1.	2.	3.			
Baryta	40.36	40.54	40.26	BaO	153	40.80
Selenious acid . . .	58.31	58.30	—	$2SeO^2$	222	59.20
	98.67	98.84			375	100.00

It has been already observed that this anhydrous salt was obtained from the mother-liquor of the above neutral salt; the analysis 3) shows that it is deposited from a solution, containing 4 mol. of acid to 1 mol. of baryta, from which it seems to follow that no salt of barium, more acid than diselenite, can be obtained.

Neutral waterfree selenite and diselenite of barium were employed by BERZELIUS to ascertain the capacity of saturation of selenious acid, of which, for this purpose, he undertook quantitative baryta determinations, showing a proportion of 57.93 in the neutral (calc. 57.96), and 40.54 in the acid salt (calc. 40.80).

SELENITES OF LEAD.

1. Neutral: $PbO^2.SeO$.

Acetate of lead being precipitated with selenite of sodium, a heavy, crystalline precipitate was obtained which, under the microscope, proved to be composed of small prisms. Almost insoluble in water, the salt is easily dissolved by warm, dilute hydrochloric acid.

Analyses:

1) 0.6915 gr. salt, fumed with sulphuric acid, gave 0.63 gr. sulphate of lead or 0.4636 gr. oxide of lead.
2) 0.516 gr. salt gave in the same way 0.47 gr. sulphate or 0.3459 gr. oxide of lead.
3) 0.6905 gr. salt was dissolved in dilute hydrochloric acid and precipitated with sulphurous acid; the selenium obtained, being washed with warm hydrochloric acid and then with water, weighed 0.159 gr. = 0.2234 gr. selenious acid.
4) 0.53 gr. gave after the same treatment 0.1235 gr. selenium or 0.1735 gr. selenious acid.

Calculated on 100 parts:

	experiment				theory		
	1.	2.	3.	4.			
Oxide of lead. . .	67.04	67.04	—	—	PbO	223	66.77
Selenious acid . .	—	—	32.35	32.74	SeO^2	111	33.23
						334	100.00

BERZELIUS, by fuming with sulphuric acid, found in the salt 66.67 pr. ct. oxide of lead.

Already on very gentle heating, this salt has a tendency to fuse, and at a commencing dull red-heat it is complety fused, forming a dark-yellow liquid, which on cooling solidifies to a yellowish white, radiately crystalline mass, not, however, without some loss of selenious acid, amounting, in one experiment, to 4.25 pr. ct. or precisely $^1/_8$ of the acid contained in the salt. At a low red heat, considerable quantities sublime; thus, after half-an-hour's heating, a loss of 21.31 pr. ct. more was observed, so that the salt had now parted with nearly $^3/_4$ of its acid. The rest, being then heated in the full flame of a BUNSEN'S gas-lamp diminished still more in weight, the total loss amounting to 39.15 pr. ct. The residue was dissolved

in nitric acid, the excess of solvents expelled, and hydrochloric acid and sulphurous acid were added, without causing any selenium to be precipitated. It was not before long continued boiling that the solution showed a slight opalization in red; no doubt, a small quantity of selenide of lead had been formed by reducing gases under the heating. This circumstance and the loss observed, too high by 6 pr. ct., induced me to repeat the experiment of heating in an atmosphere of oxygen, which proved with full evidence, that selenite of lead, at a red heat, is decomposed into subliming selenious acid and remaining oxide of lead.

1.278 gr. selenite of lead lost thus, after about an hour's heating in the flame of a Bunsen's gas-burner 0.427 gr. or 33.41 pr. ct., the calculated loss being 33.23 pr. ct.

Berzelius' statement, that the salt on heating to whiteness yields a residue of basic salt, was consequently not confirmed.

2. **Diselenite:** $Pb.O^2.SeO + SeO^2$.

As Berzelius has already remarked, selenious acid does not seem to dissolve the neutral salt of lead. By treating the salt with 1 and 3 mol. of acid, evaporating the acid solution to dryness and allowing it to act in a concentrated state, there was in both cases obtained an anhydrous salt of the above-mentioned composition, in form of microscopic, four-sided, rifled prisms.

Analyses:

1) 0.4428 gr. salt, fumed with sulphuric acid, gave 0.31 gr. sulphate of lead or 0.2281 gr. oxide of lead.
2) 0.546 gr. salt gave likewise 0.3632 gr. sulphate or 0.2673 gr. oxide of lead.

Calculated on 100 parts:

	found			calculated	
Oxide of lead	51.51	48.96	PbO	223	50.11
Selenious acid (loss)	48.49	51.04	$2SeO^2$	222	49.89
	100.00	100.00		445	100.00

As the salt, when tested, was not found to contain any water, selenious acid was determined only as a loss; the analysis 1) refers to a material, prepared without excess of acid, 2) on the contrary to a specimen, procured by adding 3 mol. selenious acid to the neutral salt, by which the

difference in the found quantities of oxide of lead is easily intelligible; the mean of both determinations agrees closely with the theory. Like the preceding bivalent metals, lead, consequently, does not give any tetraselenite.

SELENITES OF MAGNESIUM.

1. Neutral: $Mg.O^2.SeO + 6H^2O$.

If selenite of sodium be added to a tolerably concentrated solution of chloride of magnesium, so abundant a crystallization is obtained in a few moments, that the whole solution is filled with a magma of microscopic, six-sided, very thin, rhombic lamels with the acute angles truncated. On employing a more dilute solution, the precipitate is formed by very small, prismatic crystals. Both forms are of the same composition; the analyses 1—4 refer to the lamelliform, 5—6 to the prismatic salt.

Analyses:

1) 0.8283 gr. salt gave, after heating to whiteness in the blast-flame, 0.1283 gr. magnesia.
2) 0.514 gr. salt gave 0.155 gr. selenium or 0.2178 gr. selenious acid.
3) 0.5345 gr. salt gave 0.161 gr. selenium or 0.2262 gr. selenious acid.
4) 0.5855 gr. salt, kept over sulphuric acid, did not change in the weight, but lost 0.2115 gr. att 100°.
5) 0.4423 gr. salt gave, after heating to whiteness, 0.0663 gr. magnesia.
6) 0.4713 gr. salt gave likewise 0.0705 gr. magnesia.

Centesimally represented:

	experiment						theory		
Magnesia . . .	15.49	—	—	—	14.99	14.97	MgO	40	15.44
Selenious acid	—	42.37	42.32	—	—	—	SeO^2	111	42.86
Water	—	—	—	36.12	—	—	$6H^2O$	108	41.70
								259	100.00

Of the 6 mol. water of crystallization which the salt contains, 5 are expelled at 100°. Muspratt's statement: "selenite of magnesia, heated over the lamp, gives off only its water of crystallization", is more than questionable; on account of a water determination, undoubtedly performed in this

manner, he thinks himself authorized to assume the composition of the salt to be $Mg.O^2.SeO + 3H^2O$. It is, however, to be observed, that he seems to have examined it, such as it is obtained, when deposited from boiling water. It may be possible that, thus prepared, it contains 3 mol. of water of crystallization, although the behaviour of the salt, described above, seems to indicate another composition. In fact, it is very likely that in the case of Muspratt's water determination, selenious acid has also been expelled, selenite of magnesium, according to my experience, when heated, without difficulty parting with its acid.

2. **Diselenite:** $Mg.O^2.SeO + H^2.O^2.SeO + 3H^2O$.

Neutral selenite of magnesium easily dissolves in selenious acid. A solution, prepared from 1 mol. of selenite and 1 mol. of acid, having, by spontaneous evaporation, reached such a consistency that it is scarcely liquid, little by little affords the salt in pretty large aggregates in form of six-sided pyramids, composed of innumerable, small, microscopic crystalline individuals, which were but exceptionally observed isolated and then seemed to possess the form of a six-sided double-pyramid with basal end-faces, truncating the summits. The salt easily dissolves in water and is permanent even in dry air. The water of crystallization is parted with at 100° (found 16.47, calc. 16.17 pr. ct.)

Analyses:

1) 0.5 gr. salt gave 0.234 gr. selenium = 0.3288 gr. selenious acid and 0.1768 gr. sulphate of magnesium = 0.0589 gr. magnesia.
2) 0.4497 gr. salt gave 0.2106 gr. selenium = 0.2959 gr. selenious acid and 0.1605 gr. sulphate of magnesium = 0.0535 gr. magnesia.
3) 0.2155 gr. salt, kept over sulphuric acid, showed undiminished weight but at 100° lost 0.0355 gr. water.

Centesimally represented:

	found			calculated		
	1.	2.	3.			
Magnesia.	11.79	11.90	—	MgO	40	11.98
Selenious acid . .	65.76	65.81	—	$2SeO^2$	222	66.47
Water (loss). . . .	22.45	22.29	16.47	$4H^2O$	72	21.55
	100.00	100.00			334	100.00

3. Triselenite: $Mg.O^2.SeO + 2H^2.O^2SeO + 3H^2O$.

This salt crystallizes also slowly from a solution, containing 3 mol. selenious acid to 1 mol. neutral selenite, and not before this is so concentrated, that the vessel may be turned upside down, without its running out. The crystals were formed by small, shining prisms of a rhombic appearance, with the acuter edges cut by slightly developed brachydiagonal end-faces and the ends of basal end-faces.

Analyses:

1) 0.4612 gr. salt gave 0.2343 gr. selenium or 0.3292 gr. selenious acid and 0.04 gr. magnesia.
2) 0.4466 gr. salt gave 0.2308 gr. selenium or 0.3243 gr. selenious acid and 0.0385 gr. magnesia.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Magnesia.	8.67	8.62	MgO	40	8.64
Selenious acid . .	71.39	72.61	$3SeO^2$	333	71.92
Water (loss) . . .	19.94	18.77	$5H^2O$	90	19.44
	100.00	100.00		463	100.00

The extraordinary degree of saturation of this salt, not observed before for selenious acid, induced me to repeat the experiment, then allowing the crystallization to take place at 60°. Now there was obtained another

4. Triselenite: $Mg.O^2.SeO + 2H^2.O^2.SeO$.

free from water of crystallization. It crystallized from a syrupy solution in lentil-formed aggregates of crystals; probably the individual crystals were formed by rhombic lamels. In cold water, this salt dissolves pretty slowly.

Analyses:

1) 0.5645 gr. salt gave 0.3295 gr. selenium or 0.463 gr. selenious acid and 0.1875 gr. sulphate of magnesium or 0.0625 gr. magnesia.
2) 0.533 gr. salt gave 0.3117 gr. selenium or 0.4379 gr. selenious acid and 0.177 gr. sulphate of magnesium or 0.059 gr. magnesia.

Calculated on 100 parts:

	experiment 1.	experiment 2.		theory	
Magnesia	11.06	11.07	MgO	40	9.78
Selenious acid . .	82.02	82.16	$3SeO^2$	333	81.42
Water (loss)	6.92	6.77	$2H^2O$	36	8.80
	100.00	100.00		409	100.00

The values obtained indicate that the salt has been in some measure impure with an anhydrous salt, which, under the circumstances under which the crystallization took place, might easily have been formed. Thus is explained the slight excess, that the analysis presents, both of base and acid, as compared with the calculated values.

Instead of a tetraselenite, magnesium consequently forms a triselenite, and the former seem no way possible to be obtained. In the following, when accounting for the salts of glucinum and cobalt, we shall find compounds, analogous to this salt, and also formed under similar circumstances.

SELENITES OF GLUCINUM.

BERZELIUS, respecting these compounds, only mentions that "selenite of glucinum is a white powder: the biselenite is soluble and dries, when evaporated to a gummy mass, which cracks of itself. Both of them lose acid on heating". MUSPRATT says no more than: "This selenite is, in every way, analogous to that of alumina", without communicating any reasons for the statement, and; "the biselenite is extremely soluble and does not crystallize".

The only selenites of glucinum examined respecting their composition, that are known, have been described by ATTERBERG [1]). He obtained namely three basic salts of the following composition:

$$Be^3.O^5.2SeO + 6H^2O$$
$$Be^2.O^3.SeO + 4H^2O$$
$$Be^7.O^{10}.3SeO + 14H^2O.$$

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[1]) Svenska Vetensk. Akad.s handl. 1873 N:o 5 and Bull. de la Soc. chim. de Paris XXI. p. 157. (1874).

the first, on adding ammonia to a solution obtained by dissolving glucina in selenious acid, the solution being continually kept strongly sour; the second, on adding such a solution, treated with ammonia as long as the precipitate formed was again dissolved, to semisulphate of glucinum; and the third, when the mother-liquor from the second was treated with acetate of sodium. The last mentioned is, however, regarded by him with some diffidence. Neutral selenite of glucinum he has not obtained; he states, regarding it, that it does not seem to be crystallizable, and that its solution is decomposed with water, depositing basic salts.

The following selenites of glucinum have been examined by me:

1. $^2/_5$-Selenite[1]): $Be^5.O^7.2SeO+10H^2O$.

By adding neutral selenite of sodium to a very dilute solution of neutral sulphate of glucinum, a precipitate was obtained, which at first, on stirring, was once more dissolved, but on adding more selenite of sodium became permanent. If so much selenite of sodium has been added, that no more glucina exists in the solution, there will be found in the mother-liquor a considerable quantity of selenious acid in the form of acid selenite of sodium, without the solution's becoming turbid on adding sulphate of glucinum — all signs that the precipitate formed was basic.

Analyses:

1) 0.421 gr. salt gave 0.1292 gr. selenium or 0.1815 gr. selenious acid and 0.1 gr. glucina.
2) 0.2433 gr. salt gave 0.0745 gr. selenium or 0.1047 gr. selenious acid and 0.0563 gr. glucina.
3) 0.2515 gr. salt lost at 100° 0.0345 gr. of its weight.

Calculated on 100 parts:

	experiment				theory	
	1.	2.	3.			
Glucina	23.75	23.14	—	5BeO	126	23.86
Selenious acid. . .	43.12	43.03	—	$2SeO^2$	222	42.05
Water (loss) . . .	33.13	33.83	13.72	$10H^2O$	180	34.09
	100.00	100.00			528	100.00

[1]) As regards the terminology here employed in denominating the basic salts, it is to be observed, that the amount of acid contained in the basic as compared with the neutral salt is indicated by a fractional number. Such a terminalogy being adopted for the acid salt, consistency seems to me to require its extension also to the basic salts, and indeed it appears to me the only appropriate nomenclatur.

As the basic glucinum-salts, above mentioned, holds this salt also two mol. water to every atom glucinum. Of the water 4 mol. (calc. 13.64 pr. ct.) were parted with, the salt being dried at 100°; the residue has consequently this composition: $5BeO.2SeO^2 + 6H^2O$.

From the basic salt, now mentioned, the other compounds of glucinum, that will be subsequently described, have been obtained, by heating with weighed quantities of selenious acid.

2. **Neutral:** $Be.O^2.SeO + 2H^2O$.

The basic salt was treated with 63.08 pr. ct. of its weight of anhydride of selenious acid, some water added and action allowed to take place at a temperature of about 60°. The solution, after evaporating nearly to dryness and on adding a little water, afforded a salt in globular masses of a gummy nature, tough and flexible in warmth, brittle in cold. By much water, a decomposition seems to be caused, a flaky, white, amorphous precipitate being formed.

Analyses:

1) 0.3985 gr. salt gave 0.18 gr. selenium or 0.2529 gr. selenious acid and 0.0578 gr. glucina.
2) 0.3965 gr. salt gave 0.18 gr. selenium or 0.2529 gr. selenious acid and 0.0555 gr. glucina.
3) 0.7116 gr. salt, kept over sulphuric acid, did not lose anything of its weight, but diminished gradually, when heated to 100°; after being dried, for several weeks, at this temperature, it had been diminished in weight by 0.0666 gr.

Calculated on 100 parts:

	found			calculated		
	1.	2.	3.			
Glucina	14.50	14.00	—	BeO	25.2	14.63
Selenious acid . . .	63.46	63.78	—	SeO^2	111.0	64.46
Water (loss)	22.04	22.22	9.36	$2H^2O$	36.0	20.91
	100.00	100.00			172.2	100.00

Of the water, contained in the compound, it is not possible to expel quite 1 mol., by means of long heating at 100° (calcul. 10.46, experim. 9.36 proc.) the loss during the last week not exceeding 0.0005 gr.

3. $^{8}/_{5}$-Selenite: $5BeO^{2}.SeO + 3H^{2}.O^{2}.SeO + 2H^{2}O$.

The basic salt, when treated with a quantity of anhydride of selenious acid amounting to 168.2 pr. cent of its own weight, gave a solution, which, on slowly evaporating at 60° in a vessel, covered with a watch-glass, afforded a salt in microscopic crystals of complicated form and completely homogeneous. Extremely difficult to be dissolved, both in warm and cold water, it is pretty easily soluble, on heating, in water containing hydrochloric acid.

Analyses:

1) 0.4995 gr. salt gave 0.2845 gr. selenium or 0.3997 gr. selenious acid and 0.056 gr. glucina.
2) 0.421 gr. salt gave 0.24 gr. selenium or 0.3372 gr. selenious acid and 0.0475 gr. glucina.

Calculated on 100 parts:

	fonnd			calculated	
	1.	2.			
Glucina	11.21	11.28	5BeO	126	11.41
Selenious acid . .	80.04	80.10	$8SeO^{2}$	888	80.43
Water (loss). . . .	8.75	8.62	$5H^{2}O$	90	8.16
	100.00	100.00		1104	100.00

The mother-liquor from this salt was allowed under the same circumstances once more to deposit crystals. Then was obtained

4. $^{7}/_{3}$-Selenite: $3BeO^{2}.SeO + 4H^{2}.O^{2}.SeO + H^{2}O$,

deposited in microscopic, very well developed twin-crystals, in all probability formed by acute, quadratic double pyramids with strongly developed basal end-faces and one pyramid-face as twin-face. To water and muriatic acid it showed the same behaviour as the preceding salt.

Analysis:

0.4167 gr. salt gave 0.2436 gr. selenium or 0.3423 gr. selenious acid and 0.0346 gr. glucina.

In 100 parts:

	found		calculated	
Glucina	8.30	3BeO	75.6	8.02
Selenious acid . . .	82.15	$7SeO^2$	777.0	82.43
Water (loss)	9.55	$5H^2O$	90.0	9.55
	100.00		942.6	100.00

As a solution, containing diselenite of glucinum, at a somewhat higher temperature, was decomposed in such a manner, that it first deposited $^8/_5$- and then $^7/_3$-selenite, there probably remaining a small quantity of free selenious acid, according to the formula of reaction:

$$8(Be.O^2.SeO + H^2.O^2.SeO) = (5Be.O^2.SeO + 3H^2.O^2.SeO) + (3Be.O^2.SeO + 4H^2.O^2.SeO) + H^2.O^2.SeO$$

I tried, how such a solution would behave, if allowed to crystallize by means of spontaneous evaporation at the ordinary temperature. Then was

5. **Diselenite:** $Be.O^2.SeO^2 + H^2.O^2.SeO$

afforded from the syrupy solution in a completely homogeneous crystallization of oblique, four-sided lamels, showing the same comportment to water and hydrochloric acid, as the preceding acid salts.

Analyses:

1) 0.5292 gr. salt gave 0.3133 gr. selenium or 0.4402 gr. selenious acid and 0.0465 gr. glucina.
2) 0.519 gr. salt gave 0.308 gr. selenium or 0.4328 gr. selenious acid and 0.0455 gr. glucina.
3) 0.3688 gr. salt, after heating to 100°, showed no loss of weight.

Centesimally represented:

	found			calculated	
	1.	2.			
Glucina	8.78	8.76	BeO	25.2	9.50
Selenious acid	83.18	83.39	$2SeO^2$	222.0	83.71
Water (loss)	8.04	7.85	H^2O	18.0	6.79
	100.00	100.00		265.2	100.00

6. Triselenite: $Be.O^2.SeO + 2H^2.O^2.SeO$.

$^2/_5$-selenite being treated with 378.45 pr. ct. of its own weight of selenious anhydride, the solution, that consequently contained 1 mol. of glucina to 4 mol. of selenious acid, on slowly evaporation at 60° in the above-mentioned manner, afforded a salt in flat, four-sided, oblong, microscopic prisms with basal end-faces and often two opposed angles truncated. Permanent on exposure to the air, the salt was with the utmost difficulty soluble in water, warm as cold, and, not before boiling, easily dissolved in dilute hydrochloric acid.

Analyses:

1) 0.6727 gr. salt gave 0.3985 gr. selenium or 0.56 gr. selenious acid and 0.0468 gr. glucina.
2) 0.7473 gr. salt gave 0.444 gr. selenium or 0.6238 gr. selenious acid and 0.0523 gr. glucina.

Calculated on 100 parts:

	found			calculated	
	1.	2.			
Glucina	6.96	6.99	BeO	25.2	6.39
Selenious acid . .	83.25	83.48	$3SeO^2$	333.0	84.48
Water (loss)	9.79	9.53	$2H^2O$	36.0	9.13
	100.00	100.00		394.2	100.00

Any salt, more acid than this, of the metal in question, seems consequently not possible to be formed.

If we recollect from the preceding that magnesium, under similar circumstances, also gives a triselenite of an analogous composition:

$$Mg.O^2.SeO + 2H^2.O^2.SeO$$

and anticipate from the following that cobalt has in the same manner given a salt:

$$Co.O^2.SeO + H^2.O^2.SeO + SeO^2$$

and that all the metals, forming basic sesquioxides with oxygen, have not given more acid salts than diselenites with excess of selenious acid, and for the rest behave in a different manner, it seems to me, that this circumstance may be added to the reasons previously known, for considering glucina as a monoxide.

ATTERBERG [1]), from his researches on the salts of glucinum, has arrived at the conclusion, respecting the basic salts of this metal, that they may be regarded as molecular unions between neutral salt and glucinum-hydrate and in proof of this view he states that basic sulphates and selenites dried at 100° seem to contain the constituents of neutral salt and glucinum hydrate such as they are obtained, each of them, at this temperature; yet it is to be observed that the neutral selenite, that he could not obtain, has been supposed by him, when heated to 100°, to have the composition $Be.O^2.SeO + H^2O$. This view he has also extended to basic salts in general.

Such a mode of viewing the matter, however, neither contains any proper explanation of the constitution of the basic salts, because the molecular unions of modern chemistry, under which name it includes a great number of quite unexplained facts, are no way to be derived from the theory of the qvantivalence of the elementary atoms and consequently themselves in great want of an explanation, nor does it agree with the data obtained for the above basic salt of glucinum. The quantity of water, that can, under drying at 100°, be expelled from the neutral salt, does not in fact amount fully to 1 mol, so that it does not appear to be of the assumed composition

$$Be.O^2.SeO + H^2O$$

and the $^2/_5$-selenite

$$2Be.O^2.SeO + 3Be.O^2.H^2 + 7H^2O$$

loses, in this case, 4 mol. of water, so that the residue is

$$2Be.O^2.SeO + 3Be.O^2.H^2 + 3H^2O$$

but in order to agree with the theory, it ought to have parted with one more mol. of its water.

On examining the material which ATTERBERG has had at his disposal, there will be found several exceptions from the regularity in question. Not to dwell upon basic salts of those metals of which he did not know any neutral salt, and which consequently cannot lay claim to any vote in the question, such as basic chromate, molybdate and even selenite, we find one basic chloride of glucinum and two basic oxalates adduced, which do not agree with the result obtained from the examination. The first-mentioned compound is supposed, when dried at 100°, to have the composition $(BeCl^2 + 4H^2O) + 12Be.O^2.H^2$, [1]) whilst $BeCl^2 + 4H^2O$, [2]) really existing at

[1]) Loc. cit. p. 7—8.
[2]) Loc. cit. p. 14.
[3]) Loc. cit. p. 12.

ordinary temperatures, does not consist at 100°, but "loses, besides water, also hydrochloric acid, on account of which the loss of weight, on drying in warmth, considerably exceeds the amount of water in the salt". As for the oxalates [1]), one of them, which after being dried at 100° had the composition $Be.O^2.C^2O^2 + Be.O^2H^2 + H^2O$, requires a neutral salt permanent at this temperature, such as $Be.O^2.C^2O^2 + H^2O$, whereas the other, which under the same circumstances had the composition: $Be.O^2.C^2O^2 + 6Be.O^2.H^2$, makes it necessary to suppose another neutral oxalate at the same temperature, viz. $Be.O^2.C^2O^2$. The author remarks himself the latter contradiction but regards this fact as deficiency ("felaktighet") rather than in consequence of it reject a view, founded on too limited a number of not even concordant observations. The regularity in the aqueous contents of the basic salts, which he supposes himself to have observed, will accordingly not bear application to the basic salts that he had himself examined, much less can it be made the foundation of any theory of the constitution of basic salts in general.

SELENITES OF MANGANESE.

1. **Neutral:** $Mn.O^2.SeO + 2H^2O$.

Was obtained by a double decomposition of sulphate of manganese with diselenite of potassium. Then falls, at the ordinary temperature, a colourless, insoluble, amorphous salt which, on heating the solution, immediately, but at the ordinary temperatur, only gradually is converted to a crystalline powder of microscopical, probably monoclinic prisms in combination with basal- and klinodiagonal end-faces and other forms. The crystals have a slight tint of reddish-yellow. BERZELIUS has remarked the peculiar deportment of this salt, when heated in a glass-tube; the glass is namely eaten through, without assuming the slightest colour of manganese.

Analyses:

1) 0.833 gr. salt gave 0.2995 gr. selenium or 0.4208 gr. selenious acid, and 0.297 gr. manganoso-manganic oxide, after being precipitated as carbonate and heated, or 0.2762 gr. manganous oxide.
2) 0.8418 gr. salt gave 0.3 gr. selenium or 0.4215 gr. selenious acid, and 0.2781 gr. manganous oxide weighed as 0.299 gr. manganoso-manganic oxide.

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[1]) Loc. cit. p. 35.

Calculated on 100 parts:

	found 1.	found 2.			calculated
Manganous oxide . .	33.16	33.03	MnO	71	32.57
Selenious acid	50.52	50.08	SeO^2	111	50.91
Water (loss)	16.32	16.89	$2H^2O$	36	16.52
	100.00	100.00		218	100.00

MUSPRATT has undertaken a selenium determination of this salt, by melting with nitre and weighing the selenic acid formed as salt of barium, the only instance in which selenious acid has been quantitatively determined by him.

2. **Neutral:** $Mn.O^2.SeO+H^2O$.

By mixing the neutral solution of sulphate of manganese with selenite of potassium, there is also formed a white, amorphous, voluminous, insoluble precipitate, which being left under the liquor, after the lapse of some time diminishes, assuming a crystalline form. It then consisted of microscopic four-sided tablets of a slight yellowish-red colour.

Analyses:

1) 0.519 gr. salt gave 0.2011 gr. selenium or 0.2826 gr. selenious acid and 0.3935 gr. sulphate of manganese, containing 0.185 gr. manganous oxide.
2) 0.5005 gr. salt gave 0.1932 gr. selenium or 0.2715 gr. selenious acid 0.3785 gr. sulphate of manganese = 0.1779 gr. manganous oxide.
3) 0.3755 gr. salt, fumed with sulphuric acid, gave 0.282 gr. sulphate of manganese or 0.1326 gr. manganous oxide.

Calculated on 100 parts:

	experiment 1.	experiment 2.	experiment 3.		theory	
Manganous oxide. .	35.65	35.54	35.31	MnO	71	35.50
Selenious acid . . .	54.45	54.25	—	SeO^2	111	55.50
Water (loss)	9.90	10.21	—	H^2O	18	9.00
	100.00	100.00			200	100.00

It is worth notice that this salt, which is deposited from a completely neutral solution, holds less water of crystallization than the preceding, which was precipitated with diselenite at the same temperature.

3. **Diselenite:** $Mn.O^2.SeO + SeO^2$.

The preceeding salt, being treated with 55.5 pr. ct. anhydride of selenious acid and heated with some water, was converted into a beautiful rosy-coloured, heavy precipitate, forming microscopical, oblique, four-sided tablets.

Analyses:

1) 0.6016 gr. salt gave 0.3165 gr. selenium or 0.4447 gr. selenious acid and 0.3115 gr. sulphate of manganese or 0.1465 gr. manganous oxide.
2) 0.5345 gr. salt gave 0.282 gr. selenium or 0.3962 gr. selenious acid and 0.278 gr. sulphate of manganese or 0.1308 gr. manganous oxide.

Centesimally represented:

	found			calculated		
	1.	2.				
Manganous oxide	24.35	24.47	MnO	71	24.23	
Selenious acid	73.92	74.13	$2SeO^2$	222	75.77	
	98.27	98.60		293	100.00	

The same anhydrous salt was also deposited from a solution, prepared by treating the neutral selenite $Mn.O^2.SeO + H^2O$ with 165.5 pr. ct. its weight of selenious anhydride, with a view to obtain a tetraselenite; the evaporation was in this case spontaneous.

Analyses:

1) 0.512 gr. salt gave 0.271 gr. selenium or 0.3808 gr. selenious acid and 0.2563 gr. sulphate of manganese or 0.1205 gr. manganous oxide.
2) 0.4827 gr. salt gave 0.2558 gr. selenium or 0.3594 gr. selenious acid and 0.244 gr. sulphate of manganese or 0.1147 gr. manganous oxide.

In 100 parts:

	1.	2.
Manganous oxide	23.54	23.77
Selenious acid	74.38	74.46
	97.92	98.23

numbers leading to the formula above.

No tetraselenite of manganese is consequently formed either at ordinary or at higher temperatures.

SELENITES OF IRON — $\overset{II}{Fe}$.

I have not more closely examined any salt of the bivalent iron, because the difficulties, attending such a trial, would have been too great without any corresponding profit. The white, amorphous precipitate, obtained by the action of selenite of sodium on a solution of ammonium-ferro-sulphate, has namely a strong tendency to oxidate, so that its colour changes almost as speedily as that of ferro-carbonate. The precipitate grows more and more yellow-coloured and is then transformed into ferric selenite.

Berzelius remarks that ferro selenite with hydrochloric acid yields selenium, chloride of iron being formed, and that it is dissolved by more selenious acid to a diselenite which is but sparingly soluble.

Ferri-selenites will be described in connexion with salts of the other sesquioxides, here after mentioned.

SELENITES OF COBALT.

1. Neutral: $Co_2O^2.SeO + 2H^2O$.

Chlorid of cobalt treated with neutral selenite of sodium, gives a bluish-red, amorphous precipitate, which, being heated, is very soon converted into a crystalline powder of the brightest and most beautiful red-violet colour. In the microscope it proves to be composed of small, prismatic crystals Insoluble in water, it easily dissolves in selenious acid to a rosy-coloured fluid.

Analyses:

1) 0.768 gr. salt gave 0.2729 gr. selenium or 0.3835 gr. selenious acid and 0.5275 gr. sulphate of cobalt or 0.257 gr. cobaltous oxide.
2) 0.7189 gr. salt gave 0.2552 gr. selenium or 0.3586 gr. selenious acid and 0.496 gr. sulphate of cobalt or 0.2416 gr. cobaltous oxide.

In 100 parts:

	found			calculated	
	1.	2.			
Cobaltous oxide .	33.46	33.61	CoO	76	34.08
Selenious acid . .	49.93	49.87	SeO^2	111	49.78
Water (loss) . . .	16.61	16.52	$2H^2O$	36	16.14
	100.00	100.00		223	100.00

2. **Diselenite:** $Co.O^2.SeO + SeO^2$.

The neutral selenite is easily dissolved in warm, concentrated selenious acid; such a solution allowed to evaporate over sulphuric acid, affords a beautifully red, completely amorphous, glassy mass; but on slowly evaporating at 60° in a vessel covered with a watch-glass, the concentrated solution soon yields a salt in dark-violet crusts, showing a very beautiful crystallization under the microscope. The crystals consist of rhombic prisms in combination with very developed brachydiagonal end-faces and macrodiagonal dome, on account of which they assume the form of rhombic tablets, the acute angles only little truncated by the slightly developed prism-faces.

Analyses:

1) 0.504 gr. salt gave 0.265 gr. selenium or 0.3724 gr. selenious acid and 0.2608 gr. sulphate of cobalt or 0.1271 gr. cobaltous oxide.
2) 0.4847 gr. salt gave 0.2558 gr. selenium or 0.3594 gr. selenious acid and 0.251 gr. sulphate of cobalt or 0.1223 gr. cobaltous oxide.

In 100 parts:

	found			calculated	
	1.	2.			
Cobaltous oxide . . .	25.22	25.23	CoO	76	25.50
Selenious acid	73.89	74.15	$2SeO^2$	222	74.50
	99.11	99.38		298	100.00

This anhydrous salt is not at all soluble in cold, and only with difficulty in warm water.

3. **Triselenite:** $Co.O^2.SeO + 2SeO^2 + H^2O$.

One mol. of neutral selenite and 3 mol. of anhydride of selenious acid with some water being allowed slowly to evaporate at 60°, there was obtained a red-violet salt in microscopic, oblique, four-sided tablets, though complete solution never took place. To water it behaved as the preceding compound.

Analyses:

1) 0.577 gr. salt gave 0.3172 gr. selenium = 0.4457 gr. selenious acid and 0.2005 gr. sulphate of cobalt = 0.0977 gr. cobaltous oxide.

2) 0.493 gr. salt gave 0.2712 gr. selenium = 0.3811 gr. selenious acid and 0.1702 gr. sulphate of cobalt = 0.0829 gr. cobaltous oxide.

In 100 parts:

	found			calculated	
	1.	2.			
Cobaltous oxide. .	16.93	16.82	CoO	76	17.80
Selenious acid . .	77.25	77.30	$3SeO^2$	333	77.98
Water (loss). . . .	5.82	5.88	H^2O	18	4.22
	100.00	100.00		427	100.00

Consequently cobalt, as has been before shown to be the case with magnesium and glucinum, when circumstances are such that a tetraselenite might be formed, does not give any salt more acid than the triselenite just mentioned. In order to try whether the formation of a more acid salt would be caused by adding still more selenious acid, this triselenite was acted upon with as much selenious acid as it already contained and then placed to digest at 60° with a little water. Then a salt was obtained in very small, microscopical crystals, which by means of analytical determinations was proved to be the above-mentioned waterfree diselenite, for

0.529 gr. gave 0.2805 gr. selenium, corresponding 0.3941 gr. or 74.50 pr. ct. selenious acid and 0.2765 gr. sulphate of cobalt = 0.1338 gr. or 25.29 pr. ct. cobaltous oxide, numbers very closely agreeing with calculation.

SELENITES OF NICKEL.

1. Neutral: $Ni.O^2.SeO + 2H^2O$.

By treating carbonate of nickel with selenious acid at the ordinary temperature there was obtained a siskin-green deposit of microscopic, globular, insoluble crystal-aggregates.

Analysis:

0.6173 gr. salt gave 0.2162 gr. selenium = 0.3038 gr. selenious acid and 0.2045 gr. oxide of nickel, precipitated as carbonate.

In 100 parts:

	found		calculated	
Oxide of nickel . . .	33.13	NiO	75	33.79
Selenious acid	49.21	SeO^2	111	50.00
Water	17.66	$2H^2O$	36	16.21
	100.00		222	100.00

Neutral salt of nickel, precipitated with selenite of sodium, affords an amorphous, yellow-green salt, which on heating becomes crystalline; it is undoubtedly of the composition now stated.

2. **Diselenite:** $Ni.O^2.SeO + H^2.O^2.SeO + 2H^2O$.

The neutral salt, dissolved in free acid and evaporated at ordinary temperature, only gave a green gummy mass. By employing neutral salt in excess, there was obtained a solution, which, on slowly evaporating in a low vessel covered with a watch-glass, deposited a magnificently emerald salt in microscopic, four-sided, in all probability quadratic prisms with terminal faces. Permanent in the air, the salt at 100° loses part of its water. It is but slowly soluble in water.

Analyses:

1) 0.635 gr. salt gave 0.2868 gr. selenium = 0.403 gr. selenious acid and 0.1348 gr. oxide of nickel.
2) 0.4942 gr. salt gave 0.2223 gr. selenium = 0.3124 gr. selenious acid and 0.105 gr. oxide of nickel.
3) 0.3385 gr. salt, kept over sulphuric acid, did not diminish in weight, but, heated at 100°, lost 0.0132 gr. water.

In 100 parts:

	found			calculated	
	1.	2.			
Oxide of nickel	21.23	21.25	NiO	75	21.37
Selenious acid	63.46	63.22	$2SeO^2$	222	63.25
Water (loss)	15.31	15.53	$3H^2O$	54	15.38
	100.00	100.00		351	100.00

3. **Tetraselenite:** $Ni.O^2.SeO + 3SeO^2 + H^2O$.

By employing a large excess of selenious acid, and proceeding in other respects in the same manner as in the preceding case, there is obtai-

ned a salt in oblique, four-sided tablets, which are dissolved with difficulty in cold, more easily in warm water. At 100° it very slowly diminishes in weight, giving off the water.

Analyses:

1) 0.4995 gr. salt gave 0.2925 gr. selenium = 0.4109 gr. selenious acid and 0.0688 gr. oxide of nickel.
2) 0.608 gr. salt gave 0.356 gr. selenium = 0.5002 gr. selenious acid and 0.086 gr. oxide of nickel.
3) 0.4785 gr. salt did not diminish in weight, when kept over sulphuric acid, but, on very long heating at 100°, lost 0.0182 gr.

Centesimally represented:

	experiment				theory	
	1.	2.	3.			
Oxide of nickel. .	13.77	14.14	—	NiO	75	13.96
Selenious acid. . .	82.27	82.27	—	$4SeO^2$	444	82.68
Water	3.96	3.59	3.80	H^2O	18	3.36
	100.00	100.00			537	100.00

SELENITES OF COPPER.

1. **Neutral:** $Cu.O^2.SeO + 2H^2O$.

When diselenite of potassium is added to a solution of sulphate of copper a greenish-yellow precipitate falls down; the amorphous precipitate is, however, in a few moments considerably diminished and converted into small, but even to the naked eye, distinct, beautifully blue-coloured four-sided, prismatic crystals, combined with pyramid-faces. The salt, when gently heated, gave off water and became yellow; at a higher temperature it fused, selenious acid sublimed and finally oxide of copper remained as a black, glazed mass, in appearance most resembling charred sugar. The salt seems to be wholly insoluble both in cold and boiling water; when the heating had lasted for some time, the water was filtered away, but no traces of copper could be detected in it with ferro-cyanide of potassium. Also in free selenious acid it proved equally insoluble.

Analyses:

1) 1.0444 gr. salt gave 0.3554 gr. selenium = 0.4994 gr. selenious acid and 0.3627 gr. oxide of copper.
2) 1.0153 gr. salt gave 0.349 gr. selenium = 0.4904 gr. selenious acid and 0.3515 gr. oxide of copper.
3) 0.5789 gr. did not change its weight in dry air and by heating at 100°.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Oxide of copper .	34.72	34.62	CuO	79.5	35.10
Selenious acid . .	47.82	48.30	SeO^2	111.0	49.00
Water (loss) . . .	17.46	17.08	$2H^2O$	36.0	15.90
	100.00	100.00		226.5	100.00

This compound is persistent both in ordinary and dry air and when heated to 100°. MUSPRATT dried it over sulphuric acid and found it to contain 3.18—3.42 pr. ct. water and 40.55 pr. ct. oxide of copper, without mentioning the method used in either case. As he has produced selenite of copper in the same manner as that here related, only with the difference that selenite of ammonium has been used as a precipitant instead of selenite of potassium, there can be no doubt that the salt, examined by him, was identical with the above mentioned, that both his determinations are erroneous and that there exists no such salt as the one, furnished by him, viz. $3Cu.O^2.SeO + H^2O$.

2. **Diselenite:** $Cu.O^2.SeO + H^2.O^2.SeO + 2H^2O$.

By mixing hydrate of copper with an aqueous solution of selenious acid, there was obtained a gray-blue crystalline salt, which is scarcely soluble in cold water but when boiled becomes green, probably under decomposition.

Analysis:

1.05 gr. salt gave 0.251 gr. oxide of copper and 0.456 gr. selenium = 0.6407 gr. selenious acid.

Calculated on 100 parts:

	found		calculated	
Oxide of copper . .	23.90	CuO	79.5	22.36
Selenious acid . . .	61.02	$2SeO^2$	222.0	62.45
Water (loss)	15.08	$3H^2O$	54.0	15.19
	100.00		355.5	100.00

3. **Diselenite:** $Cu.O^2.SeO. + H^2.O^2.SeO.$

The foregoing neutral selenite of copper was treated with 49 and 147 pr. ct. anhydride of selenious acid and the mixture digested at about 60°. The solution, having evaporated to dryness, afforded, on treatment with water, a therein insoluble salt in form of blue-green, microscopical, four-sided prisms with obliquely cut ends. The analyses 1) and 2) refer to a material, obtained when diselenite, 3) when tetraselenite could have been formed.

Analyses:

1) 0.5805 gr. salt gave 0.28 gr. selenium or 0.3934 gr. selenious acid and 0.1435 gr. oxide of copper.
2) 0.5 gr. salt gave 0.243 gr. selenium or 0.3414 gr. selenious acid and 0.1232 gr. oxide of copper.
3) 0.596 gr. salt gave 0.2925 gr. selenium or 0.411 gr. selenious acid and 0.1425 gr. oxide of copper.

Represented in 100 parts:

	experiment			theory		
	1.	2.	3.			
Oxide of copper .	24.72	24.64	23.91	CuO	79.5	24.89
Selenious acid . .	67.77	68.28	68.96	2SeO	222.0	69.48
Water (loss). . . .	7.51	7.08	7.13	H^2O	18.0	5.63
	100.00	100.00	100.00		319.5	100.00

Consequently copper does not afford any salts, more acid than diselenite.

SELENITES OF ZINC.

1. Neutral: $Zn.O^2.SeO + H^2O$.

Solutions containing sulphate of zinc and selenite of potassium being mixed afforded an amorphous, flaky, white precipitate, which washed with water considerably diminished and was partially transformed into a crystalline state. The microscopical crystals were formed of small prisms united in bundles. In order to make this transformation complete, the salt was placed under water, without success, even after boiling. When the compound could not by this means be brought to crystallization, I tried to heat it, at the same time adding a small quantity of selenious acid, by means of which the end was easily attained and a completely homogeneous crystal-powder obtained.

Analyses:

1) 0.679 gr. salt gave 0.2525 gr. selenium = 0.3548 gr. selenious acid and 0.2625 gr. oxide of zinc.
2) 0.6332 gr. salt gave 0.2355 gr. selenium = 0.3309 gr. selenious acid and 0.2435 gr. oxide of zinc.
3) 0.4975 gr. salt did not diminish in weight when heated to 100°.

In 100 parts:

	found			calculated	
	1.	2.			
Oxide of zinc	38.73	38.66	ZnO	81	38.57
Selenious acid	52.25	52.26	SeO^2	111	52.86
Water (loss)	9.02	9.08	H^2O	18	8.57
	100.00	100.00		210	100.00

This salt is permanent both on exposure to the air and when heated to 100°.

2. Neutral: $Zn.O^2.SeO$.

The preceding salt heated with 52.86 pr. ct. anhydride of selenious acid in water, gave a slightly yellow-coloured compound in small, four-sided tablets.

Analyses:

1) 0.5192 gr. salt gave 0.215 gr selenium = 0.3021 gr. selenious acid and 0.2165 gr. oxide of zinc.
2) 0.4975 gr. salt gave 0.2068 gr. selenium = 0.2906 gr. selenious acid and 0.206 gr. oxide of zinc.

In 100 parts:

	found			calculated	
	1.	2.			
Oxide of zinc	41.70	41.41	ZnO	81	42.19
Selenious acid	58.19	58.41	SeO^2	111	57.81
	99.89	99.82		192	100.00

The mother-liquor of this salt, when left in the open air, deposited yellow, bright, shining, four-sided crystals of the tetraselenite mentioned below.

3. **Neutral:** $Zn.O^2.SeO + 2H^2O$.

When no diselenite, according to the next preceding experiment, was to be obtained by means of heating the neutral salt with an appropriate quantity of selenious acid, I tried how it would be influenced by the acid at the ordinary temperature. By adding as much anhydride as the neutral salt held before, complete solution never took place, but it was not long before the crystals were transformed into microscopical, foursided, short, probably rhombic prisms, often combined with dome and terminal faces.

Analyses:

1) 0.5168 gr. salt gave 0.1797 gr. selenium = 0.2525 gr. selenious acid and 0.1815 gr. oxide of zinc.
2) 0.4077 gr. salt gave 0.1407 gr. selenium = 0.1977 gr. selenious acid and 0.1445 gr. of zinc.

Centesimally represented:

	found			calculated	
	1.	2.			
Oxide of zinc	35.12	35.44	ZnO	81	35.53
Selenious acid	48.86	48.49	SeO^2	111	48.68
Water (loss)	16.02	16.07	$2H^2O$	36	15.79
	100.00	100.00		228	100.00

Also the mother-liquor from this salt gave very beautiful and well-formed crystals of tetraselenite. No diselenite of zinc is consequently to

be obtained and the cause of this lies no doubt in the great insolubility of the neutral salt and, on the contrary, the solubility of the tetraselenite in water. If consequently a soluble diselenite is formed for a moment, it is immediately decomposed into insoluble neutral salt and easily soluble tetraselenite.

It is remarkable that this salt, which contains 2 mol. of water of crystallization, is obtained from the neutral salt with 1 mol. of water, by the addition of free acid.

4. Tetraselenite: $Zn.O^2.SeO + 3H^2.O^2.SeO$.

WÖHLER [1]) has examined this salt. He obtained it by the action of selenious acid on metallic zinc. The thus formed selenide of zinc and free selenium being separated and the solution allowed spontaneously to evaporate, the compound crystallized in large, yellow, oblique, rhombic prisms, permanent in the air and easily soluble in water; but on heating such a solution decomposition took place, neutral salt separating.

I have obtained the same salt by treating the neutral selenite $Zn.O^2.SeO + H^2O$ with 3 mol. anhydride of acid and water, when solution readily ensued. If this solution, which on heating immediately precipitates waterfree neutral salt, is allowed spontaneously to evaporate, it affords the tetraselenite in large crystals, possessing the appearance and the properties stated by WÖHLER.

Analyses:

1) 0.5343 gr. salt gave 0.29 gr. selenium = 0.4075 gr. selenious acid and 0.075 gr. oxide of zinc.
2) 0.507 gr. salt gave 0.2775 gr. selenium = 0.39 gr. selenious acid and 0.07 gr. oxide of zinc.
3) 0.1705 gr. salt lost at 100° 0.0155 gr. of its weight.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Oxide of zinc	14.03	13.81	—	ZnO	81	13.99
Selenious acid . . .	76.27	76.92	—	$4SeO^2$	444	76.68
Water (loss)	9.70	9.27	9.09	$3H^2O$	54	9.33
	100.00	100.00			579	100.00

[1]) Ann. d. Chemie und Pharm. LXIII. S. 279 (1847).

SELENITES OF CADMIUM.

1. **Neutral:** $Cd.O^2.SeO$.

Hydrate of cadmium being dissolved on heating in selenious acid, the liquid suddenly deposited a salt in microscopical, feltred, long prisms with the ends obliquely cut off. The compound is insoluble in water.

Analyses:

1) 1.001 gr. salt gave 0.3323 gr. selenium = 0.4669 gr. selenious acid and 0.5327 gr. oxide of cadmium.
2) 1.0065 gr. salt gave 0.3333 gr. selenium = 0.4683 gr. selenious acid and 0.5355 gr. oxide of cadmium.

In 100 parts:

	found			calculated	
	1.	2.			
Oxide of cadmium . .	53.22	53.20	CdO	128	53.55
Selenious acid	46.64	46.53	SeO^2	111	46.45
	99.86	99.73		239	100.00

2. **Neutral:** $2Cd.O^2.SeO + 3H^2O$.

A solution of chloride of cadmium gave with selenite of sodium an amorphous, white, insoluble precipitate.

Analyses:

1) 0.5405 gr. salt gave 0.158 gr. selenium = 0.222 gr. selenious acid and 0.435 gr. sulphate of cadmium = 0.2677 gr. oxide of cadmium.
2) 0.5345 gr. salt gave 0.156 gr. selenium = 0.2192 gr. selenious acid and 0.432 gr. sulphate of cadmium = 0.2658 gr. oxide of cadmium.

In 100 parts:

	experiment			theory	
	1.	2.			
Oxide of cadmium	49.53	49.73	2CdO	256	48.12
Selenious acid . .	41.08	41.01	$2SeO^2$	222	41.73
Water (loss) . . .	9.39	9.26	$3H^2O$	54	10.15
	100.00	100.00		532	100.00

3. $^4/_3$-Selenite: $3Cd.O^2.SeO + H^2.O^2.SeO$.

The above mentioned amorphous selenite of cadmium dissolves easily in selenious acid at ordinary temperature. The solution, within a few moments, affords a crystalline precipitate, which, subjected to the microscope, proves to be composed of acicular crystals, united in radially arranged groups.

Analyses:

1) 0.588 gr. salt gave 0.2195 gr. selenium = 0.3084 gr. selenious acid and 0.4245 gr. sulphate of cadmium = 0.2612 gr. oxide of cadmium.
2) 0.5235 gr. salt gave 0.1967 gr. selenium = 0.2764 gr. selenious acid and 0.3785 gr. sulphate of cadmium = 0.233 gr. oxide of cadmium.

Centesimally represented:

	found			calculated	
	1.	2.			
Oxide of cadmium	44.42	44.51	$3CdO$	384	45.39
Selenious acid . .	52.45	52.79	$4SeO^2$	444	52.48
Water (loss). . . .	3.13	2.70	H^2O	18	2.13
	100.00	100.00		846	100.00

4. Sesquiselenite: $2Cd.O^2.SeO + H^2.O^2.SeO + H^2O$.

When 1 mol. of the preceding salt was digested with 2 mol. selenious acid and the solution evaporated to dryness, there remained, on treatment with water, a crystalline salt insoluble in it. Under the microscope, it proved to be composed of small, four-sided prisms, united in compact aggregates.

Analysis:

1) 0.5073 gr. salt gave 0.19 gr. selenium = 0.267 gr. selenious acid and 0.3427 gr. sulphate of cadmium = 0.2108 gr. oxide of cadmium.
2) 0.4868 gr. salt gave 0.1836 gr. selenium = 0.258 gr. selenious acid and 0.3285 gr. sulphate of cadmium = 0.2021 gr. oxide of cadmium.

Represented in 100 parts:

	found			calculated	
	1.	2.			
Oxide of cadmium . .	41.55	41.52	$2CdO$	256	40.96
Selenious acid	52.63	53.00	$3SeO^2$	333	53.28
Water (loss)	5.82	5.48	$2H^2O$	36	5.76
	100.00	100.00		625	100.00

5. **Diselenite:** $Cd.O^2.SeO + SeO^2$.

When the neutral selenite of cadmium mentioned under 1 was digested with 139.35 pr. ct. selenious anhydride, and the mixture had evaporated to dryness at about 60°, it afforded, after treatment with water, a salt insoluble therein in form of fine, microscopical, long prisms.

Analyses:

1) 0.772 gr. salt gave 0.4537 gr. sulphate of cadmium, corresponding to 0.2792 gr. oxide of cadmium, and 0.3466 gr. selenium or 0.487 gr. selenious acid.
2) 0.6435 gr. salt gave 0.3795 gr. sulphate of cadmium, corresponding to 0.2335 gr. oxide of cadmium; the selenium estimation was lost.

Represented in 100 parts:

	found			calculated	
	1.	2.			
Oxide of cadmium	36.17	36.28	CdO	128	36 57
Selenious acid	63.08	—	$2SeO^2$	222	63.43
	99.25			350	100.00

Cadmium seems consequently to be deprived of the capability of forming any salt with selenious acid, more acid than that now mentioned.

SELENITES OF MERCURY.

A) MERCUROUS SELENITES.

1. $^2/_3$-**Selenite:** $Hg^6.O^5.2SeO + 5H^2O = 2Hg^2.O^2.SeO + Hg^2.O + 5H^2O$.

Mercurous nitrate was precipitated with an excess of selenite of sodium. The obtained white precipitate proved when subjected to the microscope to be crystalline; when dried, it assumes a slightly yellowish colour. On treatment with hydrochloric acid, as already Berzelius has remarked, selenium partially separates and all the mercury is dissolved as chloride. By the analysis, the selenium thus obtained was separately determined, after which the filtrate was precipitated with hydrosulphuric acid.

The mixture of mercuric sulphide with sulphur and selenium thus precipitated was treated with a warm solution of cyanide of potassium by means of which selenium and sulphur were dissolved out and the mercuric sulphide remained. This was weighed, after washing and drying at 100°, and from the filtrate selenium was precipitated with hydrochloric acid. In this manner, in which all the mercurous selenites have been analyzed,

1) 0.514 gr. salt gave, immediately with hydrochloric acid, 0.0375 gr. selenium = 0.0527 gr. selenious acid and from the filtrate 0.457 gr. mercuric sulphide = 0.4097 gr. mercurous oxide and 0.0173 gr. selenium or 0.0243 gr. selenious acid.
2) 0.6495 gr. salt, treated with hydrochloric acid, immediately gave 0.0495 gr. selenium or 0.0695 gr. selenious acid and from the filtrate 0.58 gr. mercuric sulphide = 0.52 gr. mercurous oxide and 0.022 gr. selenium or 0.0309 gr. selenious acid.
3) 0.7795 gr. salt, heated to 100°, very slowly lost 0.0155 gr. water.

Represented in 100 parts:

	experiment			theory		
	1.	2.	3.			
Mercurous oxide .	79.71	80.06	—	$3Hg^2O$	1248	80.00
Selenious acid . .	14.98	15.47	—	$2SeO^2$	222	14.23
Water (loss) . . .	5.31	4.47	1.99	$5H^2O$	90	5.77
	100.00	100.00			1560	100.00

That the salt had this composition, may also be inferred from the quantity of selenium first precipitated with hydrochloric acid, as compared with that subsequently obtained from the solution. Their relation is namely very nearly as 2 to 1. The following equation illustrates this transposition:

$$2(Hg^6.O^5.2SeO) + 24HCl = 12HgCl^2 + 12H^2O + SeO^2 + 2Se,$$

showing that such must be the relation between these quantities of selenium.

This result not agreeing with statements of BERZELIUS and MUSPRATT, which do not, however, seem to be founded on any kind of analytical determination, that neutral, waterfree selenite ought to be formed under the circumstances under which I obtained the said basic salt, a statement, which also KÖHLER [1]), on account of a communicated analysis, asserts to be the case, I repeated the preparation of mercurous selenite, employing a much more concentrated solution of mercurous nitrate, than in the former case, so that, on adding selenite of sodium, the salt was thrown down as a thick pulpy precipitate. This was not, however, a neutral salt, but a

[1]) Ann. d Phys. u. Ch. LXXXIX. S. 146 (1853).

2. $^6/_7$**-Selenite:** $Hg^{14}.O^{13}.6SeO + 6H^2O = 6Hg^2.O^2.SeO + Hg^2O + 6H^2O$.

The precipitate, that had the same appearance and properties as the former, was extracted with least possible quantity of water, by means of BUNSEN'S method for filtering under high pressure, and, after desiccating between folds of bibulous paper, the analysis was performed in the same manner as in the case of the preceding basic salt:

0.868 gr. salt, treated with hydrochloric acid, gave a precipitate of 0.06 gr. selenium or 0.0843 gr. selenious acid, and from the filtrate from it, by precipitating with hydrosulphuric acid and digesting with cyanide of potassium, 0.7663 gr. mercuric sulphide = 0.687 gr. mercurous oxide and from the filtrate from it, hydrochloric acid precipitated 0.0465 gr. selenium or 0.0653 gr. selenious acid.

In 100 parts:

	found		calculated	
Mercurous oxide . .	79.15	$7Hg^2O$	2912	79.00
Selenious acid . . .	17.24	$6SeO^2$	666	18.07
Water (loss)	3.61	$6H^2O$	108	2.93
	100.00		3686	100.00

The composition arrived at from the values obtained, is also confirmed by the relation of the quantity of selenium, immediately precipitated from the salt with hydrochloric acid, to the quantity, afterwards obtained from the solution, the proportion of one to the other being very nearly as 7 to 5, and that this must be the case is evident from the following formula for the decomposition

$$2(Hg^{14}.O^{13}.6SeO) + 56HCl = 28HgCl^2 + 28H^2O + 5SeO^2 + 7Se.$$

The salt parted at 100° with half its water; 0.445 gr. lost namely 0.073 gr. or 1.64 pr. ct., the calculated loss being for 3 mol. of water 1.47 pr. ct.

It seems consequently to follow, that no neutral mercurous selenite is to be obtained, by employing really neutral selenite of sodium and a neutral solution of mercurous nitrate. The difference between this result and KÖHLER'S statement might possibly be explained thus, that he had not operated with neutral but with acid solutions; but it will be obvious from the following that we are not entitled to assume this. It is also impossible to judge of the accuracy of the results from his communications regarding the method employed in the analysis of the compound, as he only says: "As for the analytical methods I have only to state that mercury has been

precipitated from the solution of chloride with hydrosulphuric acid and calculated from the weight of the mercuric sulphide dried in a waterbath; further that selenium has been partly reduced with sulphite of alkali, partly precipitated as selenate of barium".

The following three salts were namely obtained from the last mentioned basic mercurous selenite, when 1, 8 and 22 mol. selenious acid were added to it, the mixture subsequently digested and dried at about 60° and the residue treated with water:

3. **$^2/_3$-Selenite:** $Hg^6.O^5.2SeO + 4H^2O = 2Hg^2.O^2.SeO + Hg^2O + 4H^2O$.

This compound was obtained in the above named manner, by treating the basic salt with 1 mol. or 3 pr. ct. selenious anhydride, in form of a white powder, which the microscope showed to be crystalline.

Analyses:

1) 0.5065 gr. salt, treated with hydrochloric acid, gave 0.038 gr. selenium or 0.0534 gr. selenious acid; in the filtrate from it 0.4545 gr. mercuric sulphide = 0.4077 gr. mercurous oxide and 0.0125 gr. selenium or 0.0175 gr. selenious acid.
2) 0.5975 gr. salt, treated with hydrochloric acid, gave 0.0443 gr, selenium or 0.0622 gr. selenious acid; from the filtrate hydrosulphuric acid precipitated 0.5365 gr. mercuric sulphide = 0.481 mercurous oxide and 0.012 gr. selenium or 0.0169 gr. selenious acid.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Mercurous oxide . . .	80.49	80.50	$3Hg^2O$	1248	80.93
Selenious acid	13.99	13.24	$2SeO^2$	222	14.40
Water (loss)	5.52	6.26	$4H^2O$	72	4.67
	100.00	100.00		1542	100.00

The relation of the selenium obtained immediately to that subsequently obtained from the solution is nearly as 3 to 1, which is also required by the formula:

$$2(Hg^6.O^5.2SeO) + 24HCl = 12HgCl^2 + 12H^2O + 3Se + SeO^2.$$

4 $^{3}/_{4}$-**Selenite:** $Hg^{8}.O^{7}.3SeO + 6H^{2}O = 3(Hg^{2}.O^{2}.SeO) + Hg^{2}O + 6H^{2}O$.

This salt was formed, when $^{6}/_{7}$-selenite was digested with 8 mol. or 24.1 pr. ct. selenious anhydride, and proved, when viewed with the microscope, to be crystallized and composed of small, fine prisms.

Analyses:

1) 0.5015 gr. salt, dissolved in hydrochloric acid, immediately gave 0.0315 gr. selenium or 0.0443 gr. selenious acid, and in the filtrate from it 0.444 gr. mercuric sulphide = 0.3981 gr. mercurous oxide and 0.022 gr. selenium or 0.0309 gr. selenious acid.
2) 0.449 gr. salt, treated with hydrochloric acid, gave an undissolved residue of 0.0295 gr. gr. selenium or 0.0415 gr. selenious acid; from the filtrate there was obtained, by precipitating with hydrosulphuric acid, digesting with cyanide of potassium and precipitating the solution with hydrochloric acid, 0.396 gr. mercuric sulphide = 0.355 gr. mercurous oxide and 0.019 gr. selenium or 0.0267 gr. selenious acid.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Mercurous oxide . . .	79.38	79.09	$4Hg^{2}O$	1664	79.05
Selenious acid	14.99	15.19	$3SeO^{2}$	333	15.82
Water (loss)	5.63	5.72	$6H^{2}O$	108	5.13
	100.00	100.00		2105	100.00

The relation of the quantity of selenium, immediately precipitated by muriatic acid, to the dissolved quantity is as 1.5 to 1, the formula:

$$Hg^{8}.O^{7}.3SeO + 16HCl = 8HgCl^{2} + 2Se + SeO^{2} + 8H^{2}O$$

requiring 2 to 1, a difference, not easily accounted for.

5. $^{9}/_{5}$-**Selenite:** $5Hg^{2}.O^{2}.SeO + 4H^{2}.O^{2}.SeO + 8H^{2}O$.

When $^{6}/_{7}$-selenite was digested with 22 mol. or 66.26 pr. ct. selenious anhydride, this salt was obtained as a yellowish powder, which, under the microscope, proved to be coarsely crystalline.

Analyses:

1) 0.83 gr. salt, treated with hydrochloric acid, gave an insoluble residue of 0.0435 gr. selenium or 0.0611 gr. selenious acid, and, from the solution, 0.585 gr. mercuric sulphide or 0.5247 gr. mercurous oxide and 0.136 gr. selenium or 0.1911 gr. selenious acid.
2) 0.7555 gr. salt, immediately with hydrochloric acid, gave 0.04 gr. selenium or 0.0562 gr. selenious acid, and, in the filtrate, 0.532 gr. mercuric sulphide or 0.477 gr. mercurous oxide and 0.124 gr. selenium or 0.1742 gr. selenious acid.

Calculated on 100 parts:

	experiment			theory	
	1.	2.			
Mercurous oxide . . .	63.22	63.14	$5Hg^2O$	2080	63.12
Selenious acid	30.39	30.50	$9SeO^2$	999	30.32
Water (loss)	6.39	6.36	$12H^2O$	216	6.56
	100.00	100.00		3295	100.00

The relation of the quantity of selenium, precipitated from the salt with hydrochloric acid, to the dissolved quantity is as 1 to 3, the formula for decomposition with hydrochloric acid requiring the relation of 5 to 13:

$$2(5Hg^2O.9SeO^2) + 40HCl = 20HgCl^2 + 20H^2O + 13SeO^2 + 5Se.$$

The relation arrived at agrees far better with a formula for a diselenite, requiring this very relation of 1 to 3, but, for the rest, the analytical values do not agree with such a mode of composition. The capability of selenious acid to afford acid salts in the most various proportions, most obvious within the next following group, makes it besides not at all doubtful that the salt really has the assumed composition.

To judge from the foregoing investigation, mercurous oxide seems to have a very great tendency to form basic salts with selenious acid; $^6/_7$-selenite, under circumstances when neutral salt could have been formed, afforded a still more basic compound, viz. $^2/_3$-selenite, and, when a diselenite could have been formed, gave $^3/_4$-selenite. This remarkable result can be derived from no other cause, than that the added selenious acid dissolved out part of the base and formed with it a very acid, soluble salt, the basic compounds in question remaining. An acid salt was obtained only in the case when a tetraselenite was to be expected. Among all the basic oxides the compounds of which with selenious acid have been treated of

in this paper, mercurous oxide is the only one, from which I have not been able to obtain a neutral selenite.

B) MERCURIC SELENITES.

1. Neutral: $Hg.O^2.SeO + H^2O$.

BERZELIUS, by dissolving mercuric oxide in selenious acid, obtained this salt in the form of a white powder, sparingly soluble in water. KÖHLER[1]), who employed oxide both precipitated and prepared by heat, was, however, not able to arrive at such a result, either in low or high temperature, but obtained a white-yellow, amorphous product, which he, on account of a communicated analysis, regards as a basic salt: $Hg^7.O^{11}.4SeO$; but it is, most surely, nothing else than a neutral salt, mixed with still unaltered mercuric oxide, and for that reason yellow-coloured.

I found, namely, BERZELIUS' statements in this respect confirmed. When precipitated mercuric oxide was treated with a solution of selenious acid, no change was observed; but when the solution was allowed to evaporate at 100°, the acid thus becoming more concentrated, the yellow colour gradually disappeared, and there was at last obtained a completely white salt of small prisms, which, under the microscope, seemed to be six-sided with obliquely cut ends. A considerable excess of selenious acid was employed for the reaction; the mother-liquor, on evaporation, afforded a product, crystallizing in large, flat, rifled prisms. On account of BERZELIUS description of a diselenite, obtained by him, I thought this to be such a salt; but, when examined, it proved to be free selenious acid, with no admixture of mercuric oxide.

The salt was prepared for analysis by washing with water and desiccating between folds of bibulous paper.

Analyses:

1) 0.766 gr. salt. dissolved in hydrochloric acid, hydrosulphuric acid being passed into the solution and the obtained precipitate digested with cyanide of potassium, gave a residue of 0.5075 gr. mercuric sulphide or 0.4725 gr. mercuric oxide, and from the filtrate 0.1714 gr. selenium or 0.2408 gr. selenious acid.
2) 0.689 gr. salt gave 0.4615 gr. mercuric sulphide or 0.4297 gr. mercuric oxide and 0.1564 gr. selenium or 0.2197 gr. selenious acid.
3) 0.8215 gr. salt did not change its weight, when dried at 100°.

[1]) Loc. cit.

Calculated on 100 parts:

	experiment 1.	experiment 2.		theory	
Mercuric oxide . .	61.67	62.37	HgO	216	62.61
Selenious acid . .	31.44	31.89	SeO^2	111	32.17
Water (loss) . . .	6.89	5.74	H^2O	18	5.22
	100.00	100.00		345	100.00

The formation of this salt in presence of an excess of selenious acid, without the mother-liquor containing any mercuric oxide, disproved the existence of the soluble diselenite which BERZELIUS states himself to have obtained. Nevertheless, the comportment of the neutral salt, when digested with 1 and 3 mol. selenious acid under the circumstances so often mentioned above, was examined; I then obtained, in both cases:

2. **$^5/_4$-Selenite:** $4Hg.O^2.SeO + H^2.O^2.SeO + 2H^2O$.

crystallized in fine microscopical needles.

Analysis 1) refers to the salt which was obtained, when 1 mol. selenious acid was added, analysis 2), when 3 mol. were added.

Analyses:

1) 0.687 gr. salt gave 0.4335 gr. mercuric sulphide or 0.4035 gr. mercuric oxide and 0.1805 gr. selenium or 0.2536 gr. selenious acid.
2) 0.697 gr. salt gave 0.443 gr. mercuric sulphide or 0.4124 gr. mercuric oxide and 0.185 gr. selenium or 0.2599 gr. selenious acid.

Calculated on 100 parts:

	experiment 1.	experiment 2.		theory	
Mercuric oxide	58.73	59.17	4HgO	864	58.66
Selenious acid	36.91	37.29	$5SeO^2$	555	37.68
Water (loss)	4.36	3.54	$3H^2O$	54	3.66
	100.00	100.00		1473	100.00

It seems consequently, that no salt, more acid than that now mentioned, is to be obtained; certainly, no diselenite is formed under the aforesaid circumstances. BERZELIUS, however, describes such a compound so distinctly, that it cannot be doubted that he has really obtained a soluble salt of mercuric oxide, formed "when selenious acid was saturated with

mercuric oxide and the solution from which the neutral salt had commenced to separate, evaporated. It crystallizes in large, rifled, prismatic crystals, containing a considerable quantity of crystallization-water, and is to a very slight extent soluble in alcohol. Potash does not completely precipitate the mercuric oxide; the salt of potassium must therefore be heated in order to be liberated from mercury. On heating, it fuses and sublimes unaltered. With sulphurous acid white mercurous selenite is first precipitated, then pink selenium."

With the selenites of mercury, the account of the salts of the bivalent metals with selenious acid is at an end. Within this class, calcium, strontium, barium seem to form a natural group, characterized by its inability to yield salts more acid than diselenites with excess of selenious acid. Their diselenites appear to be easily converted into anhydrous salts, and these are the only salts known of barium and lead. The extraordinary amount of water in the neutral selenites of calcium and strontium may also be worth notice.

Within the other group of the bivalent metals, manganese and copper approach the above, in as much as that they do not form salts more acid than diselenites, both of them anhydrous salts. Among the neutral selenites belonging to this group, there is to be found a remarkably great number with 2 mol. water of crystallization:

$$Be.O^2.SeO + 2H^2O$$
$$Mn.O^2.SeO + 2H^2O$$
$$Co.O^2.SeO + 2H^2O$$
$$Ni.O^2.SeO + 2H^2O$$
$$Cu.O^2.SeO + 2H^2O$$
$$Zn.O^2.SeO + 2H^2O.$$

Magnesium, cobalt and glucinum exhibit a remarkable phenomenon, when treated with 4 mol. selenious acid, triselenites being formed instead of tetraselenites; as these salts have been formed under precisely the same circumstances, the solutions containing 1 mol. of the base to 4 mol. of the acid, at the same temperature and by a similar treatment in other respects; they are, as already observed, very interesting with regard to the composition of glucina or the qvantivalence of glucinum, and seem to form a not unimportant addition to the evidences, before communicated, for the bivalence of glucinum. Also the diselenite and the neutral salt of glucinum are

analogous to those of the bivalent metals; but the acid salts, formed by the decomposition of the former when heated, and the basic salts, seem to partake more of the properties of the metal-group, to which we shall hereafter come; although it may be, that this circumstance only indicates that the hydrate of glucinum is possessed of very weak basic properties.

Zinc, though affording a tetraselenite, gives no diselenite, which, no doubt, is dependant upon the different solubility of the neutral salt and the tetraselenite.

The selenites of mercury make remarkable exceptions from the regularity generally observable within the group of the bivalent elements; the mercurous selenites, by their decided tendency to grow basic, even with excess of acid, and by forming an acid salt, as extraordinary as $^{9}/_{5}$-selenite; the mercuric selenites by their disinclination to yield acid salts, $^{5}/_{4}$-selenite being the only one obtained, and under circumstances when the other metals of the group have given di-, tri-, or tetraselenite.

Also cadmium is very interesting and irregular with regard to its selenites, being the only bivalent metal which has afforded an acid salt such as

$$3Cd.O^{2}.SeO + H^{2}.O^{2}.SeO,$$

a composition very characteristic of the sexivalent metals within the following group, which, almost without exception, give salts of the composition

$$R^{2}.O^{6}.3SeO + H^{2}.O^{2}.SeO + xaq.$$

Cadmium moreover gives a sesquiselenite, within this group single in its kind; also in this respect, it has a correspondency within the following group in the sesqui-selenites of aluminium, didymium and indium, whereas no bivalent metal, among the univalent only ammonium, has given such a salt. Owing to its selenites as well as its known sulphate, cadmium thus forms a notable link of union between the bivalent and the sexivalent metals, to the latter of which I shall now pass.

SELENITES OF ALUMINIUM.

Our knowledge of these compounds is as yet limited to what Berzelius and Muspratt communicate about them. The supposition of the former that the precipitate obtained from the solution of the neutral salt, is a neutral selenite of aluminium, probably does not agree with the truth, and the analytical determinations furnished by the latter to support this view, will be found from the following to be little worthy of reliance.

2. $^3/_4$-Selenite: $Al^8.O^{21}.9SeO + 36H^2O = 3(Al^2.O^6.3SeO) + Al^2.O^6.H^6 + 33H^2O$.

Sulphate of aluminium in excess was precipitated with selenite of sodium. The precipitate obtained was amorphous and very voluminous. Washed and desiccated, it formed a rather heavy powder, which, on heating, swells enormously and gives an extremely porous alumina of a cotton-like appearance: 0.1 gr. of this alumina occupied the whole cavity of a platina-crucible holding 30 cc., the cover, on heating, being lifted out of its place. Though a great excess of sulphate was employed for preparing the salt, yet there were found not inconsiderable quantities of selenious acid in the mother-liquor; already this circumstance proves the precipitate formed to be a basic salt.

Analyses:

1) 0.6332 gr. salt gave 0.2175 gr. selenium, corresponding to 0.3056 gr. selenious acid, and 0.124 gr. alumina.
2) 0.5037 gr. salt gave 0.1732 gr. selenium, corresponding to 0.2433 gr. selenious acid, and 0.1 gr. alumina.
3) 0.365 gr. salt suffered at 100° a loss of 0.0525 gr. water.

Represented in 100 parts:

	experiment				theory	
	1.	2.	3.			
Alumina	19.59	19.85	—	$4Al^2O^3$	412	20.05
Selenious acid. . .	48.26	48.30	—	$9SeO^2$	999	48.61
Water (loss) . . .	32.15	31.85	14.38	$36H^2O$	644	31.34
	100.00	100.00			2055	100.00

The loss the salt suffered at 100°, amounts to 16 mol. of water (calc. = 13.92 pr. ct.)

2. **Neutral:** $Al^2.O^6.SeO + 7H^2O$.

In order to obtain this, the preceding salt was treated with the necessary quantity of selenious acid, viz. 16.20 pr. ct. selenious anhydride. The acid being allowed to act upon it for some time, the product was crystalline; as the mother-liquor reacted strongly sour and was precipitated by ammonia the salt is soluble in water, though very sparingly.

Analyses:

1) 0.5135 gr. salt gave 0.213 gr. selenium or 0.2993 gr. selonious acid and 0.0934 gr. alumina.
2) 0.585 gr. salt gave 0.243 gr. selenium or 0.3414 gr. selenious acid and 0.106 gr. alumina.
3) 0.4935 gr. salt lost at 100° 0.017 gr. water.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Alumina	18.19	18.12	—	Al^2O^2	103	18.33
Selenious acid . .	58.28	58.36	—	$2SeO^2$	333	59.25
Water (loss). . . .	23.53	23.52	3.45	$7H^2O$	126	22.42
	100.00	100.00			562	100.00

MUSPRATT'S analytical determinations do not agree with this composition of the neutral selenite of aluminium.

In salt containing water, dried over sulphuric acid, he has found 10.71 pr. ct. of water, leading to the formula $Al^2.O^6.3SeO + 3H^2O$, without, however, more closely specifying how he arrived at this number; besides he adduces amounts of alumina and selenious acid, the latter as a loss, in a waterfree neutral salt, but leaves the reader wholly ignorant how he has obtained this compound. Regarding the preparation of the salt — it was obtained from alum with selenite of alkali — he says that "great care must be taken that the precipitant is neutral", but what precautions he himself took he has not communicated.

3. **Sespuiselenite:** $2[Al^2.O^6.3SeO] + 3H^2.O^2.SeO + 9H^2O$.

$^3/_4$-selenite of aluminium was treated in water with 81.02 pr. ct. its weight of selenious anhydride or precisely the quantity necessary to form

a diselenite. At the ordinary temperature no complete solution took place, on account of which the mixture was gently heated and then afforded, under concentration of the acid, a gummy mass, which, on adding cold water, separated a crystalline salt. The mother-liquor reacted strongly sour.

Analyses:

1) 0.7065 gr. salt gave 0.3522 gr. selenium or 0.4949 gr. selenious acid and 0.1047 gr. alumina.
2) 0.6505 gr. salt gave 0.325 gr. selenium or 0.4566 gr. selenious acid and 0.0965 gr. alumina.
4) 0.5495 gr. salt gave off 0.01 gr. water at 100°.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Alumina	14.82	14.83	—	$2Al^2.O^3$	206	14.50
Selenious acid . . .	70.05	70.19	—	$9SeO^2$	999	70.30
Water (loss)	15.13	14.98	1.82	$12H^2O$	216	15.20
	100.00	100.00			1421	100.00

4. **Diselenite:** $Al^2.O^6.3SeO. + 3H^2.O^2.SeO + 2H^2O.$

This salt was formed und the same circumstances as the sesquiselenite, excepting that it was treated with such a quantity of anhydride as was necessary in order to obtain a tetraselenite, or 210.64 pr. ct.; it exhibited also the same appearance and properties as the preceding.

Analyses:

1) 629 gr. salt gave 0.345 gr. selenium or 0.4848 gr. selenious acid and 0.071 gr. alumina.
2) 0.6395 gr. salt gave 0.3515 gr. selenium or 0.4939 gr. selenious acid and 0.0723 gr. alumina.
3) 0.6375 gr. salt parted at 100° with 0.0325 gr. water.

Centesimally represented:

	found			calculated		
	1.	2.	3.			
Alumina	11.29	11.31	—	Al^2O^3	103	11.99
Selenious acid	77.08	77.23	—	$6SeO^2$	666	77.53
Water (loss)	11.63	11.46	5.10	$5H^2O$	90	10.48
	100.00	100.00			859	100.00

SELENITES OF CHROMIUM.

1. **¾-Selenite:** $Cr^8.O^{21}.9SeO + 64H^2O = 3[Cr^2.O^6.3SeO^3] + Cr^2.O^6.H^6 + 61H^2O$.

On adding selenite of sodium to a solution of chrome-alum there is obtained a green, voluminous, amorphous precipitate. The solution of alum was employed in excess, and yet selenious acid was to be found in the mother-liquor.

Analyses:

1) 0.5293 gr. salt gave 0.136 gr. selenium or 0.1911 gr. selenious acid and 0.1134 gr. oxide of chromium.
2) 0.6183 gr. salt gave 0.1602 gr. selenium or 0.2251 gr. selenious acid and 0.1325 gr. oxide of chromium.
3) 0.51 gr. salt gave off 0.1317 gr. water at 100°.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Oxide of chromium	21.43	21.43	—	$4Cr^2O^3$	612	22.15
Selenious acid . . .	36.11	36.41	—	$9SeO^2$	999	36.15
Water (loss)	42.46	42.16	25.83	$64H^2O$	1152	41.70
	100.00	100.00			2763	100.00

At 100° the salt loses 40 mol. of water (calculated 26.06 pr. ct.); the rest then has the composition

$$Cr^8.O^{21}.9SeO^2 + 24H^2O,$$

whereas the corresponding compound of aluminium, obtained in an analogous way, then retains 20 mol. of water.

2. **Neutral:** $Cr^2.O^6.3SeO + 15H^2O$.

The preceding salt was treated at the ordinary temperature with 12.05 pr. ct. selenious anhydride in an aqueous solution. This was coloured green, the green salt was microscopically crystalline and consequently not wholly insoluble in water.

Analyses:

1) 0.7365 gr. salt gave 0.227 gr. selenium or 0.319 gr. selenious acid and 0.1445 gr. oxide of chromium.

2) 0.501 gr. salt gave 0.155 gr. selenium or 0.2178 gr. selenious acid and 0.0995 gr. oxide of chromium.
3) 0.677 gr. salt lost at 100° 0.164 gr. water.

Calculated on 100 parts:

	experiment				theory	
	1.	2.	3.			
Oxide of chromium	19.62	19.86	—	Cr^2O^3	153	20.24
Selenious acid . . .	43.31	43.47	—	$3SeO^2$	333	44.05
Water (loss) . . .	37.07	36.67	24.22	$15H^2O$	270	35.71
	100.00	100.00			756	100.00

Of 15 mol. of water 10 are lost at 100° (calc. 23.81); consequently the salt, thus treated, contains 1 mol. of water less than the corresponding salt of aluminium under the same circumstances.

3. $^4/_3$-Selenite: $Cr^2.O^6.3SeO + H^2.O^2.SeO + 12H^2O$.

$^3/_4$-selenite being treated with a quantity of acid requisite for the formation of a diselenite, or 60.25 pr. ct. selenious anhydride, solution took place neither in cold nor in heat. When the mixture, at a temperature of 60°, had been brought to evaporate to dryness, there remained a green powder of the same appearance as the basic salt. By treatment with water there was obtained a strongly sour, slightly green-coloured liquid and a microscopical crystalline, green powder, not at all soluble in water and only very slowly in hydrochloric acid. The green-coloured mother-liquor, on evaporating, deposited crystals of selenious acid, and only at a few points a green-coloured, viscid mass was to be observed in very small quantities.

Analyses:

1) 0.644 gr. salt gave 0.2425 gr. selenium or 0.3407 gr. selenious acid and 0.117 gr. oxide of chromium.
2) 0.5025 gr. salt gave 0.1917 gr. selenium or 0.2693 gr. selenious acid and 0.09 gr. oxide of chromium.
2) 0.636 gr. salt lost at 100° 0.1155 gr. water.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Oxide of chromium	18.17	17.91	—	Cr^2O^2	513	18.41
Selenious acid . . .	52.90	53.59	—	$4SeO^2$	444	53.43
Water (loss)	28.93	28.50	18.16	$13H^2O$	234	28.16
	100.00	100.00			831	100.00

This salt, which is of great importance for the estimation of the qvantivalence of the rare earth-metals, and for this purpose will be made use of in the following, loses at 100° 8 mol. of water out of 13 (calc. 17.33 pr. ct.)

4. $^5/_3$-Selenite: $Cr^2O^2.3SeO+2H^2.O^2.SeO+7H^2O$.

$^3/_4$-selenite, treated in an aqueous solution with 156.65 pr. ct. its weight of selenious anhydride, being the quantity sufficient to form a tetra-selenite, dissolves only very incompletely in cold. When the mixture had evaporated at a moderate heat — about 60° — there was obtained a viscid green mass, which treated with water deposited a green, flaky, microscopically crystalline compound, and the mother-liquor behaved precisely in the same manner, as that of the preceding salt.

Analyses:

1) 0.5825 gr. salt gave 0.2635 gr. selenium or 0.3702 gr. selenious acid and 0.102 gr. oxide of chromium.
2) 0.5497 gr. salt gave 0.2508 gr. selenium or 0.3524 gr. selenious acid and 0.0965 gr. oxide of chromium.
3) 1.0595 gr. salt parted at 100° with 0.1235 gr. water.

In 100 parts:

	found			calculated		
	1.	2.	3.			
Oxide of chromium .	17.51	17.56	—	CrO^2	153	17.58
Selenious acid	63.56	64.10	—	$5SeO^2$	555	63.80
Water (loss)	18.93	18.34	11.66	$9H^2O$	162	18.62
	100.00	100.00			870	100.00

This salt, when heated to 100°, loses 6 mol. of water out of 9 (calc. 12.23), and possesses a high theoretical interest in the same respect as the preceding.

SELENITES OF IRON — $\overset{VI}{Fe^2}$.

1. $^8/_9$-Selenite: $Fe^6.O^{17}.8SeO + 28H^2O$.

An excess of neutral sublimated chloride of iron, also employed to produce the following salt, was precipitated with selenite of sodium. The precipitate was of a dirty yellow colour, voluminous, indistinctly crystalline and very difficult to be washed. The mother-liquor held selenious acid.

Analyses:

1) 0.6235 gr. salt gave 0.2073 gr. selenium or 0.2913 gr. selenious acid and 0.1605 gr. oxide of iron.
2) 0.555 gr. salt gave 0.1856 gr. selenium or 0.2608 gr. selenious acid and 0.144 gr. oxide of iron.
3) 0.707 gr. salt, heated to 100°, lost 0.152 gr. water.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Oxide of iron	25.74	25.94	—	$3Fe^2O^3$	480	25.64
Selenious acid	46.72	47.00	—	$8SeO^2$	888	47.44
Water (loss)	27.54	27.06	21.50	$28H^2O$	504	26.92
	100.00	100.00			1872	100.00

Concerning this salt, which at 100° gives off 22 mol. of water (calc. 21.12 pr. ct), the same remark may be made in a theoretical point of view, as in the case of the two preceding salts of chromium.

2. Neutral: $Fe^2.O^6.3SeO + 9H^2O$.

Neutral chloride of iron was precipitated with an excess of selenite of sodium. The precipitate exhibited the same appearance and properties as the above-mentioned.

Analyses:

1) 0.644 gr. salt gave 0.2286 gr. selenium or 0.3212 gr. selenious acid and 0.1625 gr. oxide of iron.
2) 0.77 gr. salt gave 0.275 gr. selenium or 0.3864 gr. selenious acid and 0.1955 gr. oxide of iron.
3) 0.6517 gr. salt, when heated to 100°, lost 0.1236 gr. water.

Represented in 100 parts:

	experiment			theory		
	1.	2.	3.			
Oxide of iron	25.23	25.39	—	Fe^2O^3	160	24.43
Selenious acid	49.88	50.18	—	$3SeO^2$	333	50.84
Water (loss)	24.89	24.43	18.96	$9H^2O$	162	24.73
	100.00	100.00			655	100.00

At 100° the salt loses 7 mol. of water out of 9 (calc. 19.25).

3. **Neutral:** $Fe^2.O^6.3SeO + 7H^2O$.

The basic salt, mentioned under 1, was acted upon with the quantity of selenious anhydride necessary for forming a neutral salt or 5.93 pr. ct. Its appearance and properties did not by this treatment undergo any change that could be observed.

Analyses:

1) 0.5805 gr. salt gave 0.2193 gr selenium or 0.3081 gr. selenious acid and 0.1515 gr. oxide of iron.
2) 0 601 gr. salt gave 0.2289 gr. selenium or 0.3216 gr. selenious acid and 0.156 gr. oxide of iron.
3) 0.3815 gr. salt, when dried at 100°, gave off 0.054 gr. water.

Represented in 100 parts:

	experiment			theory		
	1.	2.	3.			
Oxide of iron	26.09	25.95	—	Fe^2O^3	160	25.85
Selenious acid	53.08	53.51	—	$3SeO^2$	333	53.80
Water (loss)	20.83	20.54	14.15	$7H^2O$	126	20.35
	100.00	100.00			619	100.00

When dried at 100°, the salt has parted with 5 mol. of water out of 7 (calc. 14.53), and the residue is consequently identical with that, which under the same circumstances was obtained from the preceding neutral salt.

4. $^4/_3$-Selenite: $Fe^2.O^6.3SeO + H^2.O^2.SeO + 7H^2O$.

By digesting the basic salt, mentioned under 1, with that quantity of acid which was required, in order that a diselenite might be formed or 59.3 pr. ct. selenious anhydride, there was obtained, after evaporation of the solution at a gentle heat, a greenish-white, crystalline salt. It is so insoluble in water that sulphocyanate of potassium does not cause any colouring, but the liquid assumed sour reaction.

Analyses:

1) 0.541 gr. salt gave 0.2267 gr. selenium or 0.3185 gr. selenious acid and 0.1155 gr. oxide of iron.
2) 0.4685 gr. salt gave 0.1955 gr. selenium or 0.2747 gr. selenious acid and 0.104 gr. oxide of iron.
3) 0.3132 gr. salt, when heated to 100°, gave off 0.0247 gr. water.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Oxide of iron	21.35	22.20	—	Fe^2O^3	160	21.39
Selenious acid	58.87	58.63	—	$4SeO^2$	444	59.36
Water (loss)	19.78	19.17	7.88	$8H^2O$	144	19.25
	100.00	100.00			748	100.00

On drying at 100°, this compound seems to give off 3 mol. of water out of 8 (calc. 7.22). With another amount of water this salt

5. $^4/_3$-Selenite: $Fe^2.O^6.3SeO + H^2.O^2.SeO + 9H^2O$.

was obtained by treating the basic salt with so much anhydride, that a tetraselenite could have been formed or 166 pr. ct. its own weight. The compound, thus obtained under the same circumstances as the preceding salt, exhibited also a similar appearance and the same properties.

Analyses:

1) 0.5555 gr. salt gave 0.2286 gr. selenium or 0.3212 gr. selenious acid and 0.1085 gr. oxide of iron.
2) 0.4495 gr. salt gave 0.1858 gr. selenium or 0.2611 gr. selenious acid and 0.0885 gr. oxide of iron.

Centesimally represented:

	found			calculated	
	1.	2.			
Oxide of iron	19.53	19.69	Fe^2O^3	160	20.41
Selenious acid	57.82	58.09	$4SeO^2$	444	56.63
Water (loss)	22.65	22.22	$10H^2O$	180	22.96
	100.00	100.00		784	100.00

That the oxide of iron in the circumstances, under which the two last mentioned compounds have been obtained, does not form any salts more acid than those, is, in a theoretical point of view, of great interest. Two of the earth-metals, yttrium and erbium, have namely, under the same circumstances also not been able to unite with more selenious acid than to form a $^4/_3$-selenite, as will be seen in the following. It is also to be remarked that the salt has taken up more water of crystallization, when a larger quantity of acid has been present.

SELENITES OF YTTRIUM [1]).

1. Neutral: $Y^2.O^6.3SeO + 12H^2O$.

From a solution of sulphate of yttrium an excess of selenite of sodium precipitated a white, heavy matter which a microscopical examination proved to consist of globular, transparent aggregates. The yttrium had so completely fallen, that a solution of oxalate only caused a scarcely perceptible precipitate in the mother-liquor. It is consequently nearly insoluble in water; a solution of selenious acid does not affect the salt, either

[1]) For the material of the rare earths that have been employed for this investigation, I am under obligation to Professor CLEVE, who has generously placed at my disposal more than sufficient quantities of the same chemically pure, spectroscopically tested material that he has himself used for his researches.

in cold or in heat, but in a concentrated state unites with it, as will be proved by the following.

Analyses:

1) 0.5219 gr. salt gave 0.1556 gr. selenium or 0.2186 gr. selenious acid and 0.3093 gr. waterfree sulphate of yttrium, containing 0.1504 gr. yttria.
2) 0.4404 gr. salt gave 0.132 gr. selenium or 0.1855 gr. selenious acid and 0.2635 gr. sulphate of yttrium, corresponding to 0.128 gr. yttria.

Centesimally represented:

	found			calculated	
	1.	2.			
Yttria	28.81	29.08	Y^2O^3	227[1])	29.26
Selenious acid	41.89	42.11	$3SeO^2$	333	42.91
Water (loss)	29.30	28.81	$12H^2O$	216	27.83
	100.00	100.00		776	100.00

2. $^4/_3$-Selenite: $Y^2.O^6.3SeO + H^2.O^2.SeO + 3H^2O$.

When the preceding salt with 42.91 pr. ct. selenious anhydride in an aqueous solution was evaporated to dryness at 60°, and the rest was treated with water, there was obtained a microscopically crystalline, insoluble salt, a considerable quantity of selenious acid remaining in the solution.

Analyses:

1) 0.4235 gr. salt gave 0.176 gr. selenium or 0.2473 gr. selenious acid and 0.26 gr. sulphate of yttrium, corresponding to 0.1264 gr. yttria.
2) 0.327 gr. salt gave 0.1365 gr. selenium or 0.1918 gr. selenious acid and 0.2015 gr. sulphate of yttrium or 0.098 gr. yttria.

Centesimally represented:

	found			calculated	
	1.	2.			
Yttria	29.85	29.95	Y^2O^3	227	30.55
Selenious acid	58.39	58.65	$4SeO^2$	444	59.76
Water (loss)	11.76	11.40	$4H^2O$	72	9.69
	100.00	100.00		743	100.00

[1]) The atomic weights for the rare earth-metals are here employed in accordance with CLEVE's recent determinations.

The same salt was also obtained by treating the neutral salt with 9 mol. or 128.73 pr. ct. selenious anhydride and, for the rest, proceeding in the manner stated above.

Analyses:

1) 0.5045 gr. salt gave 0.2125 gr. selenium or 0.2986 gr. selenious acid and 0.305 gr. sulphate of yttrium or 0.1483 gr. yttria.
2) 0.4477 gr. salt gave 0.1885 gr. selenium or 0.2648 gr. selenious acid and 0.269 gr. sulphate of yttrium or 0.1308 gr. yttria.

These values are in 100 parts:

Yttria	29.39	29.22
Selenious acid . . .	59.19	59.15,

numbers very closely agreeing with those calculated from the formula given above.

Yttrium seems consequently unable to form any salt more acid than $^4/_3$-selenite with selenious acid. The analogy with iron in this respect, has just been pointed out.

CLEVE [1]) obtained the same compound by passing newly-precipitated hydrate of yttrium into selenious acid and precipitating the nitrate of yttrium with acid selenite of sodium.

SELENITES OF ERBIUM.

1. Neutral: $Er^2.O^6.3SeO + 5H^2O$.

Nitrate of erbium, not being completely neutral, but holding free nitric acid, when treated with excess of selenite of sodium, yielded a heavy precipitate which, analogous to the above described selenite of yttrium, was composed of microscopical, globular, transparent masses.

Analyses:

1) 0.6967 gr. salt gave 0.2094 gr. selenium or 0.2942 gr. selenious acid and 0.5395 gr. sulphate of erbium or 0.3337 gr. erbia. On white-

[1]) Bih. t. Sv. Vet. Ak:s handl. I N:o 8 s. 37.

heating with addition of carbonate of ammonium, this sulphate easily afforded the calculated quantity of erbia [1]).

2) 0.7912 gr. salt gave 0.2305 gr. selenium or 0.3239 gr. selenious acid and 0.603 gr. sulphate of erbium or 0.3729 gr. erbia.

Represented in 100 parts:

	found			calculated	
	1.	2.			
Erbia	47.90	47.13	Er^2O^3	389	47.91
Selenious acid . .	42.23	40.94	$3SeO^2$	333	41.01
Water (loss)	9.87	11.93	$5H^2O$	90	11.08
	100.00	100.00		812	100.00

In consequence of the amount of water present in the salt differing from that of the corresponding selenite of yttrium, the preparation of selenite of erbium was once more undertaken, neutral sulphate being employed. Then

2. **Neutral:** $Er^2.O^6.3SeO + 9H^2O$

was obtained as an amorphous precipitate, white, with a slight inclination to rosy-colour.

Analyses:

1) 0.5655 gr. salt gave 0.1497 gr. selenium or 0.2103 gr. selenious acid and 0.249 gr. oxide of erbium, obtained by heating the sulphate to whiteness, carbonate of ammonium being added.

2) 0.5725 gr. salt gave 0.1514 gr. selenium or 0.2127 gr. selenious acid and 0.2525 gr. erbia.

In 100 parts:

	found			calculated	
	1.	2.			
Erbia	44.03	44.10	Er^2O^3	389	44.00
Selenious acid	37.19	37.15	$3SeO^2$	333	37.67
Water (loss)	18.78	18.75	$9H^2O$	162	18.33
	100.00	100.00		884	100.00

[1]) I have found this to be the case with all the sulphates of the rare earth-metals; but they can as well be quantitively determined as sulphates, because these salts, like those of magnesium and cobolt, by gentle heating not at all are decomposed.

3. $^4/_3$-Selenite: $Er^2.O^6.3SeO + H^2.O^2.SeO + 3H^2O$.

Under the same circumstances as the selenite of yttrium — i. e. by digesting the neutral salt with three and nine mol. of selenious anhydride at a temperature of 60° — selenite of erbium has also given a $^4/_3$-selenite, nor does it seem capable of taking up more acid, analogous, in this, respect, to the ferric and yttrium-salts. When the mixture had evaporated to dryness and then been treated with water, there was in both cases obtained an insoluble, microscopically crystalline powder, composed of small prisms.

Analyses:

1) 0.5085 gr. salt gave 0.1772 gr. selenium or 0.249 gr. selenious acid and 0.3407 gr. sulphate of erbium or 0.2107 gr. erbia.
2) 0.5 gr. salt gave 0.175 gr. selenium or 0.2459 gr. selenious acid and 0.333 gr. sulphate of erbium, containing 0.206 gr. erbia.
3) 0.781 gr. salt gave 0.2765 gr. selenium or 0.3885 gr. selenious acid and 0.5092 gr. sulphate of erbium, corresponding to 0.3149 gr. erbia.

Calculated on 100 parts:

	experiment			theory		
	1.	2.	3.			
Erbia	41.44	41.20	40.32	Er^2O^3	389	42.98
Selenious acid . .	48.97	49.18	49.74	$4SeO^2$	444	49.06
Water (loss). . . .	9.59	9.62	9.94	$4H^2O$	72	7.96
	100.00	100.00	100.00		905	100.00

The material for the analyses 1) and 2) was obtained by treating the neutral selenite of erbium with 3 mol., the material for the analysis 3) by treating it with 9 mol. selenious acid.

HÖGLUND [1]) has examined the same salt, obtained by the action of free selenious acid on a solution of nitrate of erbium, with or without addition of ammonia, until a permanent precipitate began to be formed.

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[1]) Bih. t. Sv. Vet. Akad:s handl. I. N:o 8 s. 38.

SELENITES OF CERIUM.

1. $^5/_6$-Selenite: $Ce^4.O^{11}.5SeO + 30H^2O$.

Sulphate of cerium being acted upon with a solution of selenite of sodium in excess, a white, insoluble, voluminous, amorphous precipitate was afforded.

Analyses:

1) 0.5625 gr. salt gave 0.127 gr. selenium or 0.1784 gr. selenious acid and 0.367 gr. sulphate of cerium, containing 0.2108 gr. oxide of cerium.
2) 0.587 gr. salt gave 0,1335 gr. selenium or 0.1876 gr. selenious acid and 0.382 gr. sulphate of cerium, containing 0.2195 gr. oxide of cerium.

In 100 parts:

	found		calculated		
	1.	2.			
Oxide of cerium . . .	37.48	37.39	$2Ce^2O^3$	648	37.17
Selenious acid	31.72	31.96	$5SeO^2$	555	31.84
Water (loss)	30.80	30.65	$30H^2O$	540	30.99
	100.00	100.00		1743	100.00

From this basic salt the following have been prepared by digesting with selenious acid at a moderate heat. The neutral salt was obtained by adding 1, $^4/_3$-selenite by adding 7, and the diselenite by adding 19 mol. andydride to 1 mol. basic salt.

2. Neutral: $Ce^2.O^6.3SeO + 12H^2O$.

In the above manner this salt was obtained in form of a white, amorphous, insoluble mass.

Analyses:

1) 0.5015 gr. salt gave 0.134 gr. selenium or 0.1883 gr. selenious acid and 0.315 gr. sulphate of cerium, containing 0.181 gr. oxide of cerium.
2) 0.4122 gr. salt gave 0.1105 gr. selenium or 0.1553 gr. selenious acid and 0.2667 gr. sulphate of cerium, containing 0.1532 gr. oxide of cerium.

Represented in 100 parts:

	found 1.	found 2.		calculated	
Oxide of cerium .	36.09	37.17	Ce^2O^3	324	37.11
Selenious acid . .	37.55	37.68	$3SeO^2$	333	38.14
Water (loss) . . .	26.36	25.15	$12H^2O$	216	24.75
	100.00	100.00		873	100.00

JOLIN [1]), by precipitating acetate of cerium with free selenious acid, obtained an amorphous, neutral salt with 3 mol. water, of which one half was lost at 100°.

3. $^4/_3$-Selenite: $Ce^2.O^6.3SeO + H^2.O^2.SeO + 5H^2O$.

This salt, produced in the above mentioned manner, afforded a microscopically crystalline, insoluble powder, the crystals being gathered together into almost globular aggregates.

Analysis:

0.58 gr. salt gave 0.2065 gr. selenium or 0.2901 gr. selenious acid and 0.3807 gr. sulphate of cerium or 0.2187 gr. oxide of cerium.

In 100 parts:

	found		calculated	
Oxide of cerium . . .	37.71	Ce^2O^3	324	36.99
Selenious acid	50.02	$4SeO^2$	444	50.68
Water (loss)	12.27	$6H^2O$	108	12.33
	100.00		876	100.00

By treating neutral salt with selenious acid, JOLIN [2]) obtained also such an acid salt, but containing only 4 mol. water of crystallization.

4. Diselenite: $Ce^2.O^6.3SeO. + 3H^2.O^2.SeO + 2H^2O$.

The basic selenite of cerium treated with 19 mol. selenious anhydride and water at a moderate heat, afforded this salt well crystallized in oblique, four-sided, microscopical laminæ, not altering in water.

Analysis:

0.8155 gr. salt gave 0.3575 gr. selenium or 0.5023 gr. selenious acid and 0.429 gr. sulphate of cerium, containing 0.2465 gr. oxide of cerium.

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[1]) Bull. soc. chim. [2] XXI. p. 539.

[2]) Loc. cit.

In 100 parts:

	found		calculated	
Oxide of cerium . .	30.22	Ce^2O^3	324	30.00
Selenious acid . . .	61.60	$6SeO^2$	666	61.67
Water (loss)	8.18	$5H^2O$	90	8.33
	100.00		1080	100.00

This salt is quite analogous in composition to the above-mentioned salt of aluminium

$$Al^2.O^6.3SeO + 3H^2.O^2.SeO + 2H^2O,$$

also formed under the same circumstances.

SELENITES OF LANTHANUM.

1. $^8/_9$-Selenite: $La^6.O^{17}.8SeO + 28H^2O$.

A solution of sulphate of lanthanum, on adding excess of selenite of sodium, yielded a white, insoluble, completely amorphous precipitate.

Analyses:

1) 0.5735 gr. salt gave 0.1562 gr. selenium or 0.2195 gr. selenious acid and 0.4015 gr. sulphate of lanthanum, which at a white heat, carbonate of ammonium being added, gave the calculated quantity of oxide of lanthanum or 0.2313 gr.
2) 0.473 gr. salt gave 0.1308 gr. selenium or 0.1838 gr. selenious acid and 0.3355 gr. sulphate of lanthanum, which, after heating to whiteness with carbonate of ammonium, gave 0.1935 gr. oxide of lanthanum, the calculated quantity being 0.1932 gr.

Centesimally represented:

	found		calculated		
	1.	2.			
Oxide of lanthanum .	40.33	40.91	$3La^2O^3$	978	41.26
Selenious acid	38.27	38.86	$8SeO^2$	888	37.47
Water (loss)	21.40	20.23	$28H^2O$	504	21.27
	100.00	100.00		2370	100.00

As to the composition this salt agrees completely with a ferric selenite, obtained from a neutral solution of chloride of iron with selenite of sodium:

$$Fe^6.O^{17}.8SeO + 28H^2O.$$

As, however none of the already mentioned earth-metals, except cerium, by decomposition of the sulphate with excess of selenite of sodium, has given any basic salts, I was induced to repeat the preparation of selenite of lanthanum. The appearance of the obtained precipitate was the same as in the preceding case. A considerable excess of selenite of sodium was employed.

Analyses:

1) 0.5097 gr. salt gave 0.1288 gr. selenium or 0.181 gr. selenious acid and 0.3312 gr. sulphate of lanthanum, containing 0.1908 gr. oxide of lanthanum.
2) 0.4683 gr. salt gave 0.12 gr. selenium or 0.1686 gr. selenious acid and 0.3095 gr. sulphate of lanthanum, containing 0.1782 gr. oxide of lanthanum.

With these values, which calculated on 100 parts may be thus represented:

	1.	2.
Oxide of lanthanum .	37.43	38.05
Selenious acid.	35.51	36.01
Water (loss)	27.06	25.94
	100.00	100.00

the following formula exhibits a tolerably close correspondence,

2. **Neutral:** $La^2.O^6.3SeO + 12H^2O.$

La^2O^3	326	37.26
$3SeO^2$	333	38.05
$12H^2O$	216	24.69
	875	100.00

although the great deficiency of selenious acid, amounting to 2—2.5 pr. ct., sufficiently proves that the precipitate was mixed with a basic salt, and that lanthanum, on mixing neutral solutions, has a great tendency to fall out as a basic selenite.

3. $^5/_3$-Selenite: $La^2.O^6.3SeO + 2H^2.O^2.SeO + 4H^2O$.

When the next-preceding salt was acted upon with an aqueous solution of 40 pr. ct. its weight of selenious anhydride, there was, already at the ordinary temperature, within a few moments obtained a crystalline compound. When subjected to the microscope, the crystals seemed to be four-sided oblique tablets; they did not alter in appearance on evaporating the solution to dryness at a moderate heat, but remained unchanged, when the residue was treated with water.

Analysis:

0.6948 gr. salt gave 0.272 gr. selenium or 0.3822 gr. selenious acid and 0.404 gr. sulphate of lanthanum, corresponding to 0.2327 gr. oxide of lanthanum.

Represented in 100 parts:

	found		calculated	
Oxide of lanthanum . . .	33.49	La^2O^3	326	32.96
Selenious acid.	55.01	$5SeO^2$	555	56.12
Water (loss).	11.50	$6H^2O$	108	10.92
	100.00		989	100.00

The correspondence between this salt and the selenite of chromium formed under the same circumstances,

$$Cr^2.O^6.3SeO + 2H^2.O^2.SeO + 7H^2O,$$

is evident. It is surprising that lanthanum has not, under the above circumstances, afforded any $^4/_3$-selenite, that being the case with all the other rare earth-metals.

5. Diselenite: $La^2.O^6.3SeO + 3H^2.O^2.SeO + 4H^2O$.

This compound was obtained, when the salt mentioned under 2 was digested with 120 pr. ct. its weight of anhydride of selenious acid, the solution allowed to evaporate to dryness at a gentle heat and the residue treated with water; the excess of acid was then extracted, leaving an insoluble crystal-powder, which under the microscope seemed to consist of four-sided prisms.

Analysis:

0.7043 gr. salt gave 0.3042 gr. selenium or 0.4274 gr. selenious acid and 0.3457 gr. sulphate of lanthanum, containing 0.1991 gr. oxide of lanthanum.

In 100 parts:

Oxide of lanthanum . . .	28.27	La^2O^3	326	29.16
Selenious acid	60.68	$6SeO^2$	666	59.57
Water (loss)	11.05	$7H^2O$	126	11.27
	100.00		1118	100.00

A before-mentioned salt of aluminium,

$$Al^2.O^6.3SeO + 3H^2.O^2.SeO + 2H^2O,$$

agrees with this diselenite, but holds only 2 mol. water of crystallization, quite in accordance with the following

5. **Diselenite:** $La^2.O^6.3SeO + 3H^2.O^2.SeO + 2H^2O$,

which was obtained by treating hydrate of lanthanum with selenious acid. The carbonic acid which the hydrate had taken up in contact with the air being expelled, there was formed a heavy powder, crystalline when viewed with the microscope; by the evaporation of the mother-liquor it was obtained crystallized in distinct, oblique, four-sided laminæ.

Analyses:

1) 0.57 gr. salt gave 0.2403 gr. selenium or 0.3376 gr. selenious acid and 0.3075 gr. sulphate of lanthanum, containing 0.1771 gr. oxide of lanthanum.
2) 0.6615 gr. salt gave 0.279 gr. selenium or 0.392 gr. selenious acid and 0.359 gr. sulphate of lanthanum, containing 0.2068 gr. oxide of lanthanum.

In 100 parts:

	found		calculated		
	1.	2.			
Oxide of lanthanum	31.07	31.26	La^2O^3	326	30.13
Selenious acid	59.24	59.26	$6SeO^2$	666	61.55
Water (loss)	9.69	9.48	$5H^2O$	90	8.32
	100.00	100.00		1082	100.00

CLEVE [1]) obtained the same salt by mixing chloride of lanthanum with selenious acid and alcohol.

SELENITES OF DIDYMIUM.

1. $^{8}/_{9}$-Selenite: $Di^{6}.O^{17}.8SeO + 28H^{2}O$.

On mixing sulphate of didymium with an excess of selenite of sodium, an amorphous, white precipitate, slightly inclining to rose, was thrown down.

Analyses:

1) 0.5083 gr. salt, gave 0.1356 gr. selenium or 0.1905 gr. selenious acid and 0.3627 gr. sulphate of didymium, containing 0.2131 gr. oxide of didymium.
2) 0.5153 gr. salt gave 0.139 gr. selenium or 0.1953 gr. selenious acid and 0.3697 gr. sulphate of didymium, containing 0.2172 gr. oxide of didymium.

Centesimally represented:

	found			calculated	
	1.	2.			
Oxide of didymium .	41.92	42.15	$3Di^{2}O^{3}$	1026	42.43
Selenious acid	37.48	37.90	$8SeO^{2}$	888	36.72
Water (loss)	20.60	19.95	$28H^{2}O$	504	20.85
	100.00	100.00		2418	100.00

Sulphate of didymium with selenite of sodium gives consequently a basic salt, analogous in composition to selenites of iron and lanthanum, treated of above.

2. $^{4}/_{3}$-Selenite: $Di^{2}O^{6}.3SeO + H^{2}.O^{2}.SeO + 8H^{2}O$.

Was obtained in the same way as the corresponding selenite of lanthanum, and formed a microscopically crystalline powder of four-sided prisms.

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[1]) Bih. t. Sv. Vet. Akad:s handl. II N:o 7 s. 45.

Analysis:

0.4435 gr. salt gave 0.147 gr. selenium or 0.2066 gr. selenious acid and 0.269 gr. sulphate of didymium, containing 0.1581 gr. oxide of didymium.

Centesimally represented:

	found		calculated	
Oxide of didymium	35.65	Di^2O^3	342	36.08
Selenious acid	46.59	$4SeO$	444	46.83
Water (loss)	17.76	$9H^2O$	162	17.09
	100.00		948	100.00

CLEVE [1]), by adding selenious acid and alcohol to the nitrate, obtained even such an acid salt, but with only 4 mol. water.

3. **Sesquiselenite:** $2[Di^2.O^6.3SeO] + 3H^2.O^2.SeO + 18H^2O$.

A solution, obtained by digesting the basic selenite of didymium, mentioned under 1, at a gentle heat in 128.5 pr. ct. its weight of selenious anhydride and water, when allowed to evaporate to dryness and treated with water, afforded this salt as a slightly lilac crystal-powder. Subjected to the microscope it exhibited four-sided prisms.

Analysis:

0.495 gr. salt gave 0.1787 gr. selenium or 0.2511 gr. selenious acid and 0.2832 gr. sulphate of didymium, containing 0.1664 gr. oxide of didymium.

In 100 parts:

	found		calculated	
Oxide of didymium . . .	33.62	$2Di^2O^3$	684	34.08
Selenious acid	50.73	$9SeO^2$	999	49.77
Water (loss)	15.65	$18H^2O$	324	16.15
	100.00		2007	100.00

That didymium has given this sesquiselenite, corresponding to the above mentioned aluminium-salt

$$2Al^2.O^6.3eO + 3H^2.O^2.SeO + 12H^2O,$$

seems in a theoretical point of view to be of great importance.

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[1]) Bih. t. Sv. Vet. Akad:s handl. II N:o 8 s. 69.

SELENITES OF INDIUM.

1. $^3/_4$-**Selenite:** $In^8.O^{21}.9SeO + 25H^2O = 3(In^2.O^6.3SeO) + In^2.O^6.H^6 + 22H^2O$.

On adding selenite of sodium in excess to neutral chloride of indium, an amorphous, slimy, insoluble salt was precipitated.

Analyses:

1) 0.419 gr. salt gave 0.1205 gr. selenium or 0.1693 gr. selenious acid and 0.176 gr. oxide of indium.
2) 0.4615 gr. salt gave 0.1305 gr. selenium or 0.1834 gr. selenious acid and 0.194 gr. oxide of indium.
3) 1.3395 gr. salt lost 0.18 gr. of its weight at 100°.

Represented in 100 parts:

	found			calculated		
	1.	2.	3.			
Oxide of indium .	42.00	42.04	—	$4In^2O^3$	1099.2	43.13
Selenious acid . .	40.41	39.74	—	$9SeO^2$	999.0	39.21
Water (loss) . . .	17.59	18.22	13.44	$25H^2O$	450.0	17.66
	100.00	100.00			2548.2	100.00

This salt is analogous to the selenites of chromium and aluminium, formed under the same circumstances. It parts at 100° with 19 mol. water (calc. 13.41 pr. ct.).

2. **Neutral:** $In^2.O^6.3SeO + 6H^2O$.

Neutral selenite of indium was obtained, when 1 mol. of the foregoing salt was digested with 3 (anal. 1—3) or 15 (anal. 4—5) or 39 (anal. 6—7) mol. selenious anhydride and some water at a temperature of about 60°, the mixture not being allowed completely to evaporate; if, on the contrary, it was wholly dried and subsequently treated with water, the acid salts, mentioned below, were obtained. In neither case, did even traces of indium occur in the solution.

The neutral salt, subjected to the microscope, exhibited a pretty developed crystallization of fine acicular prisms, which, no excess of selenious acid being present, were for the most part united in globular aggregates.

Analyses:

1) 0.5014 gr. salt gave 0.1655 gr. selenium or 0.2325 gr. selenious acid and 0.191 gr. oxide of indium [1]).
2) 0.569 gr. salt gave 0.1865 gr. selenium or 0.262 gr. selenious acid and 0.2145 gr. oxide of indium.
3) 0.5945 gr. salt lost 0.07 gr. of its weight at 100°.
4) 0.559 gr. salt gave 0.1885 gr. selenium or 0.2648 gr. selenious acid and 0.2125 gr. oxide of indium.
5) 0.719 gr. salt gave 0.242 gr. selenium or 0.34 gr. selenious acid and 0.2685 gr. oxide of indium.
6) 0.5065 gr. salt gave 0.1745 gr. selenium or 0.2452 gr. selenious acid and 0.19 gr. oxide of indium.
7) 0.5725 gr. salt gave 0.1965 gr. selenium or 0.2761 gr. selenious acid and 0.2135 gr. oxide of indium.

Represented in 100 parts:

	1.	2.	3.	4.	5.	6.	7.
Oxide of indium .	38.09	37.70	—	38.02	37.34	37.51	37.29
Selenious acid . .	46.37	46.05	—	47.37	47.29	48.41	48.22
Water (loss) . . .	15.54	16.25	11.77	14.61	15.37	14.08	14.49
	100.00	100.00		100.00	100.00	100.00	100.00

the formula requiring:

In^2O^3	274.8	38.38
$3SeO^2$	333.0	46.52
$6H^2O$	108.0	15.10
	715.8	100.00

The salt parts at 100° with $4^1/_2$ mol. of the crystallization-water (calc. 11 32 pr. ct.)

3. **Sesquiselenite:** $2[In^2.O^6.3SeO] + 3H^2.O^2.SeO + 12H^2O$.

A mixture of the basic salt and 15 mol. or 65.35 pr. ct. selenious anhydride, having evaporated to dryness at about 60°, afforded, after washing with water, a granularly crystalline salt of this composition, completely insoluble in water.

[1]) The quantity of indium-oxide was determined by evaporating the filtrate from the separated selenium to dryness, fuming the excess of formed sulphuric acid and heating the thus formed sulphate of indium, which, even in the flame of an ordinary burner, speedely loses its sulphuric acid, so that the loss of weight, on further heating to whiteness, is very inconsiderable.

Analyses:

1) 0.6735 gr. salt gave 0.2615 gr. selenium or 0.3674 gr. selenious acid and 0.2035 gr. oxide of indium.
2) 0.579 gr. salt gave 0.224 gr. selenium or 0.3147 gr. selenious acid and 0.176 gr. oxide of indium.
3) 0.4505 gr. salt lost 0.054 gr. of its weight at 100°.

Represented in 100 parts:

	experiment				theory	
	1.	2.	3.			
Oxide of indium .	30.21	30.39	—	$2In^2O^3$	549.6	30.22
Selenious acid . .	54.55	54.36	—	$9SeO^2$	999.0	54.93
Water (loss) . . .	15.24	15.25	11.99	$15H^2O$	270.0	14.85
	100.00	100.00			1818.6	100.00

Through this salt, which at 100 loses 12 mol. water (calc. 11.88 pr. ct.), so that a waterfree sesquiselenite appears to be formed, indium, as regards its relation to selenious acid, places itself by the side of aluminium and didymium, which also have given salts of an analagous composition; yet it is to be observed the didymium-salt was obtained, when a teraselenite was to be expected.

4. **Diselenite:** $In^2.O^6.3SeO + 3H^2.O^2.SeO + 4H^2O$.

Was formed, when the basic salt was digested with 39 mol. or 169.91 pr. ct. selenious anhydride, and the mixture was dried at about 60°; after treatment with water, there remained a salt, insoluble therein, which examined with the microscope proved to be distinctly crystalline.

Analyses:

1) 0.5825 gr. salt gave 0.262 gr. selenium or 0.3681 gr. selenious acid and 0.1485 gr. oxide of indium.
2) 0.609 gr. salt gave 0.2715 gr. selenium or 0.3815 gr. selenious acid and 0.157 gr. oxide of indium.
3) 0.615 gr. salt at 100°, lost 0.0635 gr. or 6 mol. water.

Represented in 100 parts:

	found				calculated	
	1.	2.	3.			
Oxide of indium. . .	25.49	25.78	—	In^2O^3	274.8	25.76
Selenious acid	63.19	62.64	—	$6SeO^2$	666.0	62.43
Water (loss)	11.32	11.58	10.32	$7H^2O$	126.0	11.81
	100.00	100.00			1066.8	100.00

Under the same circumstances as the above diselenite has been formed, aluminium, cerium and lanthanum gave salts of an analogous composition.

In its selenites, indium then shows analogy to aluminium, chromium, didymium, cerium and lanthanum, a circumstance which supports the view, that the oxide of indium is a sesquioxide, advanced by BUNSEN[1]) in his determination of the specific heat of the metal and confirmed by ROESSLER'S [2]) production of indium-ammonium-alum.

SELENITES OF URANIUM [URANYL].

1. Neutral: $[Ur^2O^2].O^2.SeO + 2H^2O$.

A solution of acetate of uranium treated with selenite of sodium in excess, afforded a slimy, amorphous, yellow precipitate, insoluble in water, but readily dissolved in hydrochloric acid.

Analyses:

1) 1.0662 gr. salt parted at 100° with 0.099 gr. water and gave 0.1885 gr. selenium or 0.2648 gr. selenious acid, and from the filtrate, after evaporation and strong heating first in contact with the air, subsequently in hydrogen, 0.6545 gr. uranous oxide or 0.6933 gr. uranic oxide.
2) 0.672 gr. salt gave in the same manner 0.1163 gr. selenium or 0.1634 gr. selenious acid and 0.414 gr. uranous oxide, corresponding to 0.4384 gr. uranic oxide.

In 100 parts:

	found			calculated	
	1.	2.			
Uranic oxide	65.02	65.24	U^2O^3	285.6	66.02
Selenious acid	24.84	24.32	SeO^2	111.0	25.66
Water (loss)	10.14	10.44	$3H^2O$	36.0	8.32
	100.00	100.00		432.6	100.00

This salt gives off the water of crystallization at 100°.

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[1]) Ann. d. Ph. u. Ch. CXLI. 1. (1871).

[2]) Journ. f. prakt. Ch. [2] VII. S. 14. (1873).

When this salt with 25.66 pr. ct. selenious anhydride and water was allowed to digest at a temperature of 60°, there was obtained, no solution taking place,

2. $^5/_3$-**Selenite:** $3(U^2O^2).O^2.SeO + 2H^2.O^2.SeO + 7H^2O$.

The compound deposited as a beautifully yellow-coloured crystal-powder which, under the microscope, proved to be completely homogeneous and composed of rhombic tablets with the acuter summits often truncated.

In the mother-liquor, that was slightly yellow, a small quantity of uranium was met with, so that it was coloured brown by ferro-cyanide of potassium; after evaporation it at last yielded colourless crystals of selenious acid. The salt was insoluble in water, with which it was washed previously to analysis.

Analyses:

1) 0.5045 gr. salt gave 0.1225 gr selenium or 0.1721 gr. selenious acid and 0.262 gr. uranous oxide or 0.2775 gr. uranic oxide.
2) 0.61 gr. salt gave 0.1488 gr. selenium or 0.2091 gr. selenious acid and 0.32 gr. uranous oxide or 0.3389 gr. uranic oxide.
3) 0.589 gr. salt, when dried at 100°, gave off 0.053 gr. water.

Calculated on 100 parts:

	experiment				theory	
	1.	2.	3.			
Uranic oxide	55.00	55.56	—	Ur^2O^3	856.8	54 44
Selenious acid . . .	34.11	34.28	—	$5SeO^2$	555.0	35.27
Water (loss)	10.89	10.16	8.83	$9H^2O$	162.0	10.29
	100.00	100.00			1573.8	100.00

The loss the salt suffered at 100°, amounts to 8 mol. water (calc. 9.24).

The same salt but with a different amount of water

3. $^5/_3$-**Selenite:** $3(Ur^2O^2).O^2.SeO + 2H^2.O^2.SeO + 5H^2O$,

was obtained by treating the neutral salt with 51.32 pr. ct. selenious anhydride under the same circumstances as in the case of the preceding salt. It formed a microscopically crystalline powder.

Analyses:

1) 0.56 gr. salt gave 0.142 gr. selenium or 0.1995 gr. selenious acid and 0.296 gr. uranous oxide, or 0.3135 gr. uranic oxide.
2) 0.779 gr. salt gave 0.1965 gr. selenium or 0.2761 gr. selenious acid and 0.41 gr. uranous oxide, corresponding to 0.4343 gr. uranic oxide.
3) 0.5295 gr. salt at 100° lost 0.035 gr. water.

Represented in 100 parts:

	experiment				theory	
	1.	2.	3.			
Uranic oxide	55.98	55.76	—	$3U^2O^3$	856.8	55.72
Selenious acid . . .	35.63	35.44	—	$5SeO^2$	555.0	36.09
Water (loss)	8.39	8.80	6.61	$7H^2O$	126.0	8.19
	100.00	100.00			1537.8	100.00

At 100° 6 mol. water of 7, contained in the salt, were expelled (calc. 7.02).

As the neutral selenite, even when treated, under the same circumstances, with 77 pr. ct. its weight of selenious anhydride, or precisely the quantity necessary to convert it into a tetraselenite, only gave the last mentioned salt, this seems to be the most acid selenite that can be formed by it. The light-yellow, crystalline compound then obtained presented namely the following values.

Analyses:

1) 0.7025 gr. salt gave 0.179 gr. selenium or 0.2515 gr. selenious acid; the uranium-determination was lost.
2) 0.548 gr. salt gave 0.14 gr. selenium or 0.1967 gr. selenious acid and 0.2855 gr. uranous oxide or 0.3024 gr. uranic oxide.
3) 0.59 gr. salt lost 0.0365 gr. water at 100°.

These values, calculated on 100 parts:

Uranic oxide . . .	—	55.18	—
Selenious acid . .	35.80	35.90	—
Water (loss) . . .	—	8.92	6.19
		100.00	

show the compound to be the same as the next-preceding.

Among the metals above treated, chromium and lanthanum have given $^5/_3$-selenites; consequently, uranium shows a certain analogy with them.

If we comprize in a general view the data resulting from the researches on the selenites of the metal-group treated of in the preceding pages, very interesting and remarkable conclusions may be arrived at.

First it is worth notice that most of the neutral solutions of these metals afford basic salts by precipitating with neutral selenite of sodium. Excluding uranium, as being single in its kind, it is only yttrium and erbium which by this treatment give neutral salts; indium like aluminium and chromium has given $^3/_4$-selenite; cerium $^5/_6$-selenite; iron didymium and lanthanum $^8/_9$-selenites. Except in the case of yttrium and erbium, neutral salts have been produced by treating the basic ones with quantities of selenious acid calculated for the purpose and weighed off to the nearest milligram in form of anhydride; the acid salts mentioned have been obtained in the same way, and the temperature at which the solutions were digested, has been the same in all experiments, the small glass-vessels in which the reaction took place, being placed immediately upon an air-bath, which always showed a temperature of 100°. The homogeneous quality of the salts was in every case determined by means of microscopical examination, so that no mixtures of salts were possible. These acid salts, all of which crystallized more or less distinctly, present a very remarkable composition. Under accurately the same circumstances, viz. 1 mol. neutral salt being treated with 3 mol. selenious acid, chromium, iron, yttrium, erbium, cerium and didymium have given $^4/_3$-selenites [it is remarkable, that this is not the case with lanthanum]; aluminium indium and didymium have afforded sesqui-selenites, both the former when diselenite, the latter when tetraselenite might have been formed; chromium and lanthanum have given $^5/_3$-selenites, when the proportion between base and acid allowed the formation of tetraselenites, and aluminium, cerium lanthanum and indium have under the last mentioned circumstances formed diselenites; but no one of the metals in question has given salts more acid than diselenite.

A better survey of these circumstances and of the corresponding composition of the products, obtained in the different cases, is given in the subjoined tables.

I have namely obtained:

1:o. From dilute solutions of the neutral salt, by treatment with neutral selenite of sodium:

a. $^3/_4$-selenite: $Al^8.O^{21}.9SeO + 36H^2O$.
$Cr^8.O^{21}.9SeO + 64H^2O$.
$In^8.O^{21}.9SeO + 25H^2O$.

b. $^5/_6$-selenite: $Ce^4.O^{11}.5SeO + 30H^2O$.

c. $^8/_9$-selenite: $Fe^6.O^{17}.8SeO + 28H^2O$.
$La^6.O^{17}.8SeO + 28H^2O$.
$Di^6.O^{17}.8SeO + 28H^2O$.

d. neutral selenite: $Y^2.O^6.3SeO + 12H^2O$.
$Er^2.O^6.3SeO + 5H^2O$.
$Er^2.O^6.3SeO + 9H^2O$.

2:o. From the basic salt, by treating with the quantity of acid necessary for forming neutral salt:

neutral selenite: $Al^2.O^6.3SeO + 7H^2O$.
$Cr^2.O^6.3SeO + 15H^2O$.
$Fe^2.O^6.3SeO + 7H^2O$.
$Ce^2.O^6.3SeO + 12H^2O$.
$In^2.O^6.3SeO + 6H^2O$.

3:o. From the basic or the neutral salt, by treating with the quantity of acid necessary for forming diselenite:

$^4/_3$-selenite: $Cr^2.O^6.3SeO + H^2.O^2.SeO + 12H^2O$.
$Fe^2.O^6.3SeO + H^2.O^2.SeO + 9H^2O$.
$Fe^2.O^6.3SeO + H^2.O^2.SeO + 7H^2O$.
$Ce^2.O^6.3SeO + H^2.O^2.SeO + 5H^2O$. [1])
$Di^2.O^6.3SeO + H^2.O^2.SeO + 8H^2O$. [2])
$Y^2.O^6.3SeO + H^2.O^2.SeO + 3H^2O$.
$Er^2.O^6.3SeO + H^2.O^2.SeO + 3H^2O$.

Like yttrium and erbium, also iron has not given salts more acid than those here mentioned.

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[1]) Obtained by JOLIN with 4 mol. H^2O.
[2]) Obtained by CLEVE with 3 mol. H^2O.

4:o. From the basic salt, by treating with the quantity of acid necessary for forming a) diselenite b) tetraselenite:

$^3/_2$-selenite: a) $2[Al^2.O^6.3SeO]+3H^2.O^2.SeO+9H^2O$.
$2[In^2.O^6.3SeO]+3H^2.O^2.SeO+12H^2O$.
b) $2[Di^2.O^6.3SeO]+3H^2.O^2.SeO+18H^2O$.

5:o. From the basic salt, by treating with the quantity of acid necessary for forming tetraselenite:

$^5/_3$-selenite: $Cr^2.O^6.3SeO+2H^2.O^2.SeO+7H^2O$.
$La^2.O^6.3SeO+2H^2.O^2.SeO+4H^2O$.

6:o. From the basic salt by treating with the quantity of acid necessary for forming tetraselenite:

diselenite: $Al^2.O^6.3SeO+3H^2.O^2.SeO+2H^2O$.
$Ce^2.O^6.3SeO+3H^2.O^2.SeO+2H^2O$.
$La^2.O^6.3SeO+3H^2.O^2.SeO+4H^2O$.
$In^2.O^6.3SeO+3H^2.O^2.SeO+4H^2O$.

As the above salts, which, within the different groups, very often agree even as to the amount of water, have been formed under circumstances that may be called absolutely the same, I think we may safely infer from this, that the metals, contained in them, have a similar quantivalence. If, consequently, iron, chromium, aluminium are unquestionably quadrivalent with sexivalent double-atoms, it is also necessary that cerium, lanthanum, didymium, yttrium, erbium, indium must share this property, on account of a correspondence in the composition of compounds, formed under the same circumstances, as complete as can be desired.

In the course of his researches on the earth-metals, Cleve [1]) says that he has found "by no means convincing reasons for the sexivalence of the metals mentioned such as the composition of didymiumchloraurate $2DiCl^3+3AuCl^3$, the amount of water in the acetates of cerium and lanthanum, the natrium-didymium-carbonate $2Di^2(CO^3)^3+3(Na^2CO^3)$ and some doublesulphates of kalium"; on account of other data, he is therefore of opinion that they are trivalent, and consequently for instance writes the formula of the chlorides RCl^3.

The palpable analogy between the above mentioned salts seems to place it, however, beyond all doubt, that they do not possess this valence,

[1]) Bih. t. Sv. Vet. Akad. handl. 2 N:o 12 s. 92.

but rather appear as sexivalent double-atoms; and it may be confidently expected that the determination of the molecular weights of some among their volatile compounds will confirm this conclusion.

Among selenites of trivalent elements, only those of bismuth and antimony have been examined; a description of them follows.

SELENITES OF BISMUTH.

1. Neutral: $Bi^2.O^6.3SeO$.

Basic carbonate of bismuth being boiled with a solution of selenious acid, this waterfree salt was formed in small, microscopical needles.

Analysis:

0.7555 gr. salt, on strong heating, gave a residue of 0.4423 gr. oxide of bismuth, during the sublimation of 0.3132 gr. selenious anhydride.

In 100 parts:

	experiment		theory	
Oxide of bismuth. .	58.54	Bi^2O^3	468	58.43
Selenious acid . . .	41.46	$3SeO^2$	333	41.57
	100.00		801	100.00

2. $^4/_3$-Selenite: $Bi^2.O^6.3SeO + H^2.O^2.SeO$.

Hydrate of bismuth, treated at the ordinary temperature with excess of concentrated selenious acid, afforded a salt in prismatic crystals, generally united in leaf-like aggregates.

Analyses:

1) 0.523 gr. salt, heated to whiteness, gave a residue of 0.266 gr. oxide of bismuth.
2) 0.4765 gr. salt gave in the same manner 0.243 gr. oxide of bismuth.
3) 0.66 gr. salt gave 0.22 gr. selenium or 0.3091 gr. selenious acid.
4) 0.8485 gr. salt gave 0.284 gr. selenium or 0.399 gr. selenious acid.
5) 0.868 gr. salt lost at 100° only 0.0005 gr. of its weight.

Calculated on 100 parts:

	experiment					theory	
	1.	2.	3.	4.			
Oxide of bismuth	50.86	50.99	—	—	Bi^2O^3	468	50.32
Selenious acid . .	—	—	46.83	47.02	$4SeO^2$	444	47.74
					H^2O	18	1.94
						930	100.00

Bismuth seems incapable of uniting with selenious acid to any salt, more acid than the above-mentioned; for being, under the same circumstances that have been often described in the preceding, digested with 2 and 8 mol. selenious acid, it remained unaltered, crystallized in small prisms, when the selenious acid had been dissolved out with water. The analyses 1—3 refer to a material, obtained when a diselenite, 4—6, when a tetraselenite might have been formed.

Analyses:

1) 0.6035 gr. salt, heated to whiteness, gave 0.3085 gr. oxide of bismuth.
2) 0.5812 gr. salt gave 0.196 gr. selenium or 0.2754 gr. selenious acid.
3) 0.502 gr. salt gave 0 1685 gr. selenium or 0.2367 gr. selenious acid.
4) 1.093 gr. salt, heated to whiteness, gave 0.54 gr. oxide of bismuth.
5) 0.5035 gr. salt gave 0.1725 gr. selenium or 0.2423 gr. selenious acid.
6) 0.505 gr. salt gave 0.174 gr. selenium or 0.2445 gr. selenious acid.

Calculated on 100 parts:

	1.	2.	3.	4.	5.	6.
Oxide of bismuth . . .	51.12	—	—	49.41	—	—
Selenious acid	—	47.40	47.15	—	48.12	48.41,

values, closely agreeing with those, calculated from the formula above.

This salt, suffering no change at 100°, shows a composition, analogous to that of the selenites that have been found above to be so characteristic for the sexivalent metals, but unlike them, contains no water of crystallization. Now, bismuth being unquestionably trivalent, such a circumstance might possibly seem to plead for the trivalence of the rare earth-metals, in opposition to what has been stated above. It suffices, however, to remark how far bismuth in its properties differs from these elements, and how much these latter resemble the undoubtedly sexivalent Fe^2, Al^2, Cr^2, in order to remove every doubt, which, owing to the said correspondence in this single selenite, might have been raised of the

truth of the conclusion regarding the quantivalence of the earth-metals in question, drown from the above stated, striking analogy between all the selenites of the latter metals and the sexivalent metals mentioned. Besides it is worth notice that the selenite of bismuth retains its amount of water at 100°, whilst the corresponding salts of erbium and yttrium [which should be the most analogous, as also these metals have not given more acid salts] according to HÖGLUND and CLEVE, part with all their water or 4 mol.

SELENITES OF ANTIMONY.

1. $^2/_3$-Selenite: $2(Sb^2.O^5.2SeO)+H^2O$.

Oxide of antimony, prepared by precipitating tartar-emetic with ammonia under warmth, when treated with a concentrated solution of selenious acid, yielded a white, brilliant salt which, on microscopical examination, proved to be crystallized in very well developed, oblique, four-sided tablets.

Analyses:

1) 0.7 gr. salt, fumed with nitric acid and heated, gave 0.412 gr. tetroxide of antimony or 0.3906 gr. trioxide.
2) 0.6235 gr. salt gave likewise 0.367 gr. tetroxid of antimony or 0.348 gr. trioxide.
3) 0.557 gr. salt gave 0.166 gr. selenium or 0.2332 gr. selenious acid.
4) 0.802 gr. salt gave 0.2435 gr. selenium or 0.3421 gr. selenious acid.
5) 0.776 gr. salt lost 0.013 gr. water at 100°.

Calculated on 100 parts:

	experiment					theory		
	1.	2.	3.	4.	5.			
Oxide of antimony	55.80	55.82	—	—	—	$2Sb^2O^3$	584	55.83
Selenious acid . . .	—	—	41.87	42.65	—	$4SeO^2$	444	42.45
Water	—	—	—	—	1.68	H^2O	18	1.72
							1046	100.00

From this basic salt, which may also be regarded as an anhydrous salt of antimonyl and at 100° parts with its water, I obtained

2. $^4/_3$-**Selenite:** $Sb^2.O^6.3SeO+SeO^2$,

after its digesting with 4 (analyses 1—2) and 10 (analyses 3—4) mol. selenious acid, or, in other words, when diselenite and tetraselenite might have been formed. After evaporating the solution at about 60° and treating the residue with water, the salt was obtained in form of prisms, discernible in the microscope, and generally of a laminar appearance.

Analyses:

1) 0.524 gr. salt gave 0.225 gr. selenium or 0.3161 gr. selenious acid.
2) 0.507 gr. salt gave 0.221 gr. tetroxide of antimony or 0.2095 gr. trioxide.
3) 0.6065 gr. salt gave 0.2555 gr. selenium or 0.359 gr. selenious acid.
4) 0.512 gr. salt gave 0.221 gr. tetroxide of antimony or 0.2095 gr. trioxide.

Calculated on 100 parts:

	experiment					theory	
	1.	2.	3.	4.			
Oxide of antimony .	—	41.32	—	40.92	Sb^2O^3	292	39.68
Selenious acid . . .	60.32	—	59.19	—	$4SeO^2$	444	60.32
						736	100.00

The above selenites of antimony have their correspondents in two sulphates, examined by Peligot: [1])

$Sb^2.O^5.2SO^2$
$Sb^2.O^6.3SO^2+H^2.O^2.SO^2$.

Lastly I have examined selenites of three quadrivalent elements, viz.: thorium, zirconium and tin, a description of which I now communicate.

SELENITES OF THORIUM.

1. **Neutral:** $Th.O^4.2SeO+8H^2O$.

Sulphate of thorium was precipitated with excess of selenite of sodium. The precipitate was very voluminous, and partly deposited very slowly. Subjected to the microscope it exhibited only traces of a crystalline form. Insoluble in water, it is readily dissolved in hydrochloric acid.

[1]) Ann. de chim. et de phys. [3] XX. p. 283.

Analyses:

1) 0.5385 gr. salt gave 0.129 gr. selenium or 0.1813 gr. selenious acid and, when the filtrate had evaporated and the sulphate been heated to whiteness with carbonate of ammonium, 0 2325 oxide of thorium.
2) 0.6317 gr. salt gave 0.1547 gr. selenium or 0.2174 gr. selenious acid and 0.2767 gr. oxide of thorium.

In 100 parts:

	found		calculated		
	1.	2.			
Oxide of thorium . .	43.17	43.80	ThO^2	266	42.09
Selenious acid	33.67	34.41	$2SeO^2$	222	35.12
Water (loss)	23.16	21.79	$8H^2O$	144	22.79
	100.00	100.00		632	100.00

2. $^7/_4$-Selenite: $2(Th.O^4.2SeO)+3H^2.O^2.SeO+13H^2O$.

When the preceding salt, at a moderate heat, was digested in an aqueous solution of 35.12 pr. ct. anhydride of selenious acid, the solution, after evaporation to dryness and treatment of the residue with water, yielded an insoluble compound, analogous in appearance to the neutral salt.

Analyses:

1) 0.5735 gr. salt gave 0.1965 gr. selenium or 0.2761 gr. selenious acid and 0.189 gr. oxide of thorium, after heating to whiteness with carbonate of ammonium.
2) 0.5415 gr. salt gave 0.1835 gr. selenium or 0.2578 gr. selenious acid and 0.1767 gr. oxide of thorium.

In 100 parts:

	found		calculated		
	1.	2.			
Oxide of thorium	32.96	32.63	$2ThO^2$	532	33.31
Selenious acid	48.14	47.61	$7SeO^2$	777	48.65
Water (loss)	18.90	19.76	$16H^2O$	288	18.04
	100.00	100.00		1597	100.00

3. $^5/_2$-**Selenite:** $Th.O^4.2SeO + 3H^2.O^2.SeO + 5H^2O$.

The neutral selenite of thorium, treated with 6 mol. or 105.36 pr. ct. anhydride of selenious acid under the above circumstances, gave a residue which seemed to have undergone no change in appearance.

Analyses:

1) 0.6 gr. salt gave 0.2467 gr. selenium or 0.3466 gr. selenious acid and 0.167 gr. oxide of thorium, after heating to whiteness with carbonate of ammonium.
2) 0.781 gr. salt gave 0.3212 gr. selenium or 0.4513 gr. selenious acid and 0.2167 gr. oxide of thorium.

In 100 parts:

	found			calculated	
	1.	2.			
Oxide of thorium. . .	27.83	27.75	ThO^2	266	27.57
Selenious acid	57.78	57.78	$5SeO^2$	555	57.51
Water (loss)	14.39	14.47	$8H^2O$	144	14.92
	100.00	100.00		965	100.00

These in composition extraordinary selenites are in such a relation one to another that the acid salt, mentioned under 2, may be regarded as a sum of the two others, the amount of water unchanged:

$$2ThO^2.7SeO^2 + 16H^2O = \begin{cases} ThO^2.2SeO^2 + 8H^2O \\ ThO^2.5SeO^2 + 8H^2O. \end{cases}$$

SELENITES OF ZIRCONIUM.

1. $^3/_8$-**Selenite:** $Zr^4.O^{11}.3SeO + 18H^2O$.

A solution of oxy-chloride of zirconium precipitated with selenite of sodium, yielded a white, slimy, amorphous, basic salt, not at all soluble in water and only with difficulty in hydrochloric acid.

Analyses:

1) 0.5775 gr. salt gave 0.119 gr. selenium or 0.1672 gr. selenious acid and 0.248 gr. zirconia, after heating to whiteness with carbonate of ammonium.

2) 0.601 gr. salt gave 0.122 gr. selenium or 0.1714 gr. selenious acid and 0.254 gr. zirconia.
3) 0.9105 gr. salt, when dried at 100°, gave off 0.208 gr. of its weight.

Calculated on 100 parts:

	experiment				theory	
	1.	2.	3.			
Zirconia	42.94	42.26	—	$4ZrO^2$	488	42.62
Selenious acid . .	28.95	28.52	—	$3SeO^2$	333	29.08
Water (loss) . . .	28.11	29.22	22.84	$18H^2O$	324	28.30
	100.00	100.00			1145	100.00

At 100° the compound loses 15 mol. water out of 18 (calc. 23.58 pr. ct.).

2. **Neutral:** $Zr.O^4.2SeO + H^2O$.

By treating 1 mol. of the basic salt with 5 mol. selenious acid at a temperature of about 60°, there was obtained a crystallized compound in small, but well formed, microscopical, four-sided prisms, obliquely cut at the ends. Totally insoluble in water, this salt dissolved only with the greatest difficulty even in boiling hydrochloric acid. As it was dissolved in this manner for the analysis, this shows a considerable loss of selenious acid, owing to volatilized chloride of selenium.

Analyses:

1) 0.523 gr. salt gave 0.2173 gr. selenium or 0.3053 gr. selenious acid and 0.1765 gr. zirconia.
2) 0.6155 gr. salt gave off 0.0225 gr. water at 100° and left 0.2645 gr. selenium or 0.3715 gr. selenious acid and 0.2052 gr. zirconia.

In 100 parts:

	found		calculated		
	1.	2.			
Zirconia	33.75	33.34	ZrO^2	122	33.70
Selenious acid	58.37	60.37	$2SeO^2$	222	61.33
Water (loss)	7.88	6.29	H^2O	18	4.97
	100.00	100.00		362	100.00

3. Neutral: $Zr.O^4.2SeO$.

After digesting the basic salt in 13 mol. anhydride of selenious acid and water at 60°, there was obtained a well crystallized compound in microscopical, oblique, four-sided prisms. After heating to whiteness, the zirconia remained crystallized in the same form as the selenite.

Analysis:

0.432 gr. salt, heated to whiteness, afforded 0.1547 gr. zirconia.

In 100 parts:

	found		calculated	
Zirconia	35.81	ZrO^2	122	35.47
Selenious acid	64.19	$2SeO^2$	222	64.53
	100.00		344	100.00

Zirconium seems consequently able to form no acid selenite and differs therein from thorium, nor has any analogy with this element been found in the

SELENITES OF TIN.

1. $^3/_5$-Selenite: $Sn^5O^{16}.6SeO + 27H^2O$.

A solution of neutral tetrachloride of tin, on adding selenite of sodium, gave a white voluminous precipitate, which in the microscope showed traces of crystallization.

Analyses:

1) 0.604 gr. salt gave 0.148 gr. selenium or 0.2079 gr. selenious acid.
2) 0.601 gr. salt gave 0.1477 gr. selenium or 0.2075 gr. selenious acid.
3) 0.609 gr. salt after heating gave a residue of 0.2445 gr. dioxide of tin.
4) 0.628 gr. salt gave likewise 0.252 gr. dioxide of tin.
5) 0.3765 gr. salt, when dried at 100°, diminished in the weight by 0.0815 gr.

Calculated on 100 parts:

	experiment					theory		
	1.	2.	3.	4.	5.			
Oxide of tin . . .	—	—	40.15	40.13	—	$5SnO^2$	750	39.43
Selenious acid . .	34.42	34.52	—	—	—	$6SeO^2$	666	35.01
Water	—	—	—	—	21.65	$27H^2O$	486	25.56
							1902	100.00

23 mol. of water, out of the 27 that the salt contains, are carried off at 100° (calc. 21.78 p. ct.).

2. Neutral: $Sn.O^4.2SeO$.

This salt was obtained by digesting the basic salt with 4 mol. selenious acid at about 60° and evaporating the solution, when it formed a microscopically crystalline residue, insoluble in water. It is dissolved in hydrochloric acid, but only when the acid is warm and in considerable excess in which it exhibits a complete analogy with selenite of zirconium. On account of this, the analyses present a very considerable loss of selenious acid. As the salt proved to be waterfree, however, the assumed composition is by no means doubtful.

Analyses:

1) 0.44 gr. salt, when heated, gave 0.176 gr. dioxide of tin.
2) 0.4465 gr. salt gave, after heating, 0.1765 gr. dioxide of tin.
3) 0.5175 gr. salt gave 0.2085 gr. selenium or 0.2929 gr. selenious acid.
4) 0.5905 gr. salt gave 0.244 gr. selenium or 0.3428 gr. selenious acid.

Calculated on 100 parts:

	found				calculated		
	1.	2.	3.	4.			
Oxide of tin . . .	40.00	39.53	—	—	SnO^2	150	40.32
Selenious acid . .	—	—	56.62	58.05	$2SeO^2$	222	59.68
						372	100.00

When the above-mentioned basic salt was digested with respectively 14 and 34 mol. selenious acid, under circumstances, consequently, when diselenites and tetraselenites could have been formed, the residue, after evaporation to dryness and treating with water, presented so viscid a consistency, that it could not be obtained in a fit state for analysis. Subjected to the microscope, its appearance proved to be completely the same as that of the above described neutral selenite, from which it may with tolerable certainty be inferred that dioxide of tin, in complete analogy with zirconia, affords no acid selenite.

Of the quadrivalent elements, the selenites of which have been here examined, zirconium and tin consequently show a correspondence, whereas thorium here, as in other cases, in its acid salts is single in its kind.

The foregoing exposition has shown the incomparably strong tendency of selenious acid to form acid and overacid salts. The mode of formulation, employed for them, points them out as compounds of neutral selenite with anhydride (the waterfree salts), or with hydrate (the salts containing water), in accordance with the most general and simple mode of viewing the salts in question. Another mode would be to set them all down as anhydrous salts, arising from new acids, formed by the union of several molecules of selenious acid under partial loss of water. In this sense, for example, the very remarkable salts of the sexivalent metals that we have become acquainted with in the preceding investigation, would have to be regarded as neutral salts of a six-basic tetraselenious acid: $H^6.O^6.Se^4O^5$, formed by 4 mol. selenious acid, with the loss of 1 mol. water. But such an assumption seems to me little authorized; it requires namely the existence of a particular acid for every new stadium of saturation, and we have seen in the preceding pages that selenious acid affords a great number of acid salts, of which the following 12 may be regarded as the representatives:

1.	$^5/_4$-selenite:	$4Hg.O^2.SeO + H^2.O^2.SeO.$
2.	$^4/_3$- „	$Fe^2.O^6.3SeO + H^2.O^2.SeO.$
3.	$^3/_2$- „	$2Am^2.O^2.SeO + H^2.O^2.SeO.$
4.	$^8/_5$- „	$5Be.O^2.SeO + 3H^2.O^2.SeO.$
5.	$^5/_3$- „	$Cr^2.O^6.3SeO + 2H^2.O^2.SeO.$
6.	$^7/_4$- „	$2(Th.O^4.2SeO) + 3H^2.O^2.SeO.$
7.	$^9/_5$- „	$5Hg^2.O^2.SeO + 4H^2.O^2.SeO.$
8.	di- „	$Mg.O^2.SeO + H^2.O^2.SeO.$
9.	$^7/_2$- „	$3Be.O^2.SeO + 4H^2.O^2.SeO.$
10.	$^5/_2$- „	$Th.O^4.2SeO + 3H^2.O^2.SeO.$
11.	tri- „	$Mg.O^2.SeO + 2H^2.O^2.SeO.$
12.	tetra- „	$Zn.O^2.SeO + 3H^2.O^2.SeO.$

On the other hand, the circumstance that the most part of these acid salts, though formed at a higher temperature, contains water, seems to be opposed to such a view. Certainly, some of them lose their whole amount of water at 100°, but this by no means proves that it is present in the compound as crystallization-water, because selenious acid itself, at this temperature, very easily becomes an anhydride. Particularly worthy of attention is the composition of the acid selenites of the univalent metals; it is namely, as already shown, of such a kind that they, without exception, may be regarded as consisting of $R^2.O^2.SeO + nH^2.O^2.SeO$. No one of them contains

more water than this, which cannot be an accident; besides, their water is only with difficulty and partially carried off at 100°.

That we may be enabled to decide with certainty which view is right, it seems to be necessary to examine more closely the nature of the water present. This question will be most simply settled by examining whether the hydrogen in the selenious hydrate, that is supposed to be present in the acid salts containing water, can be replaced by other metals. To try in this way to ascertain the constitution of the acid selenites, will be the object of a research, to which I intend immediately to devote some time, and in which consequently the double-salts of selenious acid will be examined.

ERRATA.

Page	Line	Error	Correction
1	10	a good deal of other ones	several others
2	16	supply	complete
»	28	soon be	soon to be
4	28	sulphurie	sulphuric
14	24	anhydrons	anhydrous
40	2	$Ba.O^2.SeO + SeO$	$Ba.O^2.SeO + SeO^2$
46	21	evaporated	evaporated,
99	32	$3Al^2.O^6.3eO$	$2[Al^2.O^6.3SeO]$

ESSAI

SUR LES COURANTS SUPÉRIEURS

DE L'ATMOSPHÈRE

DANS LEUR RELATION

AUX LIGNES ISOBAROMÉTRIQUES.

PAR

H.-HILDEBRAND HILDEBRANDSSON.

(Présenté à la Société Royale des Sciences d'Upsal le 28 Novembre 1874.)

UPSAL,
ED. BERLING, IMPRIMEUR DE L'UNIVERSITÉ.
1875.

Quemadmodum in mathematica, ita etiam in physica, investigatio rerum difficilium ea methodo, quæ vocatur analytica, semper antecedere debet eam quæ appellatur synthetica. — — — — — — Hypotheses enim in Philosophia, quæ circa experimenta versatur, pro nihilo sunt habendæ. NEWTON.

INTRODUCTION.

Vingt ans sont à peine écoulés depuis l'époque où la météorologie télégraphique fut organisée à l'Observatoire de Paris par son Directeur illustre, M. LE VERRIER. Ce service, poursuivi depuis ce temps dans les circonstances même les plus graves grâce à la persévérance des savants français, est à présent adopté dans presque tous les pays de l'Europe et dans les États Unis. L'étude journalière des cartes synoptiques a jeté une lumière nouvelle et surprenante sur les mouvements généraux de l'atmosphère et a renversé les anciennes théories ou hypothèses mal fondées. Cependant, vingt ans sont un temps bien court quand il s'agit d'une science, et par conséquent on ne doit pas attendre qu' on ait déjà trouvé la solution complète des problèmes difficiles et des questions compliquées de la météorologie. Une des difficultés les plus graves c'est notre ignorance presque complète de ce qui se passe dans les régions les plus hautes de l'atmosphère. Tout ce qui sert à élargir nos connaissances dans cet égard doit donc être d'une importance assez grande pour la science météorologique. C'est pourquoi nous nous sommes adonné depuis quelque temps aux études sur la relation existant entre les mouvements des courants d'air dans les régions les plus hautes de l'atmosphère et les lignes isobarométriques, et nous espérons que ces études et l'exposé que nous allons en donner ne seront pas sans utilité.

L'examen d'un grand nombre de cartes synoptiques montre que l'air se meut en spirale vers le centre d'un *minimum* barométrique, et que le sens de la rotation autour de celui-ci se fait sur l'hémisphère Nord en sens

inverse des aiguilles d'une montre. Cet examen fait voir, au contraire, que le vent s'éloigne d'un *maximum* en même temps qu'il s'établit une rotation dont le sens est contraire à celui de la précédente. Donc, on peut conclure, que l'air autour du centre d'une tempête a une composante ascendante, que par conséquent il monte et, parvenu à une certaine hauteur, s'éloigne du centre dans toutes les directions. Parvenue aux régions des maxima, cette nappe supérieure descend et alimente d'air les courants inférieurs divergents. C'est ce qui est aussi admis par M. M. Buchan, Mohn, Loomis et la plupart des météorologistes modernes. M. M. Peslin, Reye et Clement Ley sont parvenus au même résultat par leurs travaux importants.

Cependant il y a d'autres savants distingués qui par des considérations théoriques ont envisagé les choses d'une manière toute différente. Ainsi M. Marié Davy lui-même, l'un des fondateurs les plus distingués de la météorologie moderne, conduit par la considération des propriétés générales des mouvements tournants, admet que dans les cyclones l'air se trouve refoulé du centre à la circonférence par l'effet de la rotation, malgré la distribution des pressions décroissantes de la circonférence vers le centre. Mais alors une forte aspiration se produit dans le sens de l'axe et y appelle l'air surtout des régions atmosphériques situées *au-dessus* du disque tournant, vu que l'aspiration par l'extrémité inférieure de l'axe de rotation est gênée par la surface terrestre.

L'astronome savant, M. Faye, cherche aussi à démontrer [1]) l'existence d'un mouvement de haut en bas au moins dans les tourbillons d'un petit diamètre, nommés *trombes*, *tornados*, etc.

Ici comme partout dans les sciences physiques, la voie la plus sure à suivre pour trancher la question, c'est d'observer ce qui se passe actuellement dans la nature. Selon cette règle, il faut donc dans ce cas observer directement avec précision le mouvement des courants d'air dans les régions les plus hautes de l'atmosphère. Heureusement, de telles observations sont souvent possibles grâce à ces précipitations de cristaux de glace qui se montrent sous la forme de nuages qu'on appelle Cirrus, et dont la marche indique la direction du courant d'air dans lequel ils se trouvent. Ce sont, comme on sait, les nuages les plus élevés. Les voyageurs qui ont parcouru les grandes chaînes de montagnes sont unanimes pour déclarer que des plus hauts sommets leur apparence reste à peu près la même. L'aëronaute les voit aussi toujours planer au-dessus de sa tête. Les observations des nuages situés à une grande hauteur dans les régions alizées ont con-

[1]) Comptes Rend. 1873 et 74.

duit, il y a longtemps, à la découverte du contre-alizé ou alizé supérieur. Seulement les observations de ce genre sont bien rares dans nos contrées. Cependant M. CLEMENT LEY a publié [1]) 620 observations faites par lui-même sur les mouvements des Cirrus. En chaque cas il a comparé la direction du vent supérieur à la distribution simultanée de la pression de l'air à la surface terrestre, et il a trouvé *qu'en général les courants supérieurs de l'atmosphère s'éloignent des minima et convergent vers les maxima barométriques.*

J'ai pensé qu'il serait d'importance de pousser ces recherches plus loin. L'organisation d'un réseau d'observateurs, instruits à suivre autant que possible la marche des nuages de l'espèce de Cirrus, doit nous permettre de construire des cartes synoptiques comprenant les vents supérieurs. Pour atteindre ce but, j'écrivai en décembre 1873 à plusieurs observateurs de l'Observatoire d'Upsal dans toutes les parties de la Suède en les priant de vouloir bien commencer de telles observations et d'en faire parvenir les résultats pour chaque mois à l'Observatoire. On trouve à la fin de ce mémoire la liste des stations et des observateurs desquels nous avons obtenu des observations de ce genre. En outre, nous recevons de M. HOFFMEYER, Directeur de l'Institut météorologique de Danemark, des observations semblables faites aux phares de Skagen, de Fanö, de Sprogö et de Vejrö, et aussi de M. RENOU celles de son Observatoire au Parc St. Maur près de Paris.

Il est clair qu'on ne peut prendre de telles observations aux heures déterminées d'avance. Aussi les documents obtenus comprennent-ils les résultats d'observations de toutes les heures de la journée, et faites chaque fois qu'on a pu déterminer la direction du vent supérieur. Toutes ces observations sont divisées en deux parties, selon qu'elles sont prises avant ou après 2 heures du soir. Les observations de la première classe sont pour chaque jour insérées sur la carte synoptique du matin [2]). Les autres ne sont pas employées pour le travail présent faute de documents nécessaires pour la construction des cartes synoptiques du soir de toute l'Europe.

Quant aux cartes du matin j'en ai pendant mon séjour récent à Copenhague complété la plupart d'après les cartes magnifiques de l'Institut météorologique roy. de Danemark, lesquelles mon savant ami, M. le Directeur HOFFMEYER, a bien voulu mettre a ma disposition. Je prends l'occasion de

[1]) The Laws of the Winds prevailing in Western Europe. P. I. London 1872. p. 147.

[2]) Cependant ce n'est qu'aux occasions où les conditions atmosphériques ont présenté une stabilité extraordinaire que j'ai employé des observations faites plus de 4 heures avant ou après 8 h. du matin.

lui rendre mes remercîments les plus sincères pour ce service essentiel et en même temps pour l'intérêt vif qu'il a prouvé pour ce travail. Je saisis aussi avec empressement cette occasion de témoigner ma sincère gratitude à M. RUNG, au même Institut, pour le concours bienveillant qu'il m'a prêté dans la construction des cartes.

N'ayant pu trouver les moyens nécessaires à la publication de toutes les cartes, je n'en ai inséré ici qu'une seule sous la forme originale. D'après la permission bienveillante de M. HOFFMEYER elle est une copie, obtenue par un second tirage, de celle du 28 janvier 1874 dans son Atlas excellent des cartes synoptiques journalières. C'est une carte synoptique complète de l'Europe et dont les notations sont les ordinaires. Les flèches rouges, qui sont ajoutées, indiquent le mouvement des Cirrus au-dessus de chaque station d'où nous avons obtenu des observations du matin pour le jour nommé. Les 32 petites cartes ne présentent qu'une copie des *lignes isobarometriques*, et des flèches indiquant *les directions des vents supérieurs* dans les régions des Cirrus.

DISCUSSION DES CARTES.

Commençons par la grande carte. Le 28 janvier 1874, de fortes pressions se montrent sur le nord-ouest de l'Europe, et le baromètre indique 780 millim. de pression à Valentia (Irlande). De l'autre côté une bourrasque a son centre sur l'intérieur de la Russie. Son influence se fait sentir sur la Suède, la Pologne, l'Hongrie et la Turquie. Les flèches rouges indiquent l'existence d'un courant d'air dans les régions les plus hautes de l'atmosphère. Sur la côte du Golfe de Bothnie la direction générale de son mouvement est du N, sensiblement parallèle à celle des isobares et du vent inférieur. Sur la partie méridionale de la Suède il tourne au NE, et sur le Danemark il est perpendiculaire aux isobares. Par conséquent, dans ces hautes régions l'air s'éloigne du minimum et converge vers le maximum barométrique.

Les cartes I—VIII nous présentent des bords antérieurs des mouvements tournants. Dans de telles occasions le temps est en général pluvieux et le ciel couvert de bas nuages. Par conséquent, il est très difficile d'observer les mouvements des Cirrus dans les instertices souvent bien rares. Grâce à l'intérêt et à la vigilance infatiguable des observateurs nous avons pu réunir un assez grand nombre d'observations même sur ces cartes.

La carte I[ière], du 20 mars 1874, nous présente une dépression barométrique sur tout le nord de l'Europe. Dans cette dépression générale on

peut discerner deux tourbillons distincts. La première de ces bourrasques a son centre sur le Cattegat près de Gothembourg. Elle étend son action sur les parties méridionales de la Suède et de la Norvége et sur le Danemark. L'autre se trouve sur les environs de St. Pétersbourg. Les flèches, indiquant les mouvements des Cirrus, font voir que dans le voisinage du centre l'air, même dans ces hautes régions, est animé du mouvement giratoire du vent habituel dans ces phénomènes. En effet, les courants supérieurs sont sensiblement parallèles aux isobares. Sur les deux cartes suivantes, du 18 et du 19 janvier, le centre de dépression se trouve, à une plus grande distance de nos contrées, sur la mer entre l'Ecosse et la Norvége. Les flèches indiquent que les vents supérieurs ont une tendence de s'éloigner du centre. Cette tendence est le plus prononcée sur les stations les plus éloignées du centre. A Paris le 18, le mouvement des Cirrus est perpendiculaire aux lignes isobarométriques. Le 5 avril et le 31 mai le centre se trouve au NO sur la Mer de Glace. La déviation des vents supérieurs est déjà très grande. Le courant supérieur dévie constamment vers la droite de la direction du vent inférieur. Sur les trois dernières, VI—VIII, enfin, représentant l'état du ciel le 14 août, le 28 mai et le 8 juillet, les hautes pressions s'étendent déjà sur la Baltique. Les flèches sortent directement du centre en coupant les isobares à angles droits.

L'inspection des 8 cartes suivantes, IX—XVI, nous conduira au même résultat. Ces cartes représentent des cas où la Scandinavie est sous l'influence des bords postérieurs des bourrasques. Le 30 avril, un tel météore a son centre près d'Upsal. L'air à la surface terrestre est animé du mouvement giratoire habituel. Le ciel est couvert; de la pluie ou de la neige à la plupart des stations suédoises. Cependant, on a pu observer le mouvement des Cirrus à trois stations. Comme on voit, les flèches accusent que, dans le voisinage du centre, il existe même dans ces hautes régions un mouvement giratoire inverse qui est à peu près parallèle aux lignes isobarométriques. Il en est de même le 19 avril, parce que le centre est alors aussi peu éloigné de la Suède et situé sur la Finlande. Le beau temps qu'il faisait alors permit d'observer les Cirrus à un plus grand nombre de stations. Le jour d'après le centre est plus éloigné. Des vents d'entre O et SO dominent en Suède, tandis que les flèches indiquent que dans les hautes régions il existe des courants d'entre N et NO, qui vont en s'éloignant du minimum barométrique. De même à la carte suivante du 27 janvier, nous voyons les vents supérieurs couper les isobares de la même manière. A la surface terrestre une tempête très-forte souffle du NO. Sur les trois cartes des 25, 26 et 6 mars, le centre de dépression est encore

plus éloigné. tandis qu'une zone de fortes pressions règne sur l'O et le SO de la Scandinavie. Les vents supérieurs dévient fortement des minima et semblent attirés par les maxima. Enfin la carte du 10 mai nous fait voir un centre de bourrasque au SE de la Baltique. Bien que celui-ci se trouve peu éloigné de nos contrées, les vents supérieurs dans la Suède dévient fortement. En effet ils s'inclinent partout directement du centre.

Nous allons examiner quelques cas où l'air sur nos contrées a été influencé par les hautes pressions. Le plus souvent le ciel est alors clair, et par conséquent l'observation des Cirrus est très-facile. Aux cartes des 5, 11, 12 février et du 13 mars les maxima se trouvent à l'O ou au S; à ceux des 13 et 14 avril et à ceux du 1er et du 3 mars, au contraire, à l'E. Dans tous ces cas les courants dans les régions les plus élevées de l'atmosphère s'avancent ou convergent vers les maxima dans une direction presque opposée à celle du vent inférieur.

Il nous reste à examiner un cas spécial. Dans son ouvrage ci-dessus cité, M. CLEMENT LEY cherche à démontrer que l'axe des tempêtes tournantes est incliné en arrière. Cette conclusion nous semble un peu étonnante. Le contraire serait plus vraisemblable. Car nous pourrions admettre que la friction pourrait ralentir la vitesse de translation de la partie inférieure du disque tournant. Aussi pas une seule des cartes que nous avons pu construire affirme-t-elle l'opinion de M. LEY. Cependant cette opinion nous conduit à discuter une condition atmosphérique fort digne d'attention. Il arrive souvent qu'une tempête tournante est suivie de près d'une autre. Alors le vent tourne ordinairement du N successivement au NO — O — SO, comme nous l'avons démontré dans un autre Mémoire [1]). Quant aux vents supérieurs j'ai observé depuis plusieurs années que les Cirrus continuent à marcher du N pendant plusieurs heures après que le SO a commencé de souffler à la surface terrestre. Plus tard, le vent supérieur passe aussi à l'O et a l'OSO. A de telles occasions, on voit souvent les Cirrus rangés en bandes parallèles dont la direction est celle de la tangente aux isobares, (Voyez plus bas.) A cause de la perspective, ces bandes semblent converger vers deux points opposés de l'horizon. Au dessus de l'horizon de l'O s'élève un voile blanc de Cirro-Stratus, qui s'approche en prenant successivement une teinte grisâtre en même temps que les courants supérieurs tournent à l'O. Les huit cartes dernières servent à illustrer ce fait. Ces cartes des 21, 27, 28 mars, 21 avril, 8 juin, 16 mars, 25 juin et 29 août

[1]) Voyez: *Études sur quelques tempêtes.* Actes d. l. Soc. Roy. des Sciences et Belles-Lettres à Gothembourg. 1872.

nous font voir plusieurs occasions dans lesquelles deux bourrasques se succèdent après un court intervalle. Nous voyons par les directions des flèches qu'en général les courants supérieurs semblent appartenir plutôt au tourbillon qui va en s'éloignant. Le 28 mars, par exemple, il régnait jusqu'à midi un vent du N dans les régions les plus hautes de l'atmosphère, quoique le baromètre eût baissé de nouveau depuis 10 h. 30 m. du soir précédent et que le vent à 8 h. du matin soufflât du SE accompagné de pluie ou de neige sur la plupart des stations de la Suède.

Il nous semble bien probable que c'est ce phénomène assez fréquent qui a eu de l'influence sur les moyennes de M. LEY, et dont il a cherché l'explication en admettant que l'axe du tourment est incliné en arrière.

Cependant l'inspection des cartes nous donne immédiatement l'explication vraie. En effet, le courant du N dans la région des Cirrus ne doit être que la résultante du courant du NE ou de l'E, qui sort en arrière du tourment précédent, et du courant du NO ou de l'O, qui provient de celui qui s'approche.

Conclusions: Il suffit de jeter les yeux sur les cartes journalières, dont nous avons discuté quelques-unes, pour reconnaître que tous les vents supérieurs sont en liaison immédiate avec les lignes isobarométriques de la manière suivante:

1° Tout près du centre d'une dépression ou minimum barométrique les courants supérieurs se meuvent à peu près dans une direction parallèle aux isobares et aux vents inférieurs.

2° A mesure qu'on s'éloigne du centre, ils sont pliés en dehors et déviés à droite des vents inférieurs.

3° Sur les régions des maxima ils convergent vers leur centre en coupant les isobares à peu près à angles droits.

COMPARAISON DES VENTS SUPÉRIEURS AUX VENTS A LA SURFACE TERRESTRE.

Aux cartes synoptiques pour chaque matin des 8 mois de janvier — août j'ai inséré en tout 888 observations des mouvements des Cirrus. En chaque cas, j'ai comparé la direction du vent supérieur à celle de l'inférieur. Pour chacune des quatre conditions atmosphériques principales, savoir les maxima, les bords antérieurs des bourrasques, les bords postérieurs de celles-ci et enfin la crête entre deux minima, j'ai calculé le nombre des cas suivants:

1° Le courant supérieur est dévié à droite ou 2° à gauche de la direction du vent. 3° Leurs directions sont opposées, c'est à dire, la déviation dépasse 90°. 4° Les deux directions sont parallèles, et 5° un calme parfait règne à la surface terrestre.

Les résultats sont compris dans le tableau suivant:

Déviations	Maxima.	Minima		Entre deux minima.
		Bord antérieur.	Bord postérieur.	
Dév. à droite	102	**161**	**101**	**31**
Dév. à gauche	22	20	24	21
Vents opposés.....	**167**	27	13	18
Vents parallèles	36	35	36	8
Calme.....	37	9	10	10

Ces nombres prouvent clairement que les courants supérieurs de l'atmosphère s'éloignent des minima et convergent vers les maxima barométriques.

Pour étudier de plus près comment se passe ce mouvement à des distances différentes du centre d'un minimum ou d'un maximum, j'ai choisi soixante-dix cartes, représentant des conditions atmosphériques très régulières, dans lesquelles l'air sur nos contrées a été soumis à l'action d'un seul minimum ou d'un seul maximum. Sur ces cartes j'ai considéré cinq zones différentes comprenant des régions

1° d'une pression barométrique au-dessous de 745 millim., 2° comprises entre les isobares 745 et 755, 3° entre 755 et 760, 4° entre 760 et 765, et 5° au-dessus de 765 millim.

Pour chaque zone j'ai calculé le nombre dans les six cas suivants.

1° Les deux vents, le supérieur et l'inférieur, sont parallèles.

2° Le vent supérieur est dévié à droite d'un angle $< 45°$.

3° „ „ „ „ „ à gauche „ „ „

4° Les deux directions se coupent sensiblement à angles droits, c'est à dire $= 90° \pm 45°$

5° Ces directions sont opposées, ou l'angle $> 135°$, et

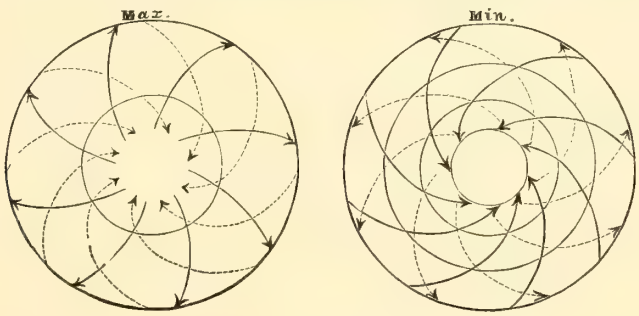
6° Un calme parfait règne à la surface terrestre.

Le nombre total d'observations qui se trouvent sur ces soixante-dix cartes est de 364. L'on trouve les résultats de cette discussion compris dans le tableau suivant.

Barom. millim.	Vents parall.	Le vent supérieur est dévié		Angle droit	Vents opposés	Calme	Somme
		à droite	à gauche				
au-dessous de 745....	**19**	16	2	6[1])	1	0	44
745—755..	17	**43**	3	4	1	3	71
755—760..	6	22	2	**23**	10	5	68
760—765..	3	20	1	**24**	23	5	76
au-dessus de 765....	2	19	1	26	**45**	12	105

Un coup d'oeil sur ce tableau suffit pour retrouver les mêmes lois auxquelles l'étude des cartes nous a conduit. En effet, tout près du centre d'un minimum, à une pression barométrique au-dessous de 745 millim. les courants supérieurs sont, dans le plus grand nombre de cas, parfaitement parallèles aux vents inférieurs. Entre les isobares 745 et 755 une déviation à droite devient prédominante dans les vents supérieurs. Entre 755 et 760, cette déviation devient plus grande; en effet, nous en avons 22 observations d'une déviation $< 45^0$, et 23 d'une déviation plus grande où les deux directions sont sensiblement à angles droits. Dans la zone suivante entre 760 et 765 millim. la déviation dépasse en général 90^0. Nous avons 24 angles droits et 23 oppositions. Enfin au-dessus de 765 millim. les oppositions ont une prépondérance absolue.

Les deux figures ci-jointes représentent d'une manière schématique le mouvement de l'air sur des contrées des maxima et des minima barométriques. Les courbes tirées en traits pleins font voir les vents à la surface de la terre, et les courbes pointillées, les courants supérieurs de l'atmosphère.



[1]) Cinq de ces cas appartiennent à la carte singulière du 10 mai (XVI) discutée plus haut.

SUR LES BANDES PARALLÈLES DE CIRRUS.

On voit souvent les nuages de l'espèce des Cirrus s'étendre en bandes parallèles sur la voûte céleste. A cause de la perspective ces bandes semblent converger vers deux points opposés de l'horizon. On les appelle en Allemagne *bandes polaires (Polarbande)*, en Angleterre tantôt *le saumon*, *(Salmon)* tantôt *l'arche de Noé.* Sous cette dernière dénomination ces formations sont aussi bien connues en Suède. On les a regardées comme signes précurseurs de mauvais temps, et sans doute elles méritent d'être étudiées à cet égard Pour étudier leur relation aux isobares j'ai inséré sur les cartes journalières des 6 mois de janvier—juin 1874 toutes les observations pendant ce temps, en tout 127. De plus j'ai employé de la même manière 44 observations faites à Upsal pendant les dernières années. Voici les résultats.

Déviation	Maxima	Minima		Entre deux minima	Somme
		Bord postérieur	Bord antérieur		
Angle > 45°	58	8	5	3	74
Angle < 45°	12	29	18	38	97
Somme	70	37	23	41	171

Par conséquent, les bandes de Cirrus sont dans les régions des maxima le plus souvent orientées dans une direction à peu près perpendiculaive aux isobares [1]), dans celles des minima, au contraire, elles sont sensiblement parallèles aux lignes isobarométriques.

CONCLUSIONS ET REMARQUES GÉNÉRALES.

Par ce qui précède nous croyons avoir démontré que *l'air s'éloigne des centres des minima et converge vers les centres des maxima dans les régions*

[1]) Pendant l'impression de ce mémoire nous avons reçu les observations sur ce phénomène qui viennent d'être publiées par M. le Directeur PRESTEL à Emden (Zeitschr. d. Oesterr. Ges. f. Meteorol. 1874 N:o 24.) Quant à la direction de ces bandes dans les régions des maxima ce savant distingué est arrivé au même résultat que nous. Nous ne regrettons qu'une chose, c'est qu'il n'ait pas observé en quel sens se meuvent ces nuages. Alors, il aurait sans doute trouvé qu'ils marchent en général *vers le centre de la plus haute pression,* et non pas dans le sens opposé, comme il le suppose.

les plus hautes de l'atmosphère. Comme on sait, c'est l'inverse qui a lieu près de la surface terrestre. *Par conséquent, un minimum doit nécessairement être le siége d'un courant d'air ascendant. Arrivé à une grande hauteur dans l'atmosphère, cet air s'éloigne partout du centre de la dépression et se déverse en nappe uniforme au-dessus des régions des maxima, où il s'abaisse graduellement vers la terre en courants descendants.* De cette manière il s'effectue sans cesse *une circulation verticale entre la surface terrestre et les limites supérieures de l'atmosphère.* Le principal agent de cette circulation doit bien être la différence de température et d'humidité à la surface terrestre, qui est plus ou moins échauffée, et dans les régions les plus hautes de l'atmosphère, où il règne une séchéresse et un froid excessifs — différence de température qui surmonte de beaucoup celle de l'équateur et des pôles. En effet chaque carte synoptique nous montre toujours des régions maxima et des minima qui doivent être regardées comme des siéges de courants verticaux, tandis que la circulation horizontale, considérée longtemps comme la seule ou du moins la principale, est, pour nous servir d'une expression mathématique, une quantité d'un ordre supérieur, dont l'existence n'est pas encore prouvée, d'une manière incontestable, par les cartes.

Quant aux courants verticaux, l'existence d'un courant dirigé de bas en haut sur les régions des minima et de haut en bas sur celles des maxima, devient, de plus, très probable par plusieurs faits. Nous n'en citerons que les deux suivants. On sait, depuis les recherches de RAMOND [1]), que la pression d'air diminue avec la hauteur plus rapidement pendant un beau temps que pendant un mauvais. Or, le beau temps accompagne les maxima, et le mauvais temps les minima, ce qui s'accorde bien à notre hypothèse. D'un autre côté, on voit souvent, surtout en hiver, un froid excessif s'établir au milieu d'un maximum barométrique. Ce froid n'y est guère transporté des latitudes élevées par un "courant polaire", *puisque la température est plus élevée de tous côtés.* (Voyez par exemple les cartes pour plusieurs jours au commencement de l'année 1874.) Par conséquent il faut admettre que l'air descendant des couches les plus élévées de l'atmosphère amène avec lui ce froid. D'après notre opinion, ces deux points sont dignes d'une étude plus approfondie, et nous espérons d'y revenir dans l'avenir.

Les observations des mouvements des Cirrus organisées chez nous sont, dans doute, insuffisantes à elles seules, si l'on veut étudier d'une manière complète les vents supérieurs et découvrir les lois qui règnent dans les hautes régions. Un seul pays est une partie de la surface terrestre

[1]) Mém. de l'inst. de France. 1808. 1. Sem.

trop restreinte pour qu'on puisse y aborder avec succès les vastes problèmes concernant les mouvements généraux de l'atmosphère. Mais à présent il suffit d'avoir indiqué les recherches les plus importantes à faire sur ce riche et fertile sujet. Nous espérons que le génie actif des amis de la science, surtout celui des Directeurs des établissements météorologiques dans toute l'Europe, poursuivra ces recherches, et nous sommes bien convaincu que la solution de chaque problème nouvel fera naître de nouvelles idées et ouvrira de nouvelles voies.

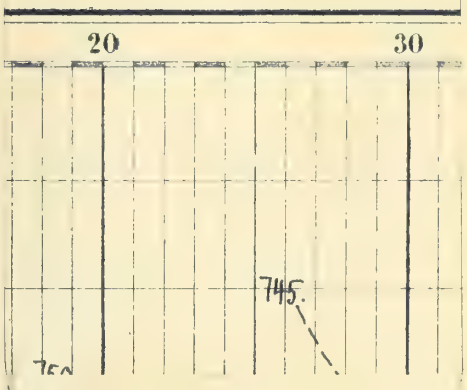
Nous nous permettons d'indiquer une contrée d'où des observations des courants supérieurs seront d'après notre opinion de la plus haute importance au point de vue de la théorie. On sait que, des parages de Portugal et de Maroc vers les Antilles, il s'étend une région d'une haute pression. Dans *l'Atlas des Mouvements Généraux de l'Atmosphère* de l'Observatoire de Paris, on trouve presque sans interruption sur cette partie de l'Atlantique un maximum d'une forme allongée. Tout autour de ce maximum, le vent circule en sens direct comme à l'ordinaire autour des maxima. En effet, au Sud nous avons l'alizé du NE, vers les Antilles il souffle de l'Est, aux côtes de l'Afrique il devient NO, et au Nord, dans les environs des Azôres il règne un vent du SO. Quant aux vents supérieurs ils sont assez bien connus de trois côtés. Au-dessus de l'alizé du NE, il règne un vent opposé du SO, le contre-alizé, dont nous avons parlé plus haut. Aux Antilles et dans l'Amérique centrale, on a aussi plusieurs fois constaté, dans les hautes régions, un vent opposé, c'est à dire un vent d'Ouest. Pour n'en citer qu'un exemple, le 1er mai 1812 des cendres tombaient aux Barbades en si grande abondance que les arbres pliaient sous leurs poids. Ces cendres provenaient d'une forte éruption du volcan de Morne-Garou situé dans l'île de Saint-Vincent, à une trentaine de lieues dans l'Ouest des Barbades. Or dans ce temps l'alizé de l'E était dans toute sa force, et il a fallu que les cendres fussent lancées par la violence de l'éruption jusque dans la couche des vents supérieurs au travers les alizés inférieurs. De même, à la côte de l'Afrique, entre le Cap Blanc et le Cap Bojador, le ciel est le plus souvent obscurci par une poussière très fine provenant sans doute de l'E de l'intérieur du continent, et par conséquent dans une direction presque opposée à celle du vent inférieur. Cependant, du côté nord de la région dont nous parlons nous n'avons pu trouver des observations des vents supérieurs. On admet, en général, qu'à une certaine distance le courant supérieur, s'abaissant graduellement vers la terre, finit par la rencontrer, et qu'au delà de ces régions, le courant supérieur abaissé à la surface continue sa route, en constituant le vent du SO dont nous venons de parler, et qui serait donc

un vrai "courant équatorial". Les résultats établis dans ce mémoire sont de nature à jeter des doutes sur cette hypothèse. Ils nous font croire, au contraire, *que des observations de la marche des Cirrus démontreront l'existence, dans ces parages, d'un courant supérieur du N ou du NO dirigé vers le centre du maximum*, comme on a trouvé des courants supérieurs de l'O au-dessus des Antilles, du SO sur le pic de Ténériffe, et de l'E dans les parages de l'Afrique.

APPENDICE:

Les mouvements des Cirrus sont observés aux stations suivantes:

Stations.	*Latitude.*	*Diff. du temps de Paris.*	*Observateurs.*
Qvickjock	66° 58′	1h. 1m.	M. M. P. Berlin
Umeå	63 50	1 12	„ E. Waldenström
Hernösand	62 38	1 2	„ S. Hjeltström
Sibo	61 14	0 57	„ A. Blombergsson
Björn (Phare)	60 40	1 2	„ C. M. Alm
Thorsång	60 28	0 53	„ R. Zätterberg
Skinnskatteberg	59 50	0 54	„ C. A. Elin
Hellefors	59 47	0 49	„ A. Giöbel
Nora	59 31	0 51	„ C. de Löwenhielm „ P. G. Malmberg
Kärne	59 28	0 44	Mll. Anne Wijkander
Tumba	59 12	1 3	„ Alma Samzelius
Svartå	59 8	0 49	M. A. Jenzén
Ede	59 4	0 54	„ M. V. von Post
Skeppsholmen	57 50	0 45	„ H. Spaak
Gothem (île Gottland)	57 35	1 6	„ L. Engbom
Löpanäs	57 2	0 51	„ B. N. Lindvall
Kalmar	56 40	0 56	„ G. L. Ideström
Halmstad	56 40	0 42	„ O. V. Brodin
Huntly	56 20	0 42	„ L. C. Dahl
Tomarp	56 6	0 48	„ C. Möllerberg.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745.

28 Janvier 1874. Le matin.

Légende explicative.

○ serein	• pluie
◔ peu nuageux	* neige
◑ demi serein	ʌ grêle
◕ nuageux	≈ brouillard
● couvert	↯ orage
⊙ l'état du temps inconnu.	

Les flèches marchent avec le vent, et le nombre de barbes en désigne la force dans un scale de 1-6.
⊙ signifie calme
La pression athmosphérique, ramenée à 0° R. au niveau de la mer, est exprimée en millimètres

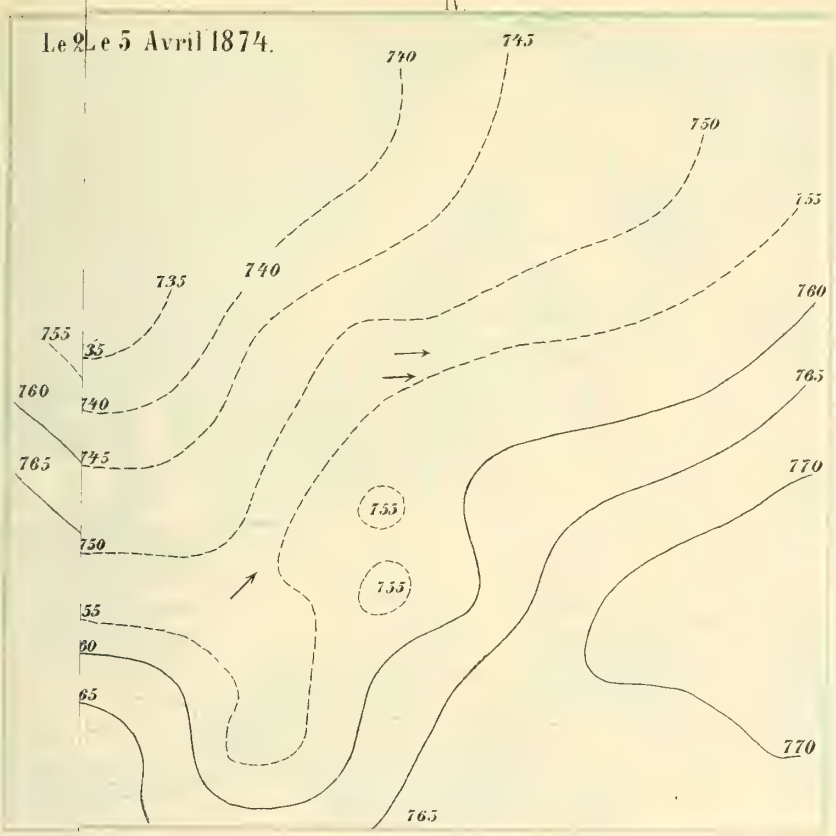
50 40 30 20 10 0 10 20 30 40 50

70 60 50 40

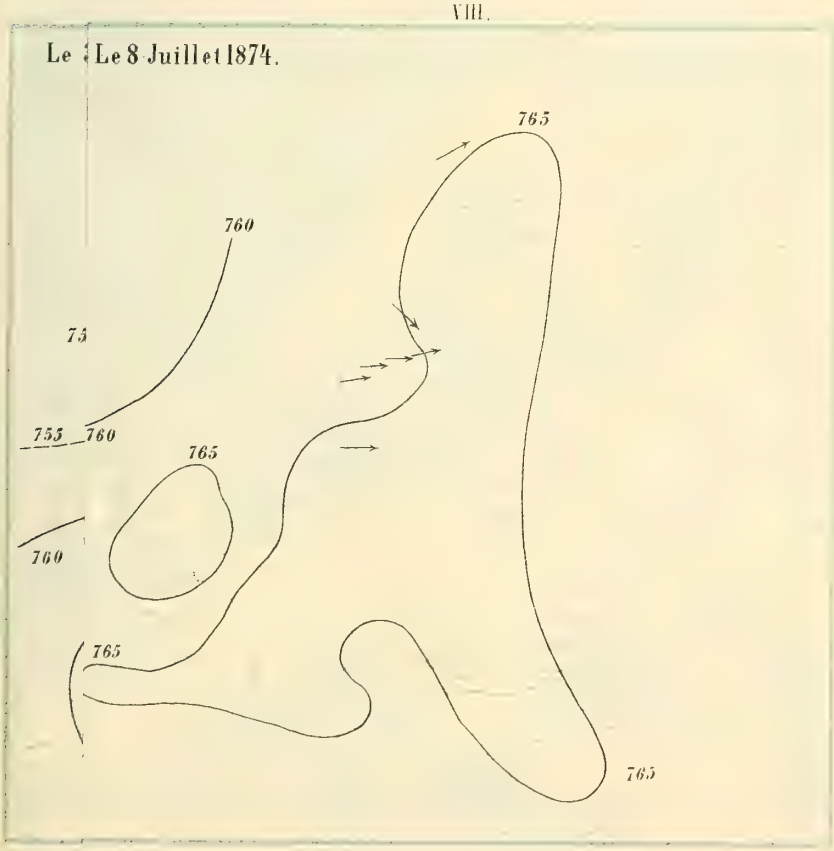
745 750 755 760 765 770 775 780 735 740

(Les flèches indiquent les mouvements des Cirrus)

IV.



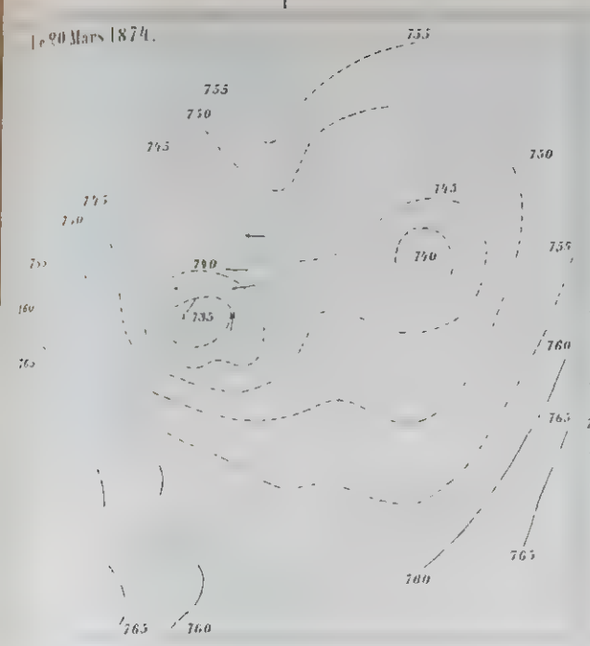
VIII.



CARTES DES LIGNES ISOBAROMÉTRIQUES ET DES VENTS SUPERIEURS

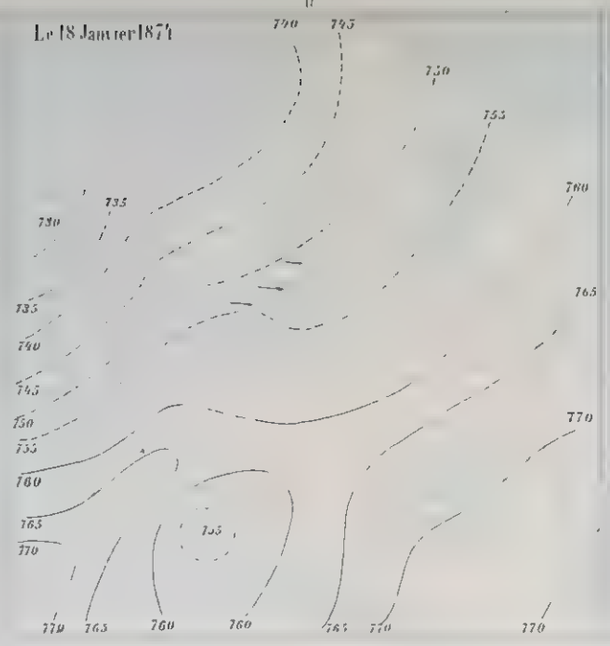
I

Le 20 Mars 1874.



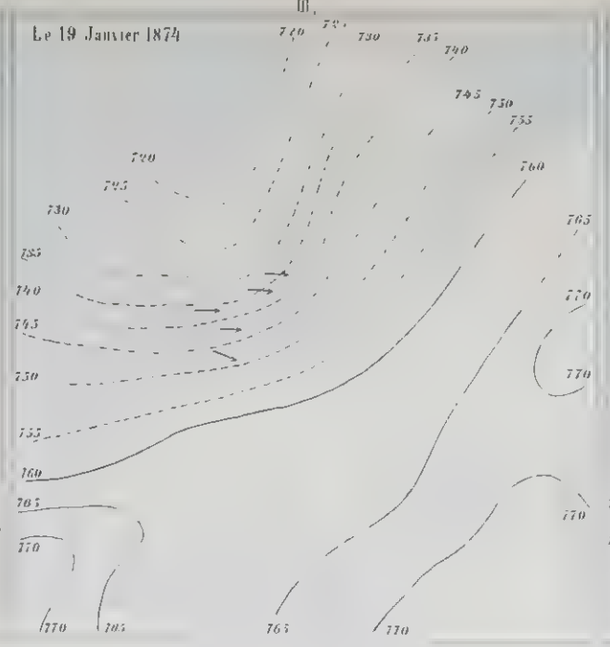
II

Le 18 Janvier 1874



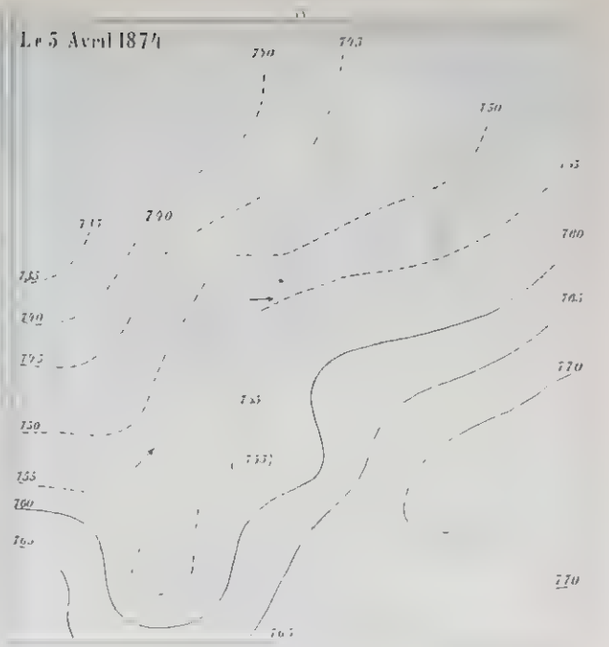
III

Le 19 Janvier 1874



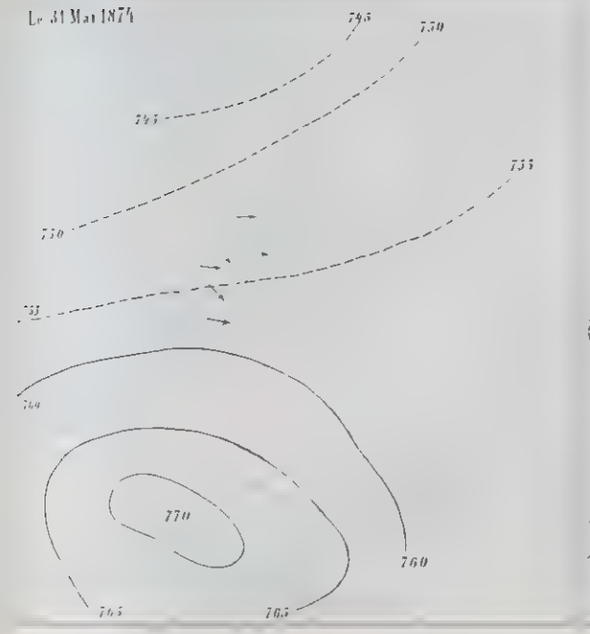
IV

Le 5 Avril 1874



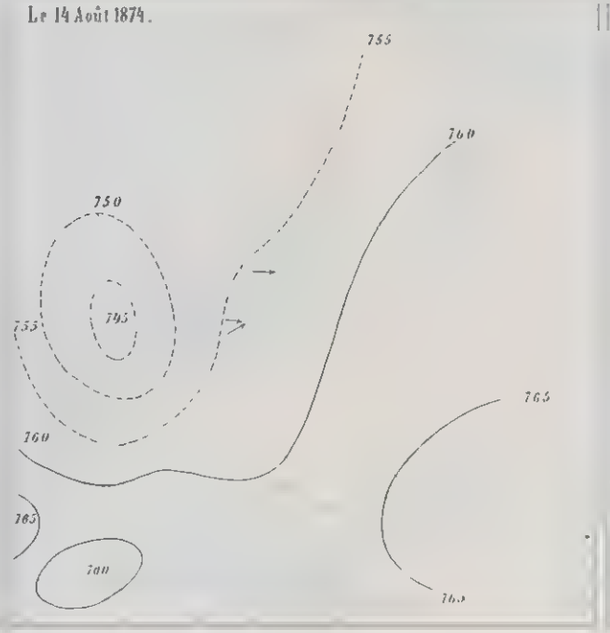
V

Le 31 Mai 1874



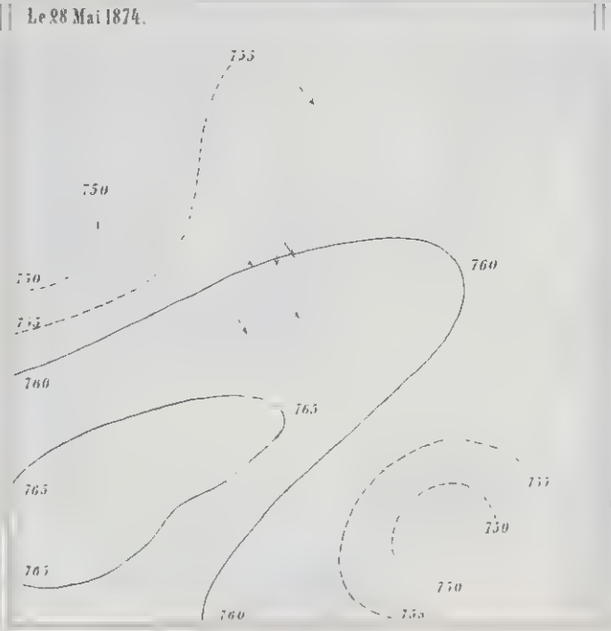
VI

Le 14 Août 1874.



VII

Le 28 Mai 1874.



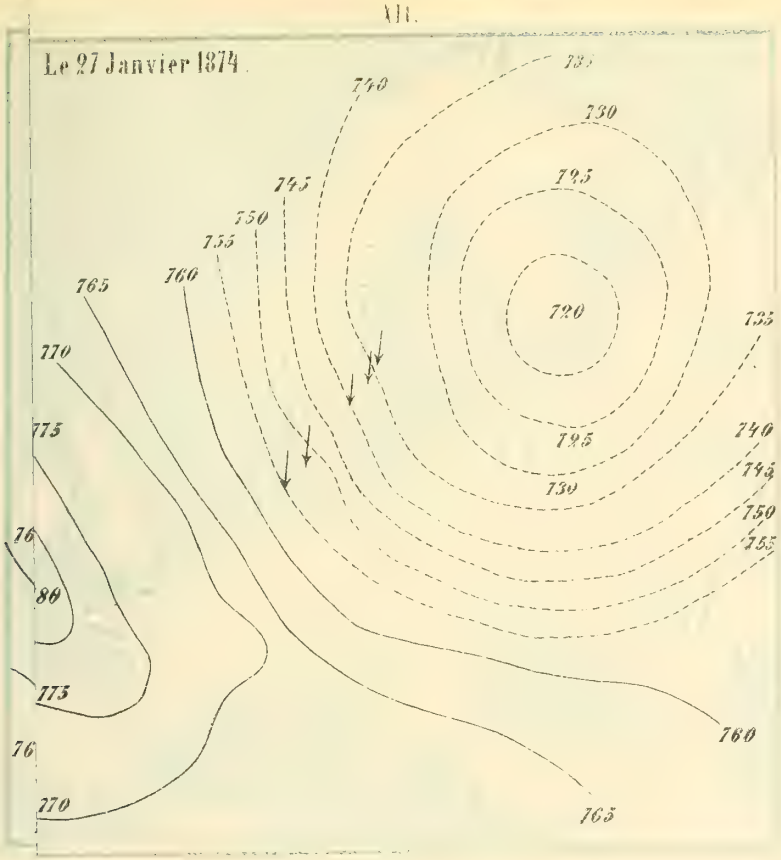
VIII

Le 8 Juillet 1874.

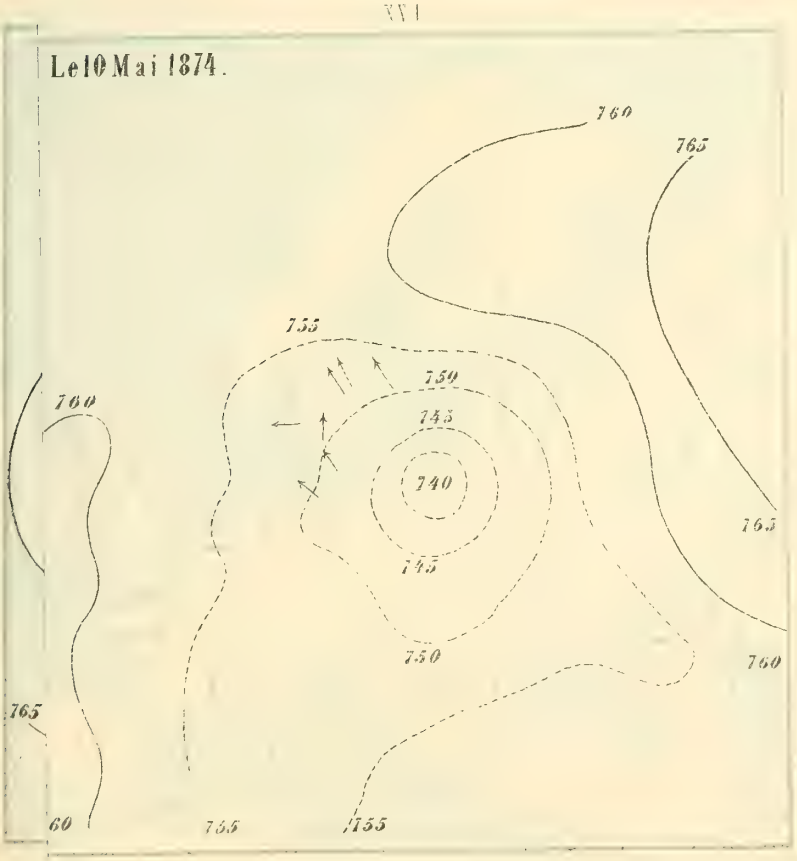
H. debrandsson [illegible]

(Les flèches indiquent [illegible]

XII.



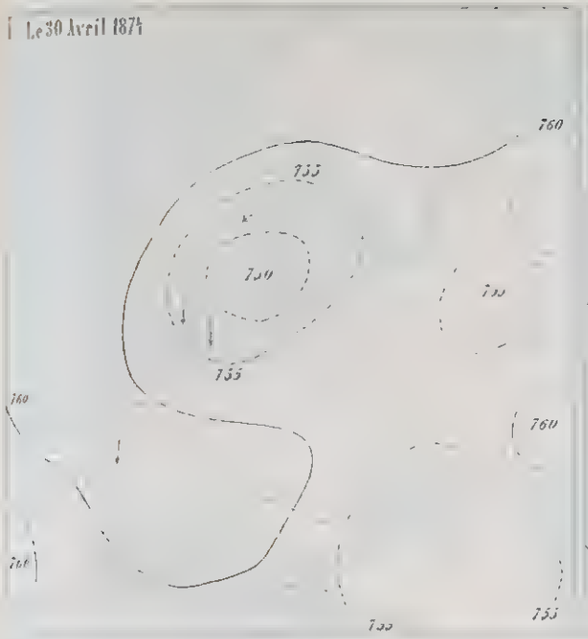
XVI



CARTES DES LIGNES ISOBAROMÉTRIQUES ET DES VENTS SUPÉRIEURS.

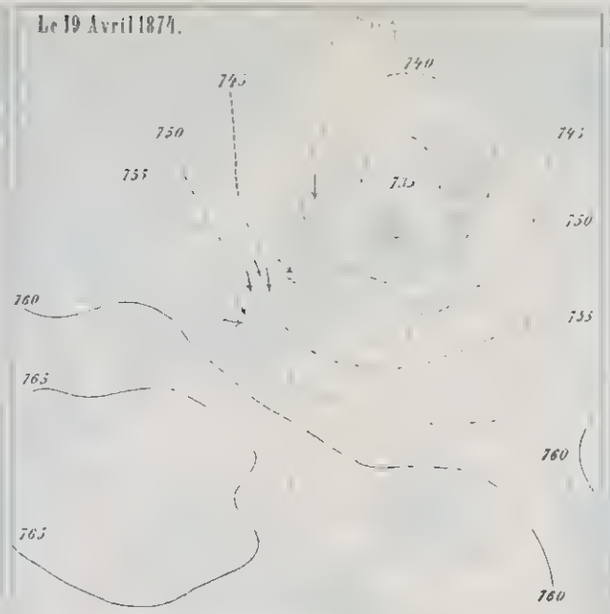
IX.

Le 30 Avril 1874



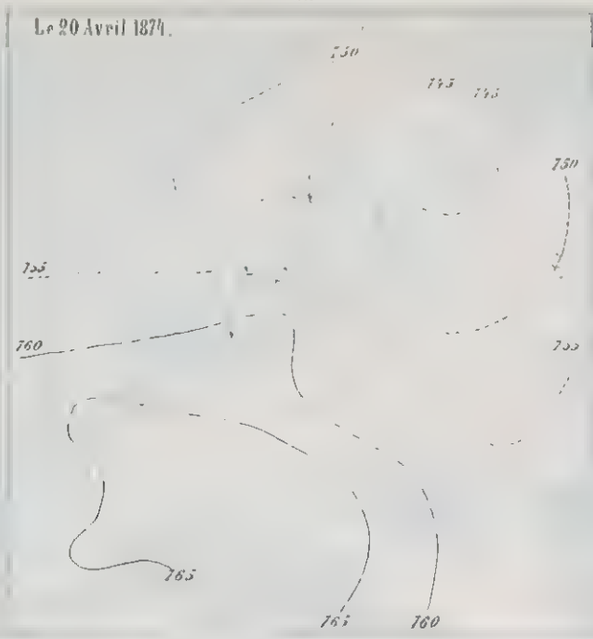
X.

Le 19 Avril 1874.



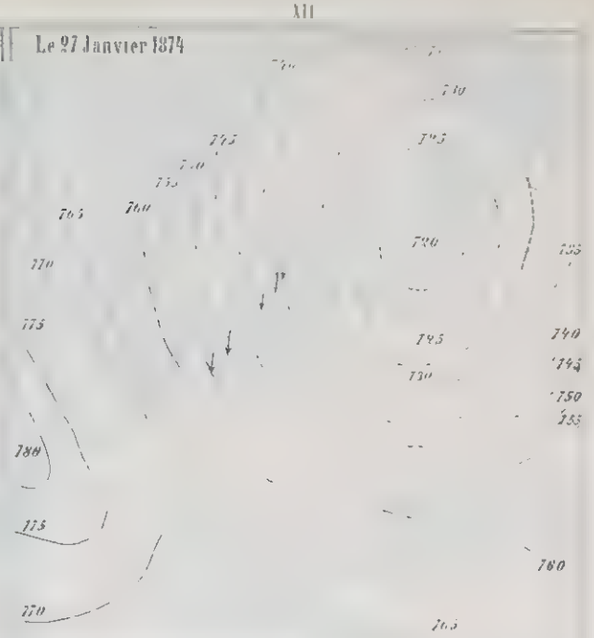
XI.

Le 20 Avril 1874.



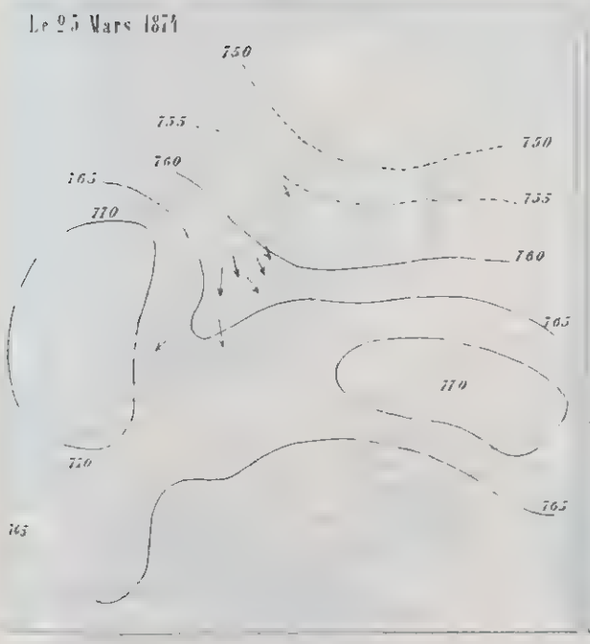
XII

Le 27 Janvier 1874



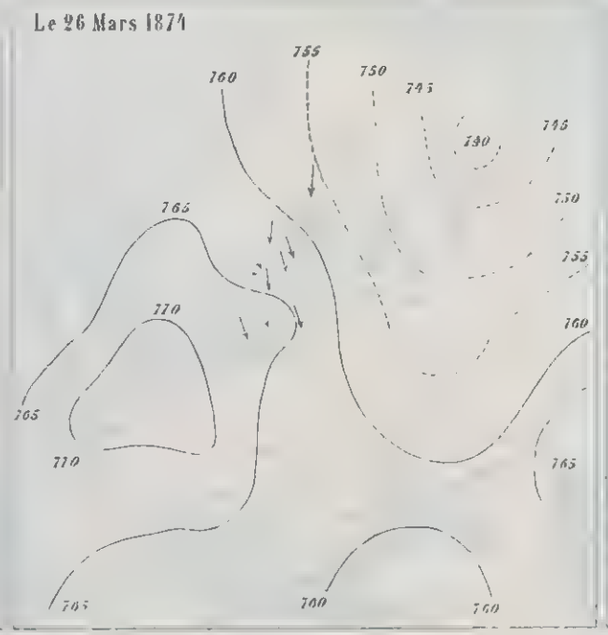
VII

Le 25 Mars 1874



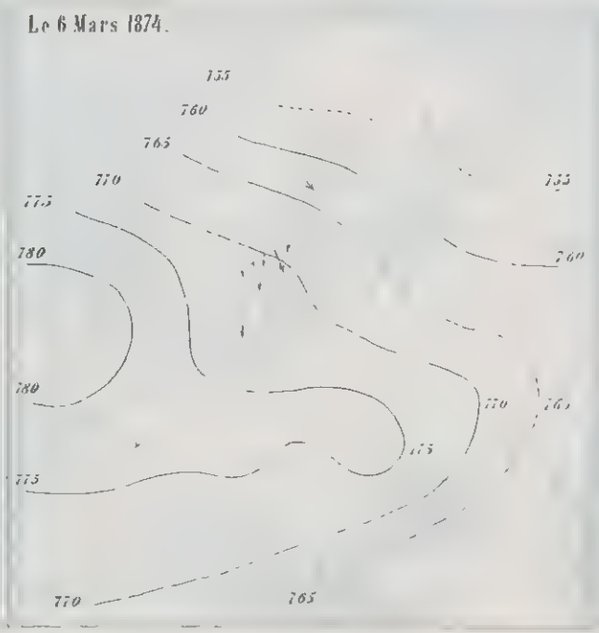
VIII

Le 26 Mars 1874

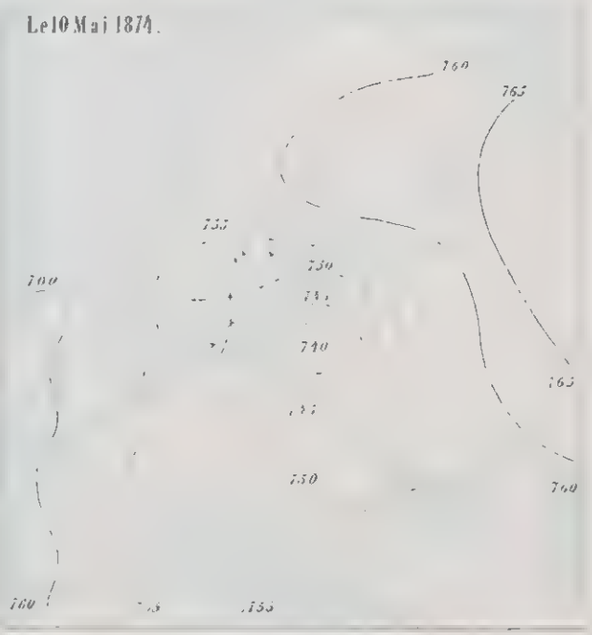


XI

Le 6 Mars 1874.

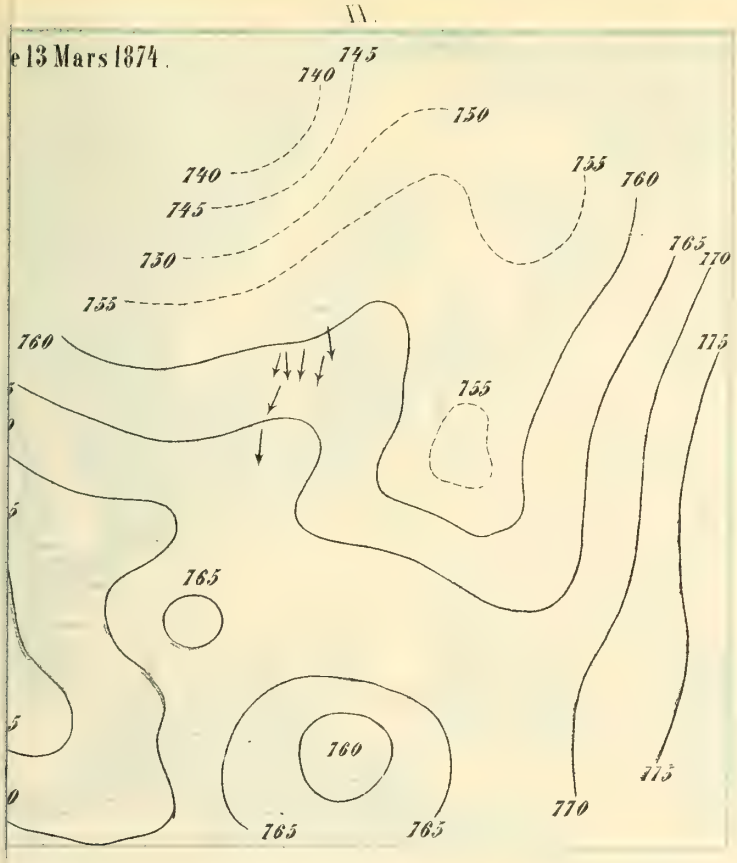


Le 10 Mai 1874.

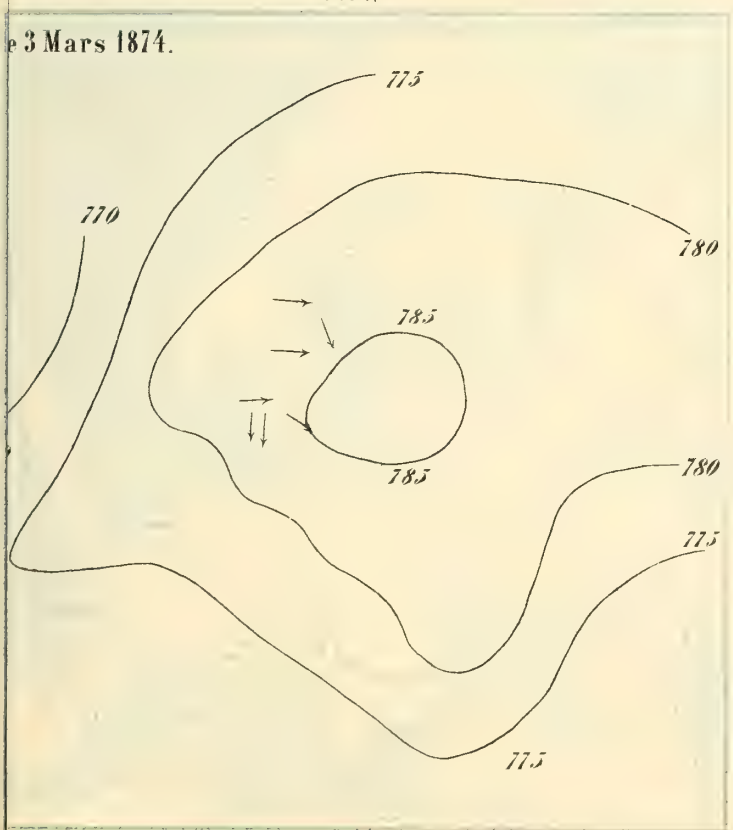


(Les f.ˢ les indiquent les mouvements des Cirrus

XX.



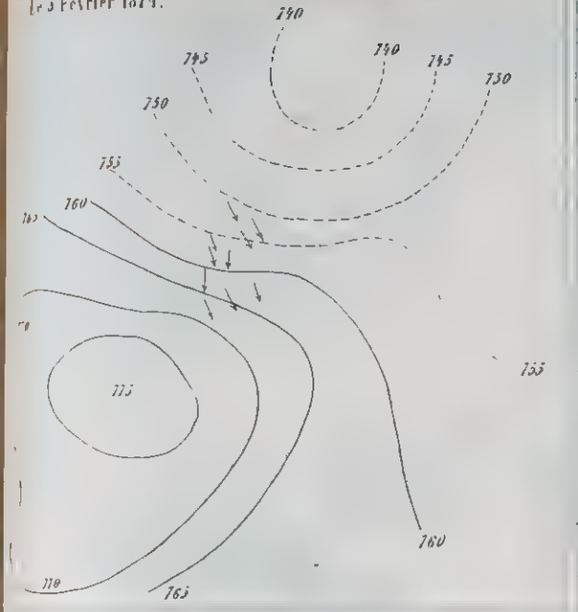
XXIX.



CARTES DES LIGNES ISOBAROMÉTRIQUES ET DES VENTS SUPÉRIEURS

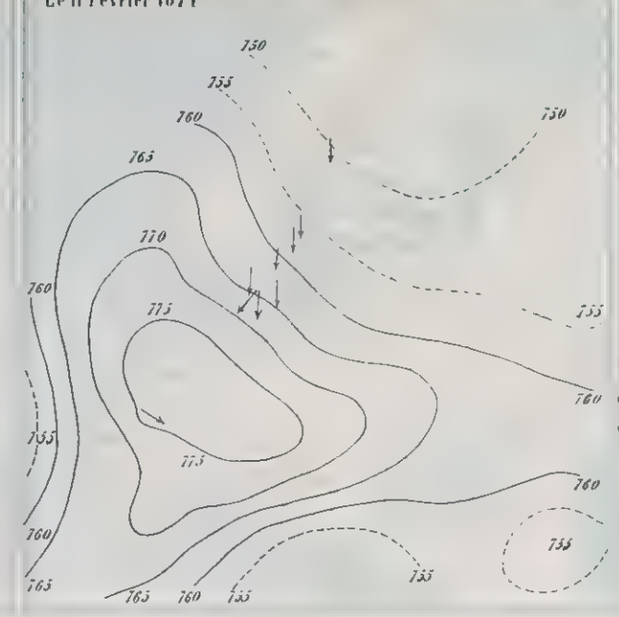
XVII

Le 5 Fevrier 1874.



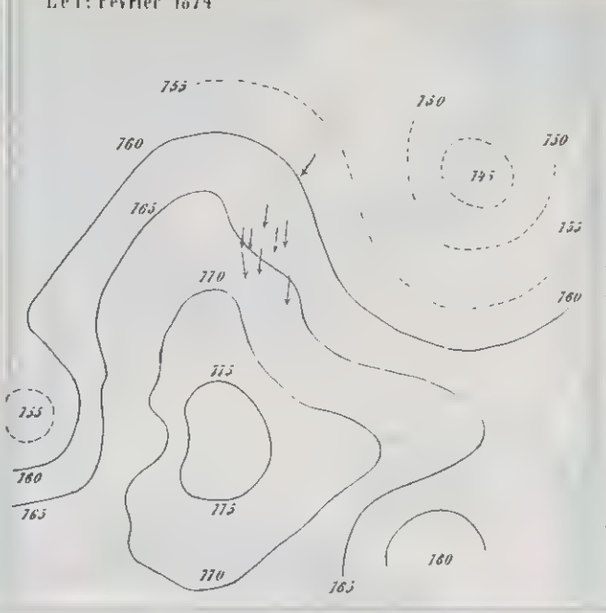
XVIII

Le 11 Fevrier 1874



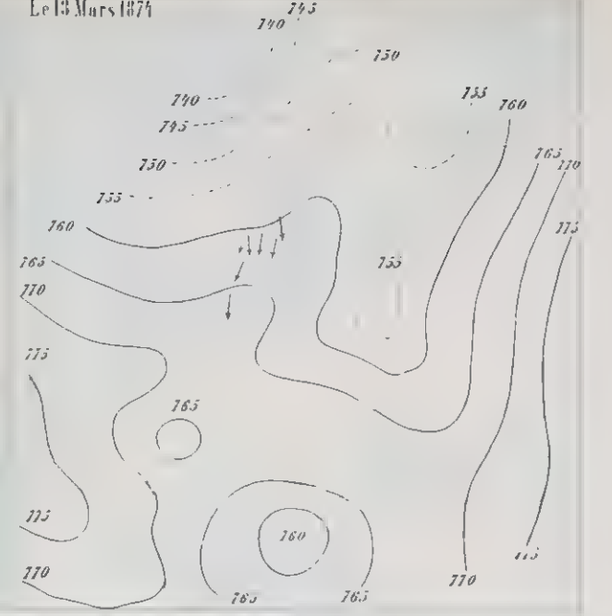
XIX

Le 1er Fevrier 1874



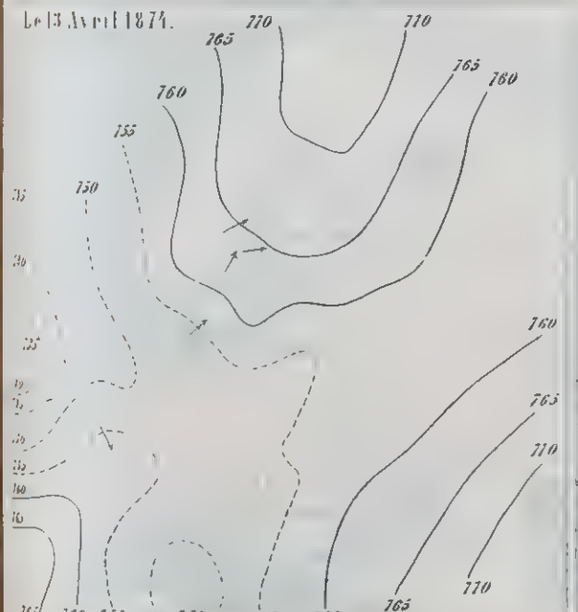
XX

Le 13 Mars 1874



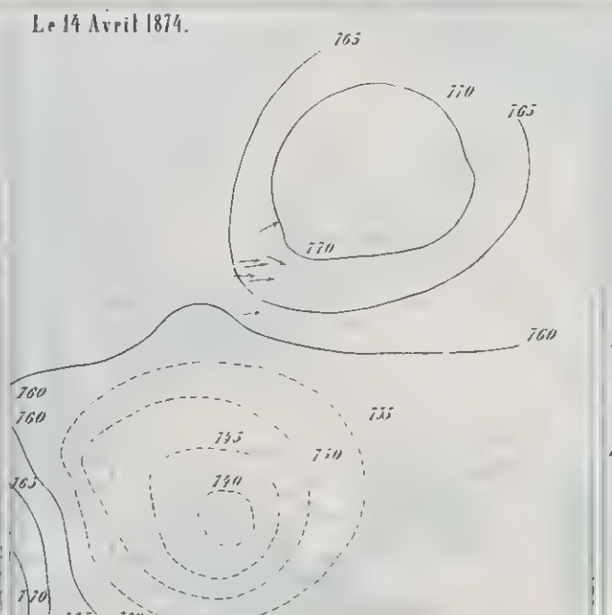
XXI

Le 13 Avril 1874.



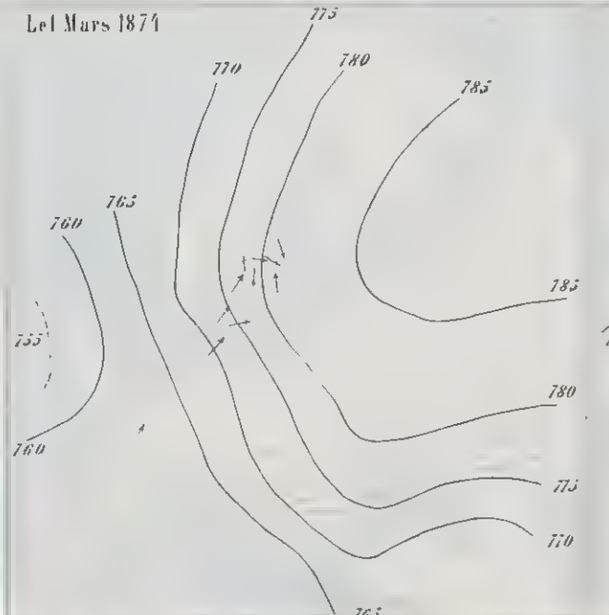
XXII

Le 14 Avril 1874.



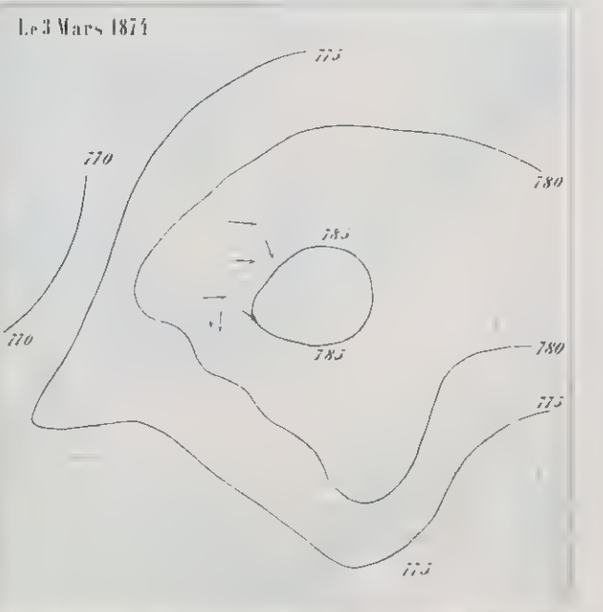
XXIII

Le 1 Mars 1874



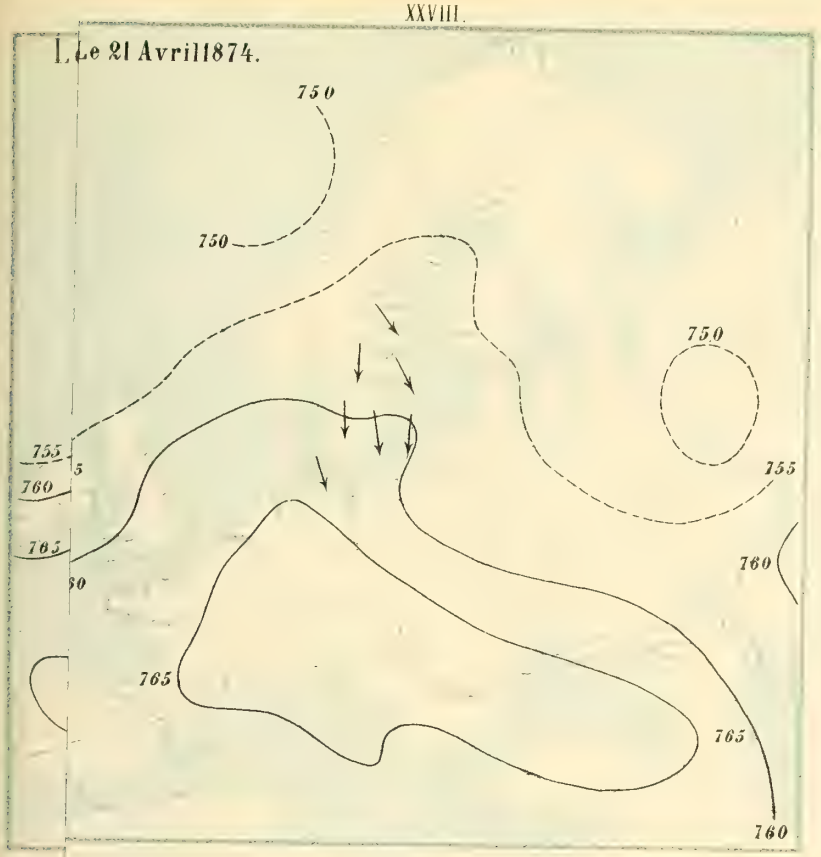
XXIV

Le 3 Mars 1874

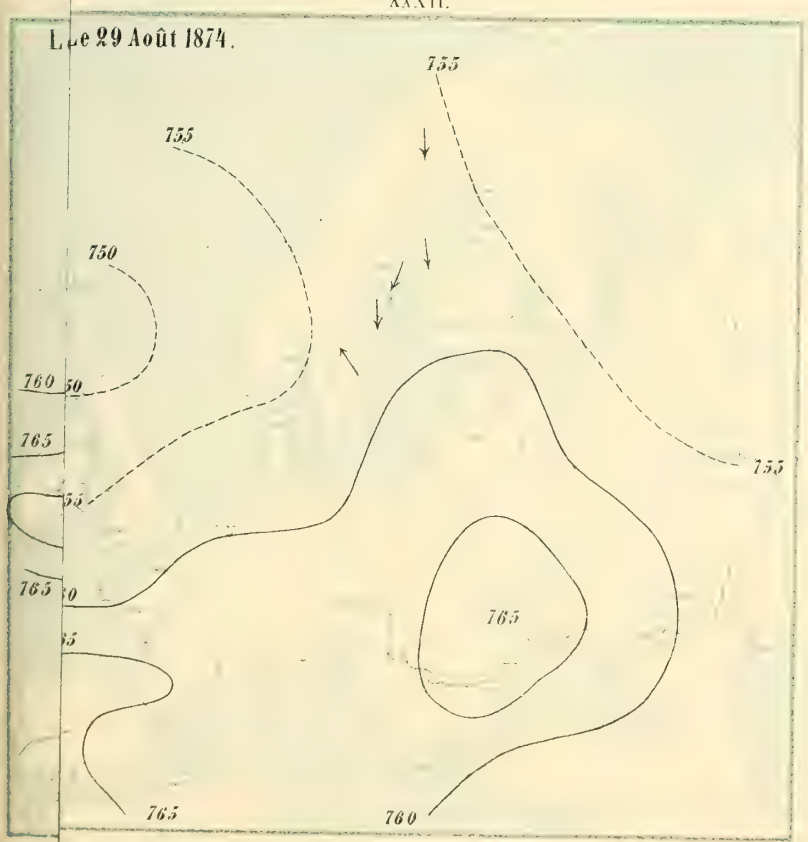


(Les flèches indiquent les mouvements des Cirrus)

XXVIII.



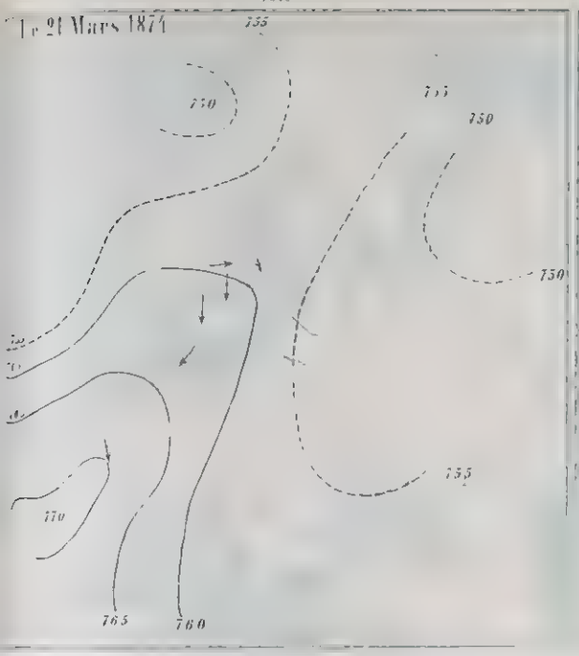
XXXII.



CARTES DES LIGNES ISOBAROMÉTRIQUES ET DES VENTS SUPÉRIEURS.

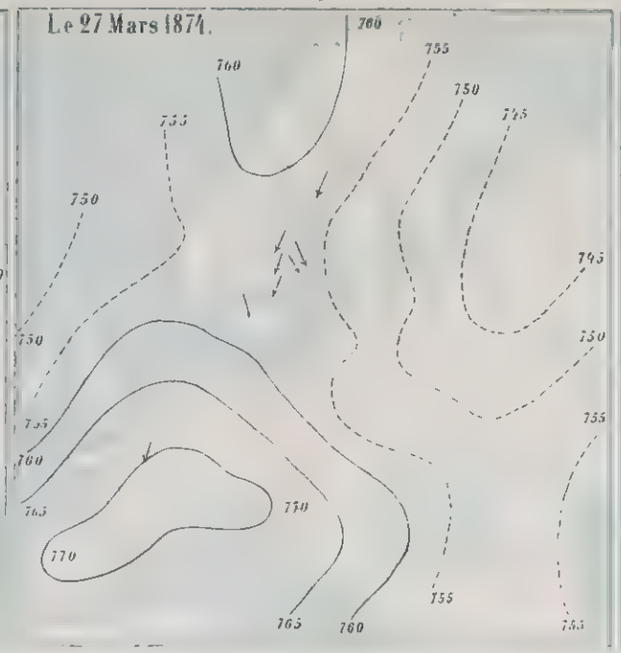
XXV.

Le 21 Mars 1874



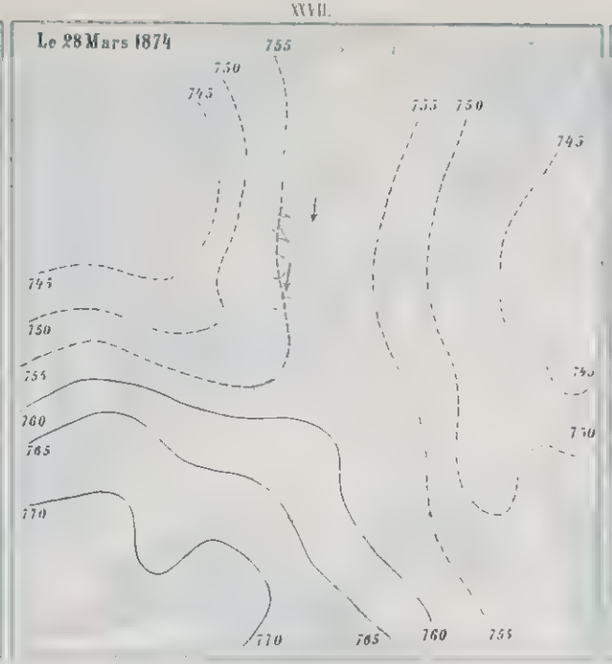
XXVI

Le 27 Mars 1874.



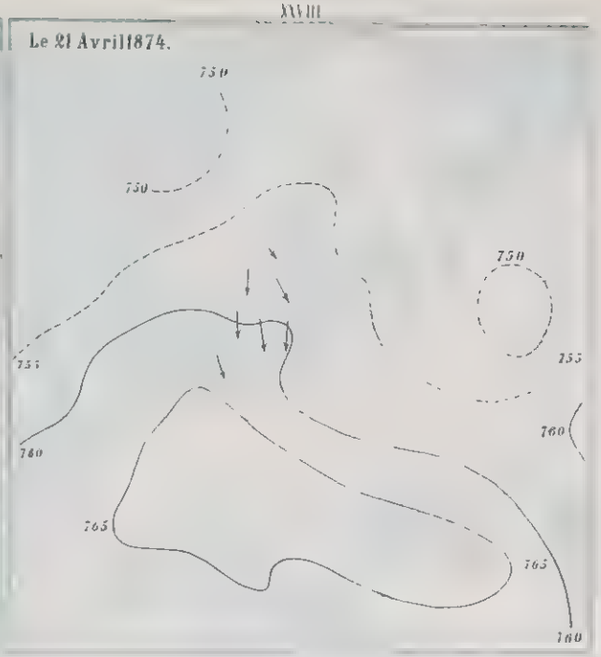
XXVII.

Le 28 Mars 1874



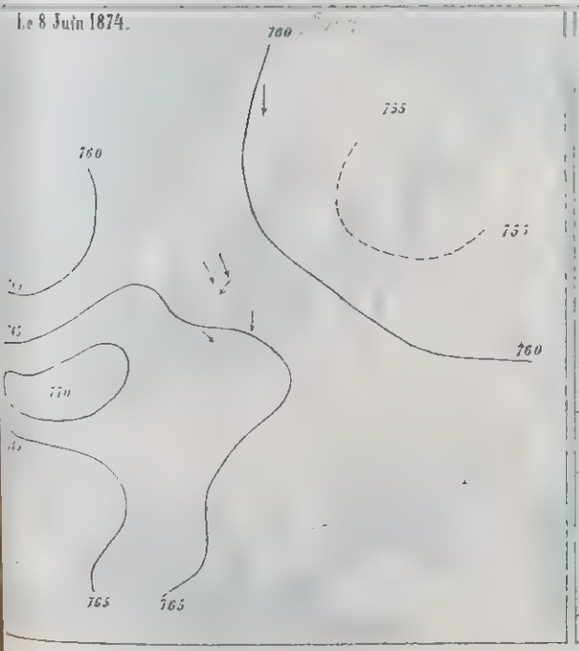
XXVIII

Le 21 Avril 1874.



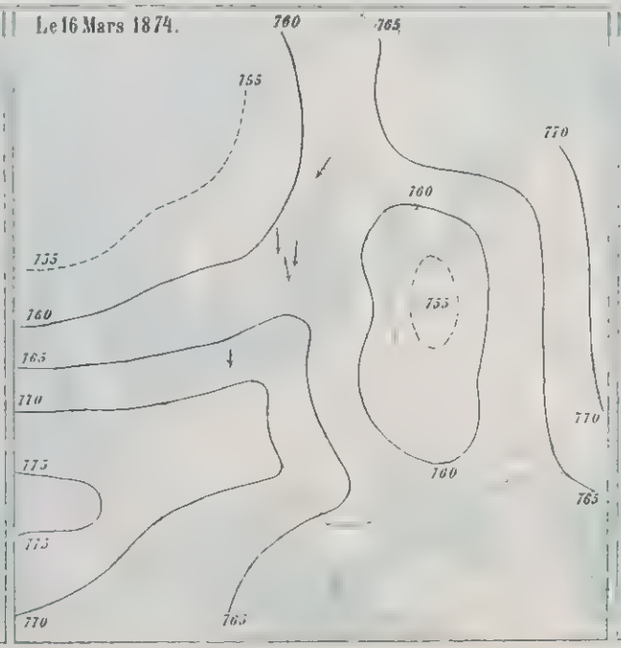
XXIV

Le 8 Juin 1874.



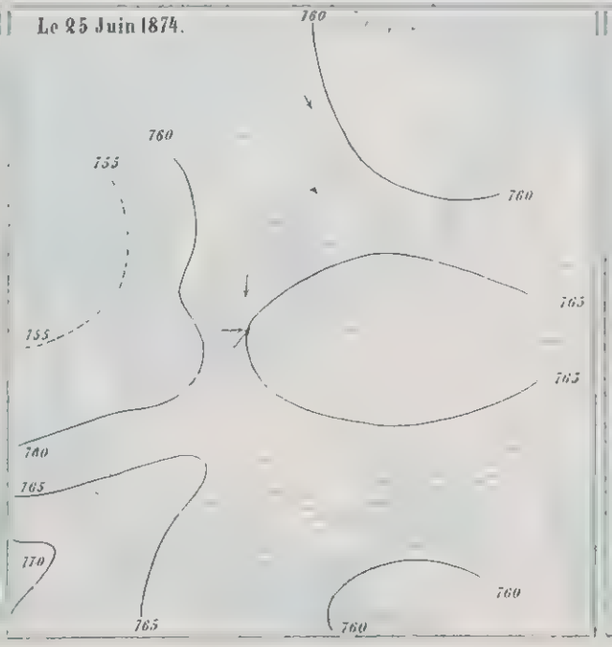
XXX

Le 16 Mars 1874.



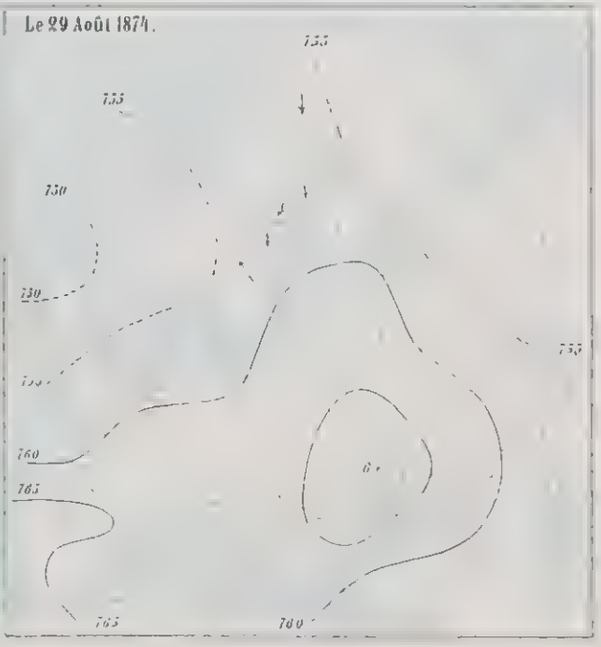
XXI

Le 25 Juin 1874.



XXII

Le 29 Août 1874.



RECHERCHES

SUR LES

SPECTRES DES MÉTALLOÏDES,

PAR

A.-J. ÅNGSTRÖM ET T.-R. THALÉN.

AVEC DEUX PLANCHES.

UPSAL,
ED. BERLING, IMPRIMEUR DE L'UNIVERSITÉ.
1875.

AVANT-PROPOS.

Lors de la mort imprévue du professeur ÅNGSTRÖM le 21 juin en 1874, ce travail, qui nous est commun et qui avait été annoncé à la *Société Roy. des Sciences d'Upsal* déjà le 5 juin 1869, était si loin avancé que la 1ière feuille, aussi que les deux planches, était imprimée. Dès le commencement, une telle convention avait été établie entre nous que nous ferions les expériences en commun, que moi, je prendrais les mesures nécessaires, tandis que lui répondrait du texte. Le texte du 2ième et du 3ième paragraphe je l'avais cependant rédigé, il y avait déjà plusieurs années, d'après des renseignements, partie écrits, partie oraux, fournis par ÅNGSTRÖM, et je l'avais soumis à son examen, mais en conséquence de la foule des occupations qui l'accablaient pendant les dernières années de sa vie, la vraie critique ne vint à s'étendre qu'à la 1ière feuille, nommée ci-dessus, qui fut imprimée en décembre 1873. Quant au reste, il avait donné quelques indications par-ci par-là, il est vrai, de la manière dont il désirait que le texte fût tourné, mais il ne lui fut jamais permis de faire une révision complète.

Si je publie néanmoins ce reste, vu qu'il faut que le travail commencé soit en quelque manière mené à fin, je le fais en déclarant expressément qu'ÅNGSTRÖM n'a pas approuvé tout ce qui va suivre, mais qu'il aurait voulu en plusieurs endroits soit changer une chose ou ajouter une autre. Si l'on est d'avis que j'aie dû compléter ainsi le travail, je n'ai besoin que de faire observer que, faute de toutes notes de nos observations antérieures, il m'aurait fallu répéter successivement toutes les expériences faites, ce qui aurait retardé trop longtemps la publication du Mémoire, et en outre il aurait probablement amené une telle refonte de l'ouvrage que je n'aurais plus osé le publier au nom d'ÅNGSTRÖM. Mais, comme j'ai cru que ce travail, renfermant en effet les dernières recherches scientifiques dont le célèbre physicien s'est occupé, pourra avoir quelque intérêt même dans sa forme actuelle, et quoique une partie en soit déjà connue par les notes d'ÅNGSTRÖM,

inserées dans les *Comptes rendus de l'Académie des Sciences* de Paris, T. 73, 1871, p. 368, et dans les *Annales de Poggendorff*, Jubelband, 1874, p. 424, je n'ai pas hésité à publier le reste, mais en le rédigeant j'ai procédé d'une manière si légère que possible pour ne pas changer inutilement ce qu'a dit ÅNGSTRÖM, parce que j'ai cru ainsi le mieux honorer la mémoire de feu mon ami.

Quant aux derniers paragraphes, depuis le 4ième jusqu'à la fin, de notre Mémoire, c'est après la mort d'ÅNGSTRÖM que je les ai écrits pour compléter en quelque sorte le travail commencé. Si le temps et d'autres circonstances le permettent, je poursuivrai peut-être une autre fois ces recherches sur les spectres des métalloïdes.

Upsal, le 20 août 1875.

ROB. THALÉN.

Il y a plus de quatre ans que nous avons terminé les recherches sur les spectres des métalloïdes dont nous donnons ici les résultats, mais le désir de les rendre encore plus complètes nous en a fait retarder la publication jusqu'aujourd'hui, et, bien que dans notre intention première, ces recherches dussent porter sur les combinaisons chimiques de la plupart des métalloïdes, nous sommes forcés de nous restreindre à présent à quelques-unes de celles du carbone et de l'azote.

Quant à notre méthode d'observation, il suffit de dire que dans nos expériences c'est toujours par l'électricité que nous avons produit l'incandescence des corps et que les spectres observés sont dus, soit à l'étincelle jaillissant entre des électrodes placés dans différents gaz, à la pression atmosphérique, soit à la lumière des gaz raréfiés dans de tubes de Geisler. Cette dernière méthode, dans tous les cas où il est possible de l'appliquer, doit être préférée pour la détermination des longueurs d'onde des raies lumineuses; mais s'il s'agit d'étudier la composition et la nature du corps dont on observe les spectres, la première méthode est la plus sûre.

Comme source d'électricité, nous nous sommes servis de la machine de M. Holtz, de la bobine de Ruhmkorff (grande dimension), ou d'une pile de Bunsen de cinquante éléments, ou bien de courants induits d'une faible tension, mais de grande quantité d'électricité, comme l'extra-courant de l'électro-aimant de Faraday.

Disons enfin que le spectroscope dont nous avons fait usage dans nos recherches, étant de grand modèle, a permis d'employer jusqu' à six prismes et que nous nous sommes servis d'un miroir tournant pour déterminer la durée des phénomènes lumineux.

§ 1. *Spectres des métaux alcalins et alcalino-terreux.*

C'est à M. WHEATSTONE qu'on doit cette découverte remarquable que les métaux, employés comme électrodes et volatilisés par l'action de l'électricité, se distinguent parfaitement les uns des autres, par la réfrangibilité des raies qu'on observe dans les spectres lumineux ainsi formés [1]). MASSON, en faisant jaillir, entre des boules métalliques, l'étincelle électrique d'une bouteille de Leyde, chargée à l'aide de la machine ordinaire, observa quatre raies communes à tous les métaux [2]), dont l'une avait déjà été découverte par FRAUNHOFER, et l'on crut d'abord que ces raies faisaient exception à la loi de M. WHEATSTONE. Mais l'un de nous fit bientôt disparaître cette soi-disant contradiction, en prouvant expérimentalement que ces raies communes proviennent exclusivement du gaz que traverse l'étincelle, de sorte que les raies du spectre ordinaire appartiendraient à deux classes; les unes, dépendant des métaux qui servent d'électrodes, les autres du gaz intermédiaire [3]). De même, il a été démontré, que les raies métalliques ne changent jamais de nature, lorsqu'on fait jaillir, dans de gaz différents, l'étincelle électrique d'une bouteille de Leyde, et en outre que les gaz composés, ainsi que les alliages (même ceux dans lesquels les métaux constitutifs se trouvent combinés chimiquement, les uns avec les autres), produisent, par l'action électrique, les spectres des corps élémentaires. Enfin, en substituant la bobine de Ruhmkorff à la machine ordinaire, M. WILLIGEN a fait une découverte, dont l'importance pour l'analyse spectrale est évidente, à savoir que les dissolutions de sels métalliques, par exemple celles de chlorures, dont on a humecté les électrodes, donnent naissance au spectre du métal, contenu dans la solution saline [4]).

Il résulte donc de ces recherches que *tout corps simple ou composé, volatilisé et rendu incandescent par l'action de l'électricité, présente un spectre discontinu, composé des raies des corps élémentaires;* de plus que *la position de ces raies dépend exclusivement des propriétés chimiques des éléments, et nullement de la température, ni de la manière d'introduire les corps dans l'étincelle*

[1]) Phil. Mag. 1835 Vol. VII, p. 299.
[2]) Annales d. Chim. et Phys., (3) T. 31, p. 295.
[3]) K. Vet. Akad:s Handl., Stockholm 1853, p. 335.
[4]) Pogg. Annalen B. 106, p. 610; B. 107, p. 473.

électrique. Cependant, bien que tout eût été ainsi préparé pour l'application chimique des observations spectrales, ce ne fut qu'après les belles découvertes des nouveaux métaux, le cæsium et le rubidium, par M. M. BUNSEN et KIRCHHOFF, que l'analyse spectrale a été réellement mise en pratique.

L'opinion que l'électricité seule nous suffit pour étudier les phénomènes d'incandescence des corps gazeux n'a pas été généralement admise. En effet, M. KIRCHHOFF estimait que pour atteindre le but proposé on devait employer un procédé beaucoup plus simple que le courant électrique, et qu'on le trouverait dans la combustion de la flamme [1]). Aussi, M. M. KIRCHHOFF et BUNSEN firent ils des experiences nombreuses sur les spectres de différents corps composés, introduits dans la flamme, et M. KIRCHHOFF prétendit avoir ainsi démontré le premier, d'une manière satisfaisante, la thèse énoncée ci-dessus, thèse qui, selon lui, n'aurait été auparavant qu'une opinion très-vague. Les résultats de leurs recherches ont été exposées dans deux mémoires insérés dans les Annales de Poggendorff [2]), et représentés séparement sur un grand tableau, publié en 1860 et donnant les spectres des métaux alcalins et alcalino-terreux.

En examinant ce tableau, que reproduisent maintenant tous les traités de physique, on remarque assurément que les spectres de sodium et de lithium, qui s'y trouvent représentés, sont entièrement conformes à ceux qu'on observe d'ordinaire à l'aide de la bobine de Ruhmkorff, mais qu'au contraire les spectres donnés comme appartenant au calcium, au strontium et au baryum diffèrent sensiblement de ceux qu'on observe pour ces corps, lorsqu'on emploie la bobine. La raison en est que ces spectres contiennent principalement les raies ou les bandes des oxydes et des chlorures, mais très peu de celles du métal lui-même.

En faisant ces remarques nous n'avons nullement l'intention de dire que la planche soit incorrecte en ce qui concerne l'apparence réelle que présentent les spectres de la flamme, nous voulons dire seulement que l'explication des spectres obtenus dans ce cas est tout-à-fait inexacte, par ce qu'on a attribué à des corps simples de raies appartenant réellement à des corps composés.

Pourquoi les spectres obtenus par la flamme s'accordent-ils si peu avec ceux des corps simples? C'est sans doute parce que la flamme n'est pas un agent si simple que l'a prétendu M. KIRCHHOFF. Bien au contraire,

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[1]) Pogg. Ann., B. 118, p. 100.

[2]) Pogg. Ann., B, 110, p. 161; B. 113, p. 374.

en employant la combustion pour rendre les corps incandescents, on ne pourra jamais être sûr d'obtenir des corps simples, et en voici la raison. Si le corps, introduit dans la flamme pour être examiné, est de telle nature qu'il ne puisse être décomposé en ses éléments par la température de la flamme, il est bien évident que les raies spectrales obtenues dans ce cas doivent appartenir nécessairement à la composition *primitive*. D'un autre côté, si la combinaison dont il s'agit se décompose déjà à une température plus basse que ne l'exige l'incandescence du corps, on comprend bien que le spectre observé est dû, ou aux *éléments* mêmes de la substance, que la flamme rend incandescente, ou aux *combinaisons* chimiques produites par la combustion de ces éléments. En un mot, lorsqu'un sel métallique quelconque est introduit dans la flamme, il détermine un spectre particulier qui peut appartenir 1:o au corps primitif composé, 2:o à ses éléments et 3:o à quelqu'une des combinaisons produites par la combustion même de la flamme, sans qu'on puisse dire à priori laquelle de toutes ces substances donne réellement naissance au spectre observé. Ces faits, évidents par eux-mêmes, ont été constatés d'une manière complète, dans les nombreuses recherches de M. MITSCHERLICH [1]) sur les spectres des oxydes métalliques et, en outre, dans celles de M. DIACON sur les spectres des chlorures [2]).

Porte-on, au contraire, un sel métallique à l'incandescence par l'action de l'électricité, les choses se passent, au moins dans les cas ordinaires, d'une manière beaucoup plus simple. En effet, lorsque la décharge se manifeste d'une manière explosive, on pourra toujours être sûr d'obtenir les spectres des corps simples, pourvu que la tension de l'appareil électrique soit suffisamment forte. Mais, afin qu'il ne reste aucun doute sur la nature des spectres obtenus, on pourra entourer les électrodes de différents gaz qui ne sauraient entrer en combinaison chimique, ni avec les électrodes eux-mêmes, ni avec les substances introduites dans l'étincelle. En opérant de cette manière, on obtiendra, nécessairement un phénomène pur d'incandescence et le spectre ne contiendra que les raies mêmes des corps simples.

En humectant les électrodes avec des dissolutions de sels de calcium, de strontium et de baryum, il sera certainement possible d'obtenir, à l'aide de l'appareil de Ruhmkorff, des spectres qui s'accordent parfaitement avec ceux qu'on obtient par la flamme dans des circonstances analogues. En effet, il suffit d'enlever le condensateur pour prolonger le temps de la décharge. Mais, il ne s'ensuit aucunement qu'on soit en droit de regarder les deux spectres comme appartenant au métal simple, contenu dans la dissolution,

[1]) Pogg. Ann. B. 121, p. 459.

[2]) Ann. d. Chim. et d. Ph. (4) Tom. VI, p. 5—25.

ni que l'explication de M. KIRCHHOFF soit satisfaisante, lorsqu'il dit que ces types différents d'un même spectre dépendraient uniquement des températures et des quantités inégales des masses du gaz employé dans l'expérience en question [1]).

Loin de nous la pensée de nier que les raies du spectre d'un gaz incandescent ne puissent apparaître en plus grand nombre, lorsque la température et peut-être même la quantité de la masse rayonnante du gaz augmente, ni que l'intensité de certaines raies ne puissent croître beaucoup plus vite que celle des autres. Mais ce qui est certain, c'est que l'assertion de plusieurs physiciens à savoir que les raies primitivement visibles peuvent entièrement disparaître à la fin, et que le spectre pourrait ainsi changer tout-à-fait d'aspect, est aussi improbable, au point de vue théorique que contraire à l'expérience. De plus, si de telles propriétés étaient réelles, il s'ensuivrait nécessairement que toutes les recherches spectrales deviendraient tout à fait impossibles, car chaque corps simple pourrait jouer alors, à l'égard de son spectre, le rôle d'un véritable Protée.

Par là nous ne nions certainement pas qu'un corps simple ne puisse dans certains cas donner différents spectres. Citons, par exemple, le spectre d'absorption d'iode qui ne ressemble en aucune façon au système des raies brillantes du même corps, obtenues au moyen de l'électricité; et remarquons de plus qu'en général tout corps simple, présentant la propriété d'allotropie, doit donner à l'état d'incandescence des spectres différents, pourvu que la dite propriété de la substance subsiste non seulement à l'état gazeux du corps, mais encore à la température même de l'incandescence.

En supposant donc qu'il y ait réellement allotropie, même pour les corps gazeux, un certain spectre d'absorption doit appartenir à chacun de ces états allotropiques; mais s'il s'arrive qu'un seul de ces états supporte la température de l'incandescence, ce qui aura toujours lieu, lorsqu'il s'agit de la décharge électrique, — on ne pourra certainement obtenir, à cette température élévée, qu'un seul spectre brillant, savoir celui des raies ordinaires.

Ainsi, l'oxygène doit présenter deux spectres différents d'absorption, l'un pour l'oxygène ordinaire et l'autre pour l'ozone. Mais, comme l'ozone est décomposé à une température peu élévée, il n'existera pour l'oxygène qu'un seul spectre à l'état d'incandescence.

Le soufre solide possède, comme on sait, plusieurs états allotropiques, et, d'après certaines observations, ce corps, même à son l'état gazeux,

[1]) Abhandl. d. Berlin. Akad. 1861: Untersuchungen über das Sonnenspectrum. p. 8.

prendrait des formes différentes. Par conséquent, en supposant que cela soit vrai, le soufre gazeux doit donner plusieurs spectres d'absorption, tandis que la possibilité d'un seul ou de plusieurs spectres brillants dépendra de la circonstance suivante, savoir si les états allotropiques plus complexes de cette substance supporteront la température de l'incandescence, avant de se décomposer.

Il est bien évident que les cas dont nous venons de parler, ne forment pas une exception à la loi générale énoncée ci-dessus, savoir que chaque corps simple ne peut donner qu'un seul spectre. En effet, si l'on suppose que l'état allotropique est dû à la constitution moléculaire du corps, soit que les molécules se combinent les unes avec les autres, soit qu'elles s'arrangent entre elles d'une certaine manière, cet état allotropique possèdera, au point de vue spectroscopique, toutes les propriétés significatives d'un corps composé, et par conséquent il doit être décomposé de la même façon que celui-ci par les effets de la décharge disruptive de l'électricité.

La chose paraîtra claire, si l'on soumet à un examen attentif les propriétés des différentes parties de l'étincelle et si l'on étudie en même temps les diverses manières, selon lesquelles la décharge électrique peut s'effectuer. Pour cela supposons qu'on supprime le condensateur, et qu'on fasse passer dans l'air atmosphérique le courant d'induction entre les électrodes de platine, on trouvera alors comme on sait, au pôle *positif*, l'étincelle électrique ordinaire, environnée d'une lumière flambante, et le pôle *négatif* entouré d'un manteau de lumière bleu-violet. M. PERROT, qui a étudié avec soin la lumière flambante au pôle positif, qu'on nomme *l'auréole*, a fait voir non seulement qu'on peut éloigner, par l'insufflation, cette auréole de l'étincelle proprement dite, mais aussi qu'elle correspond, quant à sa nature, à une décharge de quantité, tandis que l'étincelle même est une décharge de tension. Il a trouvé aussi que ces deux espèces de décharges se distinguent, l'une de l'autre, en ce que la première est accompagnée d'actions électrolytiques, qui n'existent pas dans le cas de la décharge *disruptive* [1].)

Supposé maintenant que les électrodes de platine soient humectés d'une dissolution de chloride d'un métal alcalino-terreux quelconque, l'auréole qui se trouve au pôle positif présentera alors un aspect tout autre qu'auparavant, mais la lumière restera la même à l'électrode négatif. Une recherche spectrale de l'auréole formée indique qu'elle contient les mêmes bandes brillantes que présente ordinairement la flamme, où l'on a introduit le même sel métallique. Ces bandes nuancées, appartenant presque toutes aux oxydes

[1]) Ann. d. Chim. et d. Ph. (3) Tom. LXI p. 200.

métalliques, se composent de raies excessivement fines et juxtaposées, l'une tout auprès de l'autre, et par leur aspect singulier elles se distinguent facilement des raies prononcées et dispersées sans ordre qui, étant dues à l'étincelle même, caractérisent assez nettement les corps élémentaires.

L'expérience suivante fournit encore, nous le croyons, une preuve convaincante de la différence essentielle qui existe entre les deux espèces de décharges dont nous venons de parler. Si l'on regarde dans un miroir tournant l'image de l'étincelle électrique, accompagnée de l'auréole, telle que la donne l'appareil d'induction lui-même sans condensateur, on remarque que l'image de l'étincelle proprement dite conserve sa forme d'un fil lumineux unique, lequel aspect est identique à celui qu'elle présente à la vision directe, mais au contraire que l'image de l'auréole prend la forme d'une bande lumineuse très étendue. Ces deux formes diverses des images de l'étincelle et de l'auréole nous indiquent donc que la décharge de l'électricité, dans ces deux cas, a lieu de manières tout à fait différentes, savoir qu'elle se fait *instantanément* dans l étincelle elle-même, mais que sa durée est *très appréciable* dans l'auréole.

Ainsi, puisque la décharge du courant d'induction peut s'effectuer de deux manières, soit *continuellement*, au quel cas se forme l'auréole, soit *disruptivement* dans l étincelle proprement dite, on conçoit aisément que les deux spectres, se présentent simultanément, dont l'un celui de l'auréole appartient à quelque *corps composé*, lequel dans la plupart des cas, est un oxyde métallique, et l'autre, celui de l'étincelle même, appartient aux *corps élémentaires*, c'est-à-dire aux métaux ils mêmes et aux gaz, qui se trouvent au passage de l'étincelle entre les électrodes. Ces deux spectres, ne possédant rien de commun, ne peuvent pas se transformer, l'un en l'autre.

Par conséquent, si, dans l'air, on fait jaillir, l'étincelle d'une bouteille de Leyde entre des électrodes d'aluminium, placés horizontalement devant la fente verticale du spectrocope, la partie moyenne du spectre obtenu, correspondant à l'étincelle même, contiendra les raies du métal et celles de l'air, tandis que les parties supérieures et inférieures présenteront un autre spectre, consistant en bandes nuancées et dû à l'auréole verdâtre; c'est à l'oxyde du métal en question qu'on doit attribuer ce dernier spectre.

Remarquons de plus qu'une autre manière d'obtenir le spectre d'alumine, c'est d'introduire ce métal entre les électrodes du charbon d'une pile voltaïque, très énergique où le spectre de l'oxyde se présente très nettement. Dans ce cas, on ne voit que les deux raies violettes du métal, situées entre les deux raies fraunhoférienes H mais non pas les autres.

Une chose digne de remarque, quand il s'agit des métaux alcalino-terreux, le baryum, le strontium, le calcium et l'aluminium, c'est que les raies du métal situées dans les parties vertes, bleues et violettes du spectre, se mêlent souvent avec celles de l'oxyde. Doit-on en conclure, que ces raies soient communes au corps simple et au corps composé? Nous ne le croyons pas. Ou faut-il regarder ces raies comme correspondant aux vibrations, par lesquelles ces corps se décomposent chimiquement? Pour notre part, nous sommes portés à penser que l'oxyde se trouve presque toujours, dans l'étincelle, avec des traces du métal, quelques minimes qu'elles soient, dont les raies les plus sensibles sont précisement celles qu'on observe intercalées ici entre les bandes véritables de l'oxyde.

Pour résumer tout ce que nous venons de dire des expériences qu'on a faites, non seulement par rapport au passage de l'électricité d'un corps à un autre, mais aussi relativement à sa propagation à travers des corps différents, on peut formuler les lois suivantes:

1:o Il y a deux espèces de décharge électrique, dont l'une est de *tension* et se fait par *explosion* ou *disruptivement*, et l'autre est de *quantité* et se produit par *conduction* ou d'une manière continue.

2:o Par la décharge *disruptive*, qui a toujours lieu quand la tension de l'électricité est suffisante, le corps est en général déchiré en ses moindres particules, et ainsi décomposé chimiquement en ses éléments, s'il s'agit d'un corps composé. Le phénomène d'incandescence qui accompagne soit le déchirement mécanique, soit la décomposition chimique, ne peut pas être regardé comme une conséquence de l'augmentation de la température, mais on doit dire au contraire que la haute température est due elle-même à l'influence de la force mécanique ou chimique qui a servi à la décomposition du corps. Disons de plus que, outre la décomposition produite immédiatement par la décharge disruptive, il peut aussi exister d'autres actions chimiques, qui sont pourtant d'une nature tout à fait *secondaire*.

3:o Quand l'électricité se propage par voie de *conduction*, on doit distinguer deux espèces différentes d'actions, savoir les actions *thermiques*, qui appartiennent aux *conducteurs* eux-mêmes et croissent proportionnellement au *carré de l'intensité* du courant, et en outre les actions qui se manifestent *aux surfaces de passage* d'un corps à un autre et sont *directement* proportionnelles à l'intensité du courant. Ces dernières actions consistent, si le corps est *simple*, dans des *variations de la température*, mais, si le corps est *composé*, dans des *effets chimiques*, qu'on appelle des actions *électrolytiques*. Ces deux phénomènes, celui de Peltier aussi bien que l'électrolyse ordinaire, doivent être regardés comme des manifestations différentes de la même

force; et l'une ou l'autre de ces actions se produit selon que le corps est simple ou composé.

Ces lois, dont l'évidence est démontrée, quand il s'agit des corps solides ou des fluides, doivent s'appliquer aussi aux corps gazeux, où l'on doit attendre des actions électrolytiques, aussi bien que des actions chimiques d'un ordre secondaire.

§ 2. *Sur les spectres du carbone et de ses composés.*

Des recherches nombreuses ont été faites dans ces derniers temps sur les spectres lumineux qu'on obtient des combinaisons du carbone, soit par leur combustion dans la flamme d'une lampe ordinaire à gaz, soit par l'action d'un courant électrique. Quoique nos connaissances des spectres différents, auxquels ces carbures, dans des conditions diverses, peuvent donner naissance, soient devenues ainsi beaucoup plus exactes et plus parfaites qu'elles n'étaient auparavant, cependant ce n'est pas moins vrai qu'aucune explication satisfaisante sur la formation réelle de ces spectres n'a été donnée jusqu'à présent.

Le premier qui ait fait des recherches sur le spectre, produit par la combustion des carbures d'hydrogène, assez précises pour être nommées ici, est M. SWAN [1]). Suivant lui, tous les composés, formés par le carbone avec l'hydrogène, donnent identiquement les mêmes raies brillantes, et c'est pourquoi il a regardé ces raies comme caractéristiques pour toutes les substances de la composition mentionnée. M. ATTFIELD [2]), au contraire, qui a fait aussi des recherches du même genre, a été conduit par ses études à la conclusion que les raies spectrales en question appartiennent au carbone lui-même, surtout puisque l'oxyde de carbone et l'acide carbonique donnent le même spectre que le font les carbures d'hydrogène.

On doit à M. ATTFIELD une description nette et précise sur le caractère vrai du spectre mentionné. En effet, en augmentant l'aggrandissement du spectroscope et l'intensité de la flamme, il a pu faire voir que ces raies brillantes présentent l'aspect de véritables bandes nuancées qu'on peut résoudre réellement en raies distinctes très fines et excessivement nombreuses, juxtaposées les unes auprès des autres, et que l'intensité de la lumière de ces raies varie ou, pour mieux dire, s'affaiblie d'un côté. Cependant, cette

[1]) Edinb. Phil. Transactions, T. XXI, p. 411.

[2]) Philosoph. Transactions, 1862, Vol. 152, Part I, p. 221.

description de l'aspect des raies est tout-à-fait identique à celle donnée auparavant par rapport aux raies des oxydes métalliques, et nous pensons que cette analogie remarquable ne peut être entièrement fortuite.

De plus, M. ATTFIELD a fait l'observation importante que ces bandes se présentent le plus brillantes possible, quand on fait brûler du cyanogène. Et on doit, en outre, à M. DIBBITS [1]) des dessins très exacts sur les spectres produits par la combustion d'oxyde de carbone et de cyanogène soit en l'air soit en oxygène.

Au moyen de l'électricité et par l'emploi de la bobine de Ruhmkorff, M. VAN DER WILLIGEN [2]) a réussi à observer un spectre entre des électrodes de charbon, placées l'une très près de l'autre, lequel spectre était tout-à-fait identique avec celui qu'on trouve non seulement pour des électrodes mouillées d'un liquide, contenant quelqu'un des carbures d'hydrogène, mais aussi par la combustion de gaz oléfiant. Son opinion est que ce spectre appartient au carbone.

Enfin, PLÜCKER [3]), dont les travaux importants sur les spectres des gaz raréfiés sont très connus à tous les physiciens, a aussi examiné avec un soin scrupuleux les spectres du carbone. En effet, il a étudié d'abord les spectres de la flamme du cyanogène et du gaz oléfiant à leur combustion, soit en l'air, soit en oxygène, et ensuite, au moyen du courant électrique, il a fait rougir jusqu' à l'incandescence des carbures d'hydrogène divers, comme le gaz des marais, l'hydrogène bicarboné, le méthyle et l'acétylène.

Nous regrettons de ne pouvoir entrer ici dans la description détaillée des caractères de tous ces spectres, mais nous renvoyons le lecteur aux Mémoires de PLÜCKER. Cependant, il sera bon d'énumerer ici, bien que sommairement, les donnés suivants qu'on doit regarder comme résultats de nos propres expériences et de celles d'autres expérimentateurs.

Le spectre du *cyanogène* est le plus riche en raies brillantes, car, outre les huit groupes de bandes, dont le premier se trouve entre B et C, et le dernier en dehors de H, il y a aussi, dans la région rouge du spectre, une série de bandes, chez lesquelles la lumière s'affaiblit dans une direction opposée à celle qu'on observe pour les autres huit, dont nous venons de parler. Relativement à ces bandes rougeâtres, il faut remarquer aussi que, quand on brûle le gaz en l'air, elles s'étendent beaucoup plus loin vers la partie jaune du spectre qu'à l'emploi de l'étincelle électrique.

[1]) De Spectraal-Analyse, Akademisch Proefschrift, Rotterdam 1863.

[2]) Pogg. Ann., B. 107, p. 473.

[3]) Phil. Trans. Vol. 155, Part I, 1865, p. 1.

Parmi ces huit bandes, énumérées ci-dessus, il y en a quatre qu'on retrouve aussi dans le spectre de tous les carbures d'hydrogène. C'est pourquoi nous regardons comme un fait très probable que ces bandes-ci appartiennent réellement aux *composés du carbone avec l'hydrogène.*

Si l'on fait traverser à l'étincelle des tubes de Geissler, contenant *de l'oxyde de carbone* ou de *l'acide carbonique* dans un état raréfié, on observe encore une série de bandes lumineuses, qui à l'égard de leur situation sont bien différentes des bandes mentionnées ci-dessus, et qu'on ne trouve pas dans le spectre du cyanogène. Cependant, le dessin qu'a donné PLÜCKER ne représente pas correctement les bandes de l'oxyde de carbone, puisqu'il attribue aussi à ce corps les quatre bandes qui, d'après nous, appartiennent en réalité aux carbures d'hydrogène. En effet, si le gaz a été suffisamment purifié, on ne retrouve jamais ces bandes mentionnées dans le spectre de l'oxyde de carbone.

En resumé, il y a quatre séries différentes de bandes dans les spectres des composés du carbone, savoir *deux*, situées aux extrémités du spectre qu'on trouve exclusivement à la combustion du cyanogène, une *troisième* qui est commune à tous les carbures d'hydrogène, et enfin une *quatrième* qu'on peut se procurer par de l'oxyde de carbone ou par de l'acide carbonique.

La question sera donc: *comment faut-il expliquer tous ces types différents des spectres des composés du carbone?*

En répondant à cette question, PLÜCKER a parti de l'hypothèse qu'il existe un certain nombre de corps élémentaires qui peuvent donner, à leur échauffement, chacun à leur tour, deux espèces de spectres lumineux ayant des caractères tout-à-fait différents, qui ne possèdent rien de commun. Si l'on applique cette théorie au cas présent, il s'ensuivra que le carbone, à des températures inégales, pourra donner des spectres différents, et de plus que ce corps simple devra, même en état gazeux, se présenter simultanément sous des variétés très diverses. Mais cette explication, quelque ingénieuse qu'elle soit, ne nous paraît pas assez satisfaisante pour qu'on puisse l'accepter. C'est pour cette raison que nous voulons la remplacer ici par une autre, qui sera, nous l'espérons, plus naturelle et plus conforme aux expériences faites.

D'abord, on doit se rappeler que tous les types précédents des spectres des composés du carbone se ressemblent, les uns aux autres, vu qu'ils consistent entièrement, comme nous venons de le dire, en bandes résolubles en raies isolées très-fines, de la même nature que celles des spectres des oxydes métalliques, mais qu'il existe entre eux cette différence essentielle

que la direction, dans laquelle s'affaiblie la lumière, n'est pas la même. Tous ces types appartiennent à la première classe, comme le dit PLÜCKER [1]), et l'on pourra donc soupçonner à bon droit qu'ils sont dus exclusivement aux corps composés et pas du tout aux corps élémentaires.

Cependant, par rapport au carbone, il existe actuellement un spectre du second ordre, pour parler conformément aux vues de PLÜCKER, observé par nous déjà en 1863, dont il faut rendre compte ici. C'est ce spectre seul qui, d'après nous, appartient au carbone lui-même, tandis que tous les autres types dérivent des composés différents de ce corps. Voici d'abord le caractère du spectre en question.

Si l'on s'imagine que la région rouge du spectre soit située à droite, le spectre mentionné contient une raie double très forte, qui est située tout auprès, mais à droite de la raie fraunhoférienne C, de plus quelques raies fines, mais distinctes entre *D* et *E*, d'ailleurs trois raies marquées à gauche de *b*, et enfin une bande à gauche de *G*. Cette dernière bande, s'étendant de plus en plus, à mesure que l'intensité de la décharge disruptive augmente, est remarquable en ce qu'elle ressemble beaucoup aux deux bandes d'hydrogène, situées à *F* et près de *G*. En effet, quand on opère avec de l'oxyde de carbone raréfié, elle se présente sous la forme d'une raie isolée et parfaitement linéaire, tandis que dans les circonstances ordinaires elle prend l'aspect d'une bande très large qui se dégrade près des bords.

La raie rouge a été observé aussi par M. HUGGINS [2]) qui indique qu'on l'obtient très bien en opérant avec de l'acide carbonique, mais non avec des carbures d'hydrogène. La cause vraisemblable du dernier phénomène est que les carbures ne donnent, en général, que les spectres des combinaisons; cependant, si la décharge, par l'action d'un grand condensateur, devient disruptive, les raies du carbone paraîtront aussi, ce qui prouve évidemment que le spectre en question est réellement dû au carbone. Veut-on en avoir quelques autres preuves, nous pouvons citer les expériences que voici.

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[1]) [Ces dénominations des spectres du premier et du deuxième ordre, données par PLÜCKER, furent toujours regardées par ÅNGSTRÖM comme très impropres et même incorrectes. Car, abstraction faite de la circonstance qu'aucun corps ne possède en général, selon lui, plus d'un seul spectre, il pensait qu'il aurait été plus juste d'appeler spectre du *premier* ordre celui qui consiste exclusivement en *raies*, et spectre du *deuxième* ordre celui qui contient des *bandes nuancées*, puisque les spectres des raies ont été découverts plus tôt que les autres. — R. T.]

[2]) Phil. Transact.: Vol. 154, P. II, 1864, p. 145. WATTS: Index of spectra, 1872, p 20, Spectrum n:o IV; Phil. Mag. Ser. 4, Vol. 38, p. 249; Vol. 41, p. 12.

Si l'on fait éclater l'étincelle entre des électrodes de charbon, les raies obtenues s'éteignent au milieu du champ, de la même manière que le font les raies véritables des métaux. De plus, à la décharge disruptive à travers quelqu'une des combinaisons du carbone, on obtient non seulement ces raies du carbone, dont il est question, mais aussi celles de l'oxygène, de l'hydrogène, ou de l'azote; en un mot, toutes les raies des corps élémentaires qui constituent le carbure employé.

Autour des électrodes du charbon, on peut aussi observer, à la décharge disruptive, une auréole qui nous indique qu'il existe simultanément une décharge continue. Le spectre de l'auréole dépend de la nature du milieu dans lequel éclate l'étincelle: dans *l'azote*, on trouve les groupes bleus et violets qui sont caractéristiques pour le cyanogène; dans *l'hydrogène*, c'est le spectre des carbures d'hydrogène qu'on observe; et ensuite, dans *l'oxygène* se présente le même spectre que donne l'oxyde de carbone dans un tube de Geissler.

Les groupes nuancés du cyanogène, situés dans les parties bleues et violettes du spectre, se montrent aussi, soit quand l'étincelle traverse la partie luisante d'une flamme à gaz, soit dans l'arc voltaïque entre des électrodes du charbon d'une pile puissante. Cependant, ce dernier spectre du cyanogène, qui est remarquable par l'éclat vif des raies, n'est pas pur, mais mêlé des raies des carbures d'hydrogène, dont la splendeur est encore plus magnifique.

D'après ces observations, on peut regarder comme un fait démontré que *l'auréole donne respectivement les spectres du cyanogène, des carbures d'hydrogène, de l'oxyde de carbone ou de l'acide carbonique, selon que les électrodes du charbon sont entourées d'azote, d'hydrogène ou d'oxygène.* La supposition la plus naturelle sera donc que les spectres produits appartiennent réellement à ces corps composés, d'autant plus que l'aspect même de ces spectres indique leur origine des corps composés et non pas des corps élémentaires.

On sait bien que l'acide carbonique se décompose par le courant électrique et que le spectre observé est dû exclusivement à l'oxyde de carbone qui se produit à ce moment-là, et l'on pourrait donc supposer que l'acide carbonique lui-même ne puisse donner aucun spectre particulier. Cependant, quand l'acide carbonique se forme directement, par exemple à la combustion du cyanogène, il nous semble comme très probable que ce corps même peut déterminer un spectre, et cette opinion a été aussi confirmée par les observations de PLÜCKER. En effet, il a trouvé que les bandes nuancées rouges du cyanogène, brûlé dans l'air ou dans l'oxygène, deviennent

plus fortes et s'élargent beaucoup, à mesure que la combustion devient plus vive, tandis que, suivant une expérience faite par nous au moyen de l'étincelle qui a traversé le gaz cyanogène, circulant dans un tube de verre et ainsi purgé peu à peu de l'air, ces bandes rouges ne s'étendraient qu'à la bande première des carbures d'hydrogène et s'évanouirairent même pendant quelques moments de l'expérience. Remarquous de plus que la cause vraisemblable de ce que, aussi dans cette expérience, le spectre des carbures d'hydrogène pouvait se présenter, doit sans doute être cherchée dans l'impossibilité de dessécher parfaitement le gaz du cyanogène, quand on le prépare de cyanure de mercure.

Il nous paraît être beaucoup plus difficile d'expliquer d'une manière satisfaisante la naissance du spectre des carbures d'hydrogène qui se présente toujours à la combustion d'une combinaison quelconque de carbone avec de l'hydrogène, et même, suivant M. ATTFIELD, à celle de carbure de soufre. Quoique ce spectre ait été considéré, par quelques observateurs, comme appartenant au carbone, nous ne pouvons, pour notre part, accepter cette opinion, et en voici la raison. En employant le condensateur, le spectre du gaz d'éclairage détermine non seulement les raies ordinaires des carbures d'hydrogène, mais aussi celles du carbone et de l'hydrogène. Remarquons en outre que l'aspect même des bandes nuancées, étant semblables à celles du cyanogène, indique immédiatement, comme nous venons de le dire plusieurs fois, que le corps, duquel elles dérivent, est composé.

La difficulté, il nous semble, devait en grande partie s'évanouir, si l'on pouvait démontrer que, à la combustion de carbures d'hydrogène de combinaisons très variées, le même produit chimique se formerait toujours. Mais, c'est précisément là ce qu'a démontré M. BERTHELOT. Suivant lui, il se produira une quantité d'acétylène toutes les fois qu'une combustion imparfaite des carbures d'hydrogène, de l'éther etc. a lieu, et même quand l'étincelle électrique passe entre des électrodes du charbon au travers de l'hydrogène. Il sera donc très probable que c'est l'acétylène qui donne naissance au spectre qu'on retrouve pour tous les carbures d'hydrogène.

Quant à l'observation de M. ATTFIELD que l'oxyde de carbone donne le spectre ordinaire des carbures d'hydrogène, nous devons remarquer que cela ne s'accorde pas bien avec nos propres expériences. Dans un tube de Geissler, contenant de l'oxyde de carbone ou de l'acide carbonique, on peut certainement trouver des traces des spectres des carbures d'hydrogène, puisque le gaz n'est jamais parfaitement sec, mais d'après les observations mêmes de PLÜCKER le spectre particulier de l'oxyde de carbone n'en a aucune ressemblance.

Disons ensuite qu'on observe ordinairement dans le spectre de l'oxyde de carbone et à gauche de la raie fraunhoférienne *G* une raie très forte qui appartient en réalité au carbone lui-même. Ici, nous trouvons donc la même chose que nous venons d'indiquer par rapport aux oxydes des métaux, nous voulons dire que le spectre de l'oxyde est mêlé souvent de certaines raies du corps simple lui-même.

§ 3. *Les spectres de l'azote et de ses combinaisons.*

Pour résumer tout ce qu'on savait depuis bien longtemps sur les spectres auxquels l'azote peut donner naissance, il faut se rappeler que la *décharge disruptive* de l'étincelle électrique en plein air entre des métaux quelconques produit deux spectres qui se superposent, savoir celui du métal employé comme électrodes et celui de l'air. C'est à l'un de nous [1]) qu'on doit la première description de ce spectre de l'air, contenant les raies de l'azote, de l'oxygène et de l'hydrogène, qui par leur forme et leur situation sont bien définies. Au contraire, si la décharge devient *continue*, ce qui aura lieu toujours, si l'on fait passer le courant d'induction au travers d'une corde mouillée, ou si l'on approche suffisamment les électrodes, l'une de l'autre, on aperçoit, supposé que les électrodes ne soient pas oxydables, mais qu'elles soient de platine, un tout autre spectre de l'air, qui contient une multitude de bandes nuancées, dont la ressemblance avec celles des oxydes des métaux est surprenante, C'est précisément ce dernier spectre qu'a observé, le premier, déjà en 1858 M. VAN DER WILLIGEN [2]), mais nous devons remarquer que ses dessins sont un peu imparfaits.

Pour l'azote raréfié, renfermé dans un tube de Geissler, PLÜCKER [3]) a observé deux spectres qui diffèrent beaucoup par leur aspect, l'un de l'autre, puisque l'un contient des bandes nuancées, et l'autre des raies aiguës. D'après ses observations, ces spectres se présentent alternativement, selon que le courant d'induction est permis de traverser directement le tube, ou, après l'interposition d'un condensateur, d'y arriver sous la forme d'une étincelle. Il suppose que ces spectres appartiennent, tous les deux, à l'azote lui-même, mais que la naissance de l'un ou de l'autre dépend de l'intensité de l'incandescence du gaz; et c'est par ce motif qu'il les a nommés spectres du premier ou du second ordre.

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[1]) ÅNGSTRÖM: K. Sv. Vet. Ak:s Handl., 1853, p. 335; Pogg. Ann., B. 94, p. 141.

[2]) Pogg. Ann., B. 106, p. 610.

[3]) Phil. Trans., Vol. 155, Part I, p. 6.

Puisque PLÜCKER n'a pas senti les liens qui unissaient ses propres observations à celles qui avaient été faites d'avance, il croyait qu'il avait découvert deux spectres tout-à-fait nouveaux, mais c'est loin d'en être ainsi. Au contraire, les deux spectres obtenus par lui sont réellement identiques à ceux que nous venons de mentionner ci-dessus. En effet, le spectre du second ordre, suivant la dénomination de PLÜCKER, n'est autre chose que le spectre ordinaire de l'air, si l'on exclut naturellement les raies qui appartiennent à l'oxygène et à l'hydrogène, dont le nombre n'est cependant pas grand. De plus, le spectre du premier ordre est le même que celui de l'auréole, observé déjà par M. VAN DER WILLIGEN. L'identité de ces derniers spectres a été indiqué par l'un de nous [1]) à l'Assemblée des Naturalistes Scandinaves à Stockholm en 1863 et de plus par M. BRASACK [2]).

Par ce que nous venons de dire, on trouve ainsi que les observations, faites par PLÜCKER, n'ont pas conduit en réalité à la connaissance de spectres nouveaux par rapport à l'air, mais certainement qu'il a donné une méthode nouvelle pour les obtenir, — méthode qui facilite beaucoup l'étude de l'apparence de ces spectres et de même la détermination des longueurs d'onde des raies. On peut remarquer aussi que le spectre de l'azote, qui consiste en raies aiguës, aurait dû s'appeler le spectre du premier ordre plutôt que le spectre des bandes nuancées, puisque celui-là fut observé bien avant celui-ci.

Quant à l'interprétation de ces deux spectres, nous pensons qu'ils dépendent seulement des différentes manières desquelles se fait la décharge électrique, et qu'ils sont dus aux corps tout-à-fait divers. Ainsi, le spectre des raies distinctes, occasionné par la décharge *disruptive*, doit être considéré comme dû à *l'azote* lui-même, puisqu'il se présente aussi dans les tubes de Geissler dans les mêmes circonstances que quand la décharge est disruptive; mais les bandes nuancées appartiennent sans doute à quelqu'une des *combinaisons* de l'azote, formée par la décharge de *quantité* ou par *conduction*.

Partant de cette opinion de la naissance des deux spectres, nous allons décrire le spectre obtenu à la décharge continue, soit que cette décharge se fasse sous la forme d'une auréole en plein air, ou dans de l'azote raréfié dans un tube de Geissler.

Au moyen du prisme, on trouve dans l'auréole au pôle *positif* un grand nombre de bandes nuancées, parmi lesquelles on peut distinguer deux séries tout-à-fait différentes, dont l'une se trouve dans la partie la moins

[1]) ÅNGSTRÖM: Förhandl. vid de Skand. Naturf:s 9:e möte, Stockh. 1863, p. 138.
[2]) Abhandl. d. Naturf. Ges. zu Halle, B. X, p. 36, etc.

réfrangible du spectre, et l'autre dans les parties verte, bleue et violette. L'aspect de ces deux séries étant très inégal, on pourrait soupçonner à bon droit qu'elles appartiennent réellement à des corps différents Quoi qu'il en soit, ce qui est vrai, c'est que leur intensité varie beaucoup dans des circonstances diverses et d'une manière tout-à-fait différente.

Au pôle *négatif*, on observe le manteau bleu-violet, dont les couleurs spectrales, situées dans les parties verte, bleue et violette, ne changent pas avec la nature des électrodes.

Dans les tubes de Geissler, contenant de l'azote raréfié, on retrouve, à la décharge continue, les mêmes spectres que donne l'auréole en plein air. Cependant, il faut dire que la lumière du pôle positif, dont l'intensité est très grande, se présente ici non seulement dans le voisinage du pôle lui-même, mais aussi le long du tube capillaire. Au pôle négatif, le manteau bleu-violet s'agrandit et devient très brillant à mesure que le gaz est plus raréfié.

Quelles sont maintenant les combinaisons d'azote qui donnent les spectres mentionnés à la décharge continue? C'est là ce qu'on ignore tout-à-fait par rapport à la matière incandescente au pôle négatif. Quant au gaz qui se trouve au pôle positif, on peut prouver d'une manière directe par l'emploi d'une dissolution de sulphate de fer qu'il se produit là du bioxyde d'azote. De plus, puisqu'on sait bien que l'étincelle électrique, en passant au travers d'une masse d'air renfermé dans un vase, donne naissance aux vapeurs rutilantes qui indiquent l'existence d'acide hypoazotique, il s'en suit que l'azote se combine dans ces circonstances avec l'oxygène. La seule question sera donc, d'où vient l'oxygène, quand il s'agit d'un tube de Geissler rempli d'azote. À cet effet, on doit se rappeler qu'en se procurant de l'azote, on ne pourra jamais chasser parfaitement l'air, ou du moins on ne dégagera pas entièrement le gaz de la vapeur d'eau, ce qui prouve la présence perpétuelle de la raie d'hydrogène C, fait qui expliquera suffisamment la possibilité des combinaisons mentionnées. Ensuite, puisque le spectre lumineux en question n'a aucune ressemblance avec le spectre d'absorption produit par de l'acide hypoazotique, on pourra en conclure qu'il sera le bioxyde d'azote qui à l'état d'incandescence donne les bandes nuancées qu'on trouve au pôle positif soit dans l'auréole ordinaire, soit dans le tuyau capillaire d'un tube de Geissler contenant de l'azote raréfié.

§ 4. *Expériences.*

Qu'il me soit permis d'indiquer ici quelques expériences qui puissent confirmer à un certain degré les énoncés ci-dessus.

Expériences par rapport au *carbone.*

Remarquons d'abord que, quand une étincelle puissante, par l'emploi d'un condensateur, éclate entre des électrodes du charbon de cornu, soit en l'air, soit en oxygène, les raies spectrales du carbone se présentent très distinctes. Cependant, afin qu'on voie clairement ces raies, la distance entre les électrodes ne doit pas être trop petite, et il faut aussi que la dispersion du spectroscope soit suffisamment grande.

Première expérience. Dans un tube de verre, muni d'électrodes de charbon, j'ai fait circuler un courant d'oxygène, produit par la décomposition électrolytique d'eau. L'étincelle du condensateur a donné, dans ce cas, un spectre très beau qui se composait des raies du *carbone* et de celles de *l'oxygène* qu'on trouve sur la planche I, fig. V.

Deuxième expérience. En employant deux bouteilles de Leyde comme condensateurs, j'ai fait éclater l'étincelle entre des électrodes de platine, dont la distance, l'une de l'autre, était de $35^{mm.}$, placées dans un courant *d'acide carbonique.* Le miroir tournant a prouvé que la décharge était *instantanée*, et dans le spectre j'ai vu, outre les raies de platine, celles du *carbone* et de *l'oxygène*, savoir: les raies du carbone 5144 (tout le groupe), 4266; celles de l'oxygène 6170, 4940, 4703, 4645, 4593, 4476 (faible), 4416 et 4350.

Troisième expérience. En répétant l'expérience précédente sans condensateur, et en diminuant la distance entre les électrodes jusqu'à $5^{mm.}$, j'ai analysé, par le miroir et par le spectroscope, l'auréole vert-pâle qui se présente dans ce cas entre les électrodes. L'image de l'auréole, vue dans le miroir tournant, était très allongée, ce qui indique que la décharge se faisait par *conduction.* Dans le spectre, qui était très faible, j'ai pu distinguer les raies suivantes de *l'oxyde de carbone:* 5608, 5197 et 4834.

Quatrième expérience. Dans un courant de *gaz d'éclairage* que j'ai fait passer au travers d'un tube de verre, muni d'électrodes d'alumine, dont la distance était d'environ $10^{mm.}$, l'étincelle du condensateur a donné les raies *d'hydrogène* C, F (large), et 4340 près de G (très large); de plus, les

raies du *carbone* 6580 (raie double), 5144 (raie triple) et 4266, et enfin les trois bandes suivantes de *l'hydrogène carburé* 5633, 5164, 4736. La décharge était *disruptive* suivant les indications du miroir tournant.

Cinquième expérience. J'ai fait passer un courant de *cyanogène*, produit par la décomposition du cyanure de mercure, à travers le tube mentionné dans l'expérience précédente. Le courant, étant faible, n'a pas permis d'employer le condensateur, mais comme l'a prouvé le miroir, la décharge fut néanmoins *instantanée*. Les raies spectrales que j'ai observées étaient: la raie C de *l'hydrogène* (le gaz n'était par séché), le groupe 5144 du *carbone*, la raie double 5003 de *l'azote*, les quatre bandes de *l'hydrogène carburé* 6187, 5633, 5164, 4736, et enfin quelques faibles bandes dans la partie violette du spectre, appartenantes sans doute au *cyanogène*.

Ajoutons que, dans la 4ième et la 5ième expérience, une multitude de charbon en poudre a été précipitée au voisinage des électrodes sur les parois du tube de verre.

Toutes ces cinq expériences ont été faites sous la pression atmosphérique.

Voici de plus quelques expériences sur *l'azote*.

Première expérience. Dans un flacon de verre d'un diamètre de 100mm., rempli à moitié d'une dissolution concentrée de sulphate de fer descendaient deux fils de platine renfermés entièrement, sauf leurs points les plus extrêmes, dans des tubes de verre. L'air du flacon fut mis en communication avec l'air atmosphérique. Chacun des fils était en contact avec son conducteur de la machine électrique de Holtz (grand modèle), dont les bouteilles de Leyde ont été éloignées. Le pôle positif se trouvait à une distance de 15mm. au-dessus de la surface du liquide; le pôle négatif, au contraire, se plongeait entièrement au-dessous de la dite surface.

En tournant la machine, j'ai vu se produire au pôle positif une faisceau de lumière de couleur pourpre ou rouge-violacé, et le courant, étant dirigé perpendiculairement à la surface du liquide, la déprimait notablement. Après avoir fait fonctionner la machine en 15 minutes, et durant ce temps il ne se produisait pas d'étincelles, mais seulement la lumière jaillissante mentionnée, je vis les parties les plus hautes du liquide jusqu'à une profondeur de 20mm. colorées en brun assez fort, ce qui prouvait que le liquide avait absorbé une certaine quantité de bioxyde d'azote.

J'ai obtenu le même résultat en employant sans l'intervention d'un condensateur le courant d'induction de la bobine de Ruhmkorff.

J'ai repété cette expérience sous une autre forme. En effet, j'ai introduit dans l'une des boules d'un tube ordinaire de Geissler une quantité de dissolution de sulphate de fer, et ce tube a été mis en communication avec la machine pneumatique à mercure. Le tube, étant rempli d'air atmosphérique, avait son pôle positif à la boule supérieure, et le pôle négatif dans la boule inférieure un peu au-dessus de la surface du liquide. Les deux pôles étaient en contact avec les électrodes de la bobine de Ruhmkorff.

a) *Pression du gaz égale à 137*^{mm}.

L'aspect général: Dans les deux boules, on trouve des filets lumineux d'une teinte rougeâtre; dans le tube capillaire la lumière est assez homogène; au pôle négatif apparaît un point lumineux de la couleur violette.

Le spectre est composé de celui de l'azote, dont les raies sont d'une intensité demi-forte, et de celui du bioxyde d'azote. Les bandes nuancées de cette combinaison, quoiqu'elles ne fussent pas fortes, étaient cependant assez distinctes. Entre elles, les bandes violettes étaient plus faibles que celles de la partie la moins réfrangible du spectre.

Le miroir tournant prouvait qu'il y avait dans toutes les parties du tube les deux espèces de décharge, savoir la décharge disruptive et la continue, ce qu'on a pu conclure du fait que l'image allongée commençait avec un trait fort brillant.

Le liquide avait noirci considérablement pendant l'expérience.

b) *Pression du gaz égale à 42*^{mm}; toutes choses égales d'ailleurs.

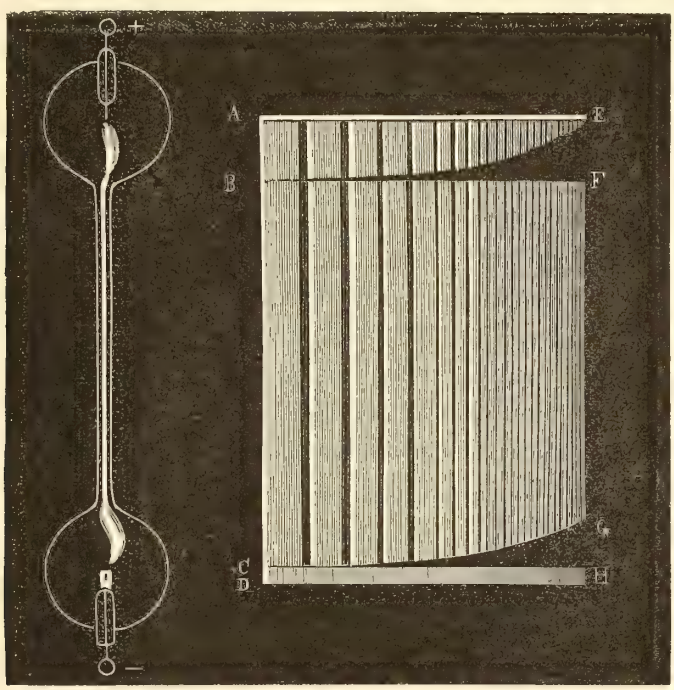
L'aspect général: L'auréole rougeâtre apparaissait le long des deux boules sous forme d'un seul filet assez large. Au bout extrême du pôle positif, on observe un point incandescent d'une teinte rouge-blanchâtre, ce qui indique que le fil rougit en ce point. Gaîne violette existe au pôle négatif. Intervalle noir entre cette gaîne violette et la partie inférieure du filet de l'auréole.

Le spectre assez fort ne contient que les bandes du bioxyde d'azote soit dans les boules, soit dans le tube capillaire. Au pôle négatif, on voit très distinctement les bandes de la lumière bleuviolet.

Le miroir n'indique aucune étincelle ou décharge disruptive proprement dite, mais en le tournant rapidement on voit une multitude de décharges partielles ou intermittentes. Au bord du commencement, l'image se divise en sillons isolés, dont j'ai pu compter cinq ou six au moins; les suivantes étaient confondues et se dégradaient. Voyez la figure. Chacun de ces premiers sillons ou cannélures, dont la largeur individuelle était assez

grande, avait le bord du commencement un peu plus intense que le reste; cependant, il n'existait pas d'étincelle proprement dite. Le pôle positif, qui rougissait à son extrémité, donna une image continue A E. C'est la même continuité que présente l'image C D H du pôle négatif. A la fin de la décharge, c'est seulement dans le tube capillaire et près du pôle positif que l'auréole possède assez d'intensité pour être vue, d'où viennent les espaces obscures B E F et C G H.

Le liquide, qui s'était noirci encore plus, était d'une couleur brun-noirâtre.



c) *Pression du gaz égale à 15*^{mm}.

Le liquide entra en ébullition, les bulles des vapeurs d'eau remplissaient en partie le tube capillaire, et par suite on ne pouvait pas observer le phénomène d'incandescence du gaz. Le courant électrique, se propageant vraisemblablement par la couche d'eau répandue le long des parois du tube, produit dans le centre du tube une faible lumière appartenante à l'hydrogène.

De ces expériences on pourra certainement conclure qu'il y aura formation de *bioxyde d'azote*, sitôt que la décharge devient continue, c. à. d. toutes les fois qu'il se produit une auréole au pôle positif, soit en plein air, soit en azote, et de plus il est bien probable que le spectre des bandes nuancées qu'on observe dans cette auréole appartient réellement à la combinaison mentionnée.

Remarquons de plus que, si la décharge devient disruptive, l'étincelle ordinaire est accompagnée presque toujours d'une auréole, mais, à cause de la grande intensité des raies aiguës, il sera bien difficile de distinguer les bandes nuancées qui sont en général très faibles.

Deuxième expérience. Un tube ordinaire de Geissler, rempli d'air, fut mastiqué hermétiquement par ses tubes d'issue d'une part avec un petit flacon contenant de l'acide phosphorique solide et d'autre part avec la machine pneumatique à mercure. L'électrode de la boule supérieure, que nous appellerons A, fut en contact avec le pôle positif de la machine de Ruhmkorff, et celle C de la boule inférieure avec le pôle négatif. Le long du tube d'issue inférieur se trouvait d'ailleurs un fil de platine très fin, dont le bout, nommé B, fut éloigné de celui de C à peu près de 18^{mm}. Par cette disposition, on pouvait à volonté faire circuler le courant d'induction soit de A à C au travers des deux boules et le long du tube capillaire, ou de B à C dans la boule inférieure seule.

Quand le courant d'induction passa entre A et C, la pression du gaz étant d'abord égale à 90^{mm}, l'aspect général de la lumière était exactement le même que dans l'expérience sus-dite à l'emploi de la dissolution de sulphate de fer, et le prisme annonçait, même dans ce cas, que le spectre appartient au bioxyde d'azote. Le miroir tournant résolvait l'image du tube illuminé en six sillons isolés, dont le premier seul possède son bord du commencement un peu plus intense que le reste, mais au delà des sillons mentionnés l'image était continue comme à l'ordinaire et s'affaiblit graduellement. L'extrémité de l'électrode A rougit; celle de C fut environnée de la gaîne bleuviolet.

En diminuant la pression jusqu'à 48^{mm}, on n'observe pas d'autre changement essentiel, excepté que l'image dans le miroir est devenue plus étendue.

À une pression de 12^{mm}, au contraire, on voit aux deux pôles et l'auréole brun-jaunâtre et la gaîne de la lumière bleuviolet, cependant celle-là est prépondérante en A, et celle-ci en C. Ceci devient plus évident encore, quand la pression descend à 4^{mm}. Alors, le fil A est environné

d'une faible lumière bleuâtre, contenant les bandes 4709 et 4281, c. à. d. les couleurs spectrales ordinaires de la lumière au pôle négatif, et on voit de plus entre le bout du fil et le tube capillaire l'auréole brun-rougeâtre appartenante au bioxyde d'azote. Le pôle C, au contraire, est entouré d'une auréole double, savoir, d'une gaîne de lumière bleuâtre très intense tout près du fil et d'un cylindre plus grand d'une lumière bleue et un peu faible prenant la forme d'un sphéroïde aplati.

Le phénomène mentionné s'explique immédiatement par la supposition que les deux courants d'induction traversent le tube, l'un après l'autre, et dans des directions contraires. C'est là ce que j'ai trouvé confirmé en faisant fonctionner lentement à la main l'interrupteur de l'appareil d'induction: l'une fois, à l'interruption du courant, la lumière violette se montra en C, l'auréole étant en A; l'autre fois, à la fermeture du courant, c'est l'inverse qui a eu lieu [1]).

Si l'on fait traverser le courant entre B et C, le même phénomène sera plus marqué encore. Les gaînes violettes qui apparaissent aux deux pôles augmentent leur intensité à mesure qu'on diminue la pression et elles deviennent enfin excessivement brillantes. Chaque pôle est fourni de plus d'une auréole brun-rougeâtre sous la forme d'un pinceau lumineux et tout-autour de ces auréoles il y en a encore une qui est plus faible, mais plus grande, et s'étende d'un pôle à l'autre.

Les indications du spectroscope étaient en harmonie parfaite avec ce que nous venons de dire par rapport à l'aspect général de la lumière. Ajoutons seulement que dans les spectres des auréoles, c. à. d. dans celui du bioxyde d'azote, les bandes violettes sont les plus intenses.

Dans son Mémoire "Sur le spectre de l'auréole boréale", ÅNGSTRÖM [2]) a rendu compte d'une expérience faite par lui à l'aide d'une bouteille contenant de l'air, dont le fond était recouvert d'une couche d'acide phosphorique. Après avoir fait le vide jusqu'à une pression de quelques millimètres, il a obtenu, par le courant d'induction, des deux électrodes un spectre contenant presque uniquement les bandes violettes de la lumière qui se présente ordinairement au pôle négatif. Cette expérience qui a attiré l'attention particulière de M. HERSCHEL [3]), s'explique parfaitement par ce que nous venons de dire sur les deux courants d'induction. Dans la bouteille, employée par ÅNGSTRÖM, où la lumière violette du pôle négatif possède une intensité ex-

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[1]) Voy. WIEDEMANN: Galvanismus, 1ière Ed., B. II, p. 895.

[2]) Pogg. Ann.: Jubelband, p. 426; Journal de physique: T. III, p. 212.

[3]) Phil. Mag., Ser. 4, Vol. 49, p. 66.

traordinairement grande, on voit passer alternativement les deux courants d'induction, si l'on ferme ou interrupte lentement le courant primitif. Au commencement de l'expérience, je n'ai pu voir aucune trâce de l'auréole brun-jaunâtre, mais le courant ayant traversé quelque temps l'air renfermé, on peut distinguer les bandes nuancées de l'auréole, quelque faibles qu'elles soient. Est-ce donc, peut-on demander, que la vapeur d'eau sera nécessaire à la formation du bioxyde d'azote qui donne l'auréole au pôle positif; — c'est là ce qu'a pensé quelquefois ÅNGSTRÖM sans pouvoir décider la question. Quoi qu'il en soit, l'étude de la lumière violette au pôle négatif vient de devenir d'une importance extraordinaire par le fait indiqué par ÅNGSTRÖM, nous voulons dire que les bandes faiblement éclairées du spectre de l'aurore boréale coïncident avec celles du spectre au pôle négatif.

Pour le présent je me restreins à annoncer les expériences ci-dessus; une autre fois je pourrai peut-être les continuer.

§ 5. *Sur la détermination des longueurs d'onde des raies et des bandes spectrales.*

Après avoir étudié soigneusement quelles raies doivent être attribuées à chacun des gaz en question, les déterminations des longueurs d'onde des raies et des bandes spectrales ont été faites au moyen de deux méthodes un peu différentes, savoir par l'enregistrement dans le spectre solaire et par des mesures micromètriques. Aux déterminations définitives, lorsqu'il s'est agi des spectres de l'oxyde de carbone, du bioxyde d'azote et de celui de la lumière au pôle négatif, l'incandescence du gaz a eu lieu dans des tubes de Geissler. Puisque, dans ces tubes, l'intensité de la lumière électrique est en général très faible, on conçoit qu'en faisant coïncider le fil vertical du réticule avec les raies brillantes du spectre, il fut nécessaire d'exclure du champ de vision du spectroscope la lumière solaire et de déterminer ensuite par la situation trouvée du fil entre les raies fraunhoférienes la longueur d'onde de la bande ou de la raie en question. Or, puisque dans ce cas la lumière solaire devait passer au travers du tuyau étroit du tube de Geissler, ce tuyau fonctionnant comme une lentille cylindrique, une parallaxe plus ou moins grande pouvait avoir lieu quelquefois, et par suite il faut remarquer que cette méthode n'est pas en général susceptibel dans le cas actuel de la même exactitude qu'à l'ordinaire. Cependant, afin de diminuer autant que possible les erreurs provenues de la parallaxe, j'ai employé les moyennes des déterminations faites.

En employant le micromètre, dont le tambour portait cent divisions égales, j'ai répété plusieurs fois les mesures, dont les déterminations individuelles s'accordaient entre elles jusqu'à quelques centièmes d'un pas de vis. En outre, la lunette d'observation du spectroscope restant fixe dans sa position primitive, j'ai déterminé et avant et après chaque série des mesures, au moyen du micromètre, les positions des raies fraunhoférienes les plus importantes qui se trouvaient dans le champ de vision, et ensuite par l'interpolation graphique il fut donc aisé d'obtenir, au moyen du "spectre normal du soleil" par ÅNGSTRÖM, les longueurs d'onde des raies brillantes du gaz.

Pour indiquer approximativement l'exactitude qu'on peut obtenir dans ces mesures, je donne ici quelques valeurs du pas de vis, exprimées en longueurs d'onde pour les parties différentes du spectre:

Longueur d'onde:	Pas du vis:
5000	13,0
5600	19,6
6200	25,9
6800	35,0

Si l'on suppose que l'erreur commise à la mesure n'ait pu surpasser un dixième d'un pas de vis, et en prenant pour la longueur d'onde le dix-millionième d'un millimètre comme unité, on obtient des erreurs égales au plus à 1,4 dans le voisinage de *b*, à 2,0 dans celui de *D* et à 3,2 dans celui de *C*. Cependant, je suis bien sûr que les erreurs actuellement commises n'ont jamais monté à cette grandeur. Disons enfin que les nombres, donnés dans les tableaux suivants, sont des moyennes des deux espèces de déterminations.

Quant à l'aspect général des spectres différents, je renvois le lecteur aux planches données. C'est seulement par rapport au spectre du bioxyde d'azote qu'il ne sera peut-être pas superflu d'en donner une description détaillée. Ce spectre se compose 1° de quatre groupes de bandes nuancées d'une forme singulière, situées entre le rouge extrême et l'endroit au-delà de la raie *b*: et 2° des bandes isolées depuis *b* jusqu'à l'extrémité violette du spectre. Toutes ces bandes sont dégradées vers le côté du spectre le plus réfrangible. Dans les quatre groupes, chaque bande consiste en trois raies nuancées, dont la première est la plus intense et par suite la seule qu'on

voie aux minima. Les deux premières raies de chaque groupe se trouvent à une distance, l'une de l'autre, qui est plus petite que celle entre la deuxième et la troisième raie. Quelquefois, quand la lumière a été suffisamment grande, j'ai cru voir au voisinage de D la deuxième raie décomposée en deux et la troisième en trois petits maxima, mais comme ceci n'a été observé qu'exceptionnellement, je ne l'ai pas indiqué sur la planche.

En comparant entre elles les distances des raies du même ordre, dont se composent les bandes particulières, on trouve qu'elles diminuent assez régulièrement du côté rouge du spectre vers le côté violet. Cependant, dans le groupe situé au voisinage de $\lambda = 5500$, il y a plusieurs séries secondaires qui proviennent probablement, comme le prouvent les différences mentionnées, d'une superposition des bandes des groupes différents, et c'est pourquoi j'ai indiqué aussi bien sur la planche que dans le tableau à quel groupe on doit attribuer chacune de ces bandes.

Sur la planche II, on trouve quelques-unes des bandes nuancées, appartenantes aux spectres du carbure d'hydrogène et de l'oxyde de carbone, résoutes en raies isolées. Dans la partie de bande qui est située du côté le moins réfrangible du spectre, ces raies sont si excessivement serrées, qu'elles se confondent presque entièrement, et c'est pour cette raison que l'intensité de la bande augmente considérablement dans cet endroit. Mais à cause de ces petits intervalles, les observations sont très pénibles, si l'on n'emploie que le courant de la bobine de Ruhmkorff, et par suite il ne m'a pas été possible de déterminer ici les positions des premières raies de la bande. Cependant, tout ce qui a été mesuré actuellement se trouve indiqué dans les tableaux.

Je devais maintenant comparer les déterminations des longueurs d'onde faites par nous avec celles des autres observateurs, mais dans les circonstances actuelles je m'abstiens d'une révision complète et je me borne aux remarques suivantes.

Parmi les déterminations récentes, celles de M. VOGEL[1]) ont été mises au premier rang. Si l'on les compare avec les nôtres, on trouve aisément que les raies obtenues par lui au moyen des tubes de Geissler contenant de l'oxygène ou de l'hydrogène s'accordent très bien avec celles que nous venons de signaler comme appartenantes à l'oxyde de carbone, mêlées des raies connues C, F, etc. de l'hydrogène; et de plus que les raies de l'azote, suivant M. VOGEL, sont les mêmes que les raies les plus importantes du bi-

[1]) Pogg. Ann., B. 146, p. 569.

oxyde d'azote, données dans nos tableaux. Une telle comparaison montrera aussi que plusieurs des raies du bioxyde d'azote se trouvent aussi parmi celles qu'a attribuées M. VOGEL à la lumière du pôle négatif. Quant aux longueurs d'onde trouvées, il y a certainement quelques différences entre ses observations et les nôtres, mais elles ne sont pas en général plus grandes qu'on pourra bien les attribuer aux méthodes différentes d'observation.

M. CAPRON [1]) a aussi fait quelques déterminations par rapport aux raies spectrales qu'ont données des tubes de Geissler contenant de l'air ou d'autres gaz. Mais comme ses planches ne possèdent pas la précision désirable, il sera presque impossible d'en conclure, si ses observations confirment ou contredisent les énoncés d' ÅNGSTRÖM à l'égard de la coïncidence des raies de l'aurore boréale avec celles de la lumière du pôle négatif. Voilà pourquoi je me dispense pour le moment d'entrer dans une discussion quelconque sur ce sujet.

[1]) Phil. Mag., Ser. 4, Vol. 49, p. 249.

EXPLICATION DES PLANCHES.

Planche I^ière.

En tête de la planche I^ière se trouvent les raies fraunhoférienes C, D, etc. du spectre solaire pour servir de points de repère, et de plus quelques-unes des bandes d'absorption telluriques les plus importantes.

Fig. I représente le spectre du *carbure d'hydrogène*;

Fig. II „ celui de *l'oxyde de carbone;*

Fig. III „ celui du *bioxyde d'azote;*

Fig. IV „ celui de la lumière du *pôle négatif;*

Fig. V „ au bord supérieur le spectre de *l'oxygène* et au bord inférieur celui du *carbone.* Outre ces raies se trouvent là celles de *l'hydrogène*, représentées par der raies ponctuées.

Planche II.

Fig. I représente quelques raies isolées des groupes principaux du spectre du *carbure d'hydrogène*, et

Fig. II „ celles de *l'oxyde de carbone.*

LONGUEURS D'ONDE DES RAIES BRILLANTES

DE QUELQUES MÉTALLOÏDES ET DE SES COMBINAISONS,

EXPRIMÉES EN DIX-MILLIONIÈMES DE MILLIMÈTRE.

1. *Carbure d'hydrogène* (planche I, fig. I) [1].

COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.
	1ier *Groupe.*		2ième *Groupe.*		3ième *Groupe.*
Orangé	6187,3	Jaune	5633,0	Vert	5164,0
	6119,0		5583,0		5128,0
	6056,3		5538,0		5097,5
	6000,8		5500,0		
	5953,5		5466,0		
	4ième *Groupe.*		5ième *Groupe.*		
Bleu	4736,0	Indigo	4311,0 large.		
	4714,0				
	4697,0				
	4682,0				

[1]) On retrouve ces cinq groupes de bandes du carbure d'hydrogène dans l'ouvrage de M. MARSCHALL WATTS, intitulé: Index of spectra, pages 18 et 19. Voir: *Carbon*, Spectrum n:o 1, α, γ, δ, ε et f.

Mesures micrométriques.

RAIE.	LONGUEUR d'onde.	RAIE.	LONGUEUR d'onde.	RAIE.	LONGUEUR d'onde.
Le groupe *jaune* du *carbure d'hydrogène* (planche II, fig. I).					
α	5633,0		5568,3		5509,5
	.....		5564,8		5504,3
	5604,0		5561,4	δ	5500,0
	5602,0		5557,6		5496,0
	5600,0		5553,5		5491,5
	5597,5		5549,8		5486,0
	5594,5		5546,1		5479,5
	5592,0		5542,3		5476,0
	5589,0	γ	5538,0		5471,0
	5585,5		5534,5	ε	5466,0
β	5583,0		5530,6		5461,0
	5580,4		5526,7		5455,5
	5577,2		5522,3		5450,0
	5574,3		5517,7		5444,5
	5570,9		5513,6		.....
Le groupe *vert* du *carbure d'hydrogène* (planche II, fig. I).					
α	5164,0		5092,1		5029,0
	.....		5089,3		5024,5
	5144,0		5085,9		5021,5
	5142,5	Intenses.	5082,4		5016,7
	5141,0		5079,2		5012,5
	5139,2		5076,0		5008,5
	5137,3		5072,7		5004,5
	5135,5	Intenses.	5069,4		5000,0
	5133,8		5066,5		4996,0
	5132,0		5062,8		4991,5
	5129,7		5059,5		4987,5
β	5128,0	Intenses.	5055,6		4983,0
	.....		5052,2		4978,0
	5106,3		5048,5		4973,5
	5103,0		5044,7		4969,0
	5100,0	Intenses.	5040,2		4964,0
γ	5097,7		5036,7		.....
	5095,5		5033,0		.....

2. *Oxyde de carbone* (planche I, fig. II).

COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.
Groupe (a) *):		*Groupe* (b) *):	
Rouge	6622,0	Orangé	6398,5
Orangé	6078,0	Jaune	5817,0
Jaune	5607,5		5397,5
Vert	5197,0	Vert	5015,0
Bleu	4833,5	Bleu	4697,0
Indigo	4509,0	Indigo	4394,0
Violet	4209,0	Violet	4131,0

Mesures micrométriques: (planche II, fig. II).

LONGUEUR d'onde.	LONGUEUR d'onde.	LONGUEUR d'onde.		
Groupe *jaune* (a):	Groupe *vert* (a):	Groupe *bleu* (a):		
5607,5	5197,0	4833,5	4796,0	
.....	.....	.....	4792,5	
5591,8	.....	.....	4788,8	intenses.
5588,3	5186,5	4822,5	4785,5	intenses.
5585,5	5183,5	4820,3	4780,6	
5582,5	5181,5	4818,1	4776,4	
5578,5	5178,5	4816,0	4772,2	
5574,0	5175,0	4813,5	4767,8	intenses.
5570,5	5172,5	4811,0	4762,8	intenses.
5566,5	5169,5	4808,6	4757,7	
5562,2	5166,2	4805,7	4753,0	intenses.
5557,5	5162,0	4802,8	4748,0	intenses.
.....	.....	4799,4	.....	

*) Plusieurs de ces bandes ont été données dans l'index de M. MARSCHALL WATTS, Spectrum N:o II, p. 20.

Outre les bandes énumérées dans le tableau ci-dessus d'une intensité assez grande, il y en a d'autres très faibles, dont les situations approximatives ont été indiquées sur la planche I, fig. II.

3. *Bioxyde d'azote* (planche I, fig. III).

COULEUR des rayons.	LONGUEUR D'ONDE. 1ière raie.	2ième raie.	3ième raie.	INTENSITÉ.
Rouge	6870,0	»	»	Minima
	6785,7	6778,6	6760,0	
	6701,0	6693,4	6673,5	
	6621,8	6614,2	6594,7	
	6542,3	6533,8	6516,3	
Orangé	6465,5	6458,6	6440,6	
	6392,5	6384,8	6366,8	
	6321,0	6313,8	6294,9	Maxima
	6249,2	6242,6	6225,5	
	6183,2	6175,1	6158,2	Minima
	6125,4	6118,8	6102,1	
	6066,3	6060,6	6043,3	
	6011,8	6004,6	5987,8	
	5957,3	5950,5	5933,3	
	5904,6	5897,5	5882,5	
Jaune	5853,0	5846,1	5830,5	Maxima
	5801,8	5795,3	5780,6	
	5752,0	5745,6	5730,7	
	5703,8	»	5682,5	Minima
	5657,9	»	5637,2	
	5612,6	»	5594,2	
	5567,9	5563,0	5551,8	
	5525,2	5518,7	5506,0	
	5482,8	»	»	
Jaune	5513,4	»	5493,7	
	5476,9	5472,6	»	
	5441,9	5437,0	5422,1	
	5406,4	5401,7	5387,4	Maxima
	5371,7	5366,7	5353,2	
Vert	5339,7	»	»	Minima
	5306,3	»	»	
	5273,8	»	5256,3	
	5244,6	5239,3	5226,5	
	5213,1	5207,7	5196,1	Maxima
	5183,4	5179,3	5165,8	

COULEUR des rayons.	LONGUEUR D'ONDE. 1ière raie.	2ième raie.	3ième raie.	INTENSITÉ.
Vert	5153,7	5149,0	5138,7	
	5126,5	»	»	
	5097,7	»	»	
	5065,6	»	»	
	5032,0	»	»	

(*Suite* du spectre *du bioxyde d'azote*).

COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.
Vert	4972,0	Bleu	4574,0		4144,0
	4919,0		4489,0	Violet	4098,0
Bleu	4813,0	Indigo	4417,0		4063,0
	4722,0		4346,0		4002,0
	4666,0		4271,0	Violet extrême	3952,0
	4649,0	Violet	4203,0		

4. *La lumière du pôle négatif* [1]) (planche I fig. IV).

COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.	COULEUR des rayons.	LONGUEUR d'onde.
1ier *Groupe.*		2ième *Groupe.*		3ième *Groupe.*	
Vert	5227,5	Bleu	4709,3	Indigo	4281,0 [2])
	5150,0		4653,5	Violet	4239,0
	.....		4601,2		4203,0
		Indigo	4555,2		4175,0
			4516,5		

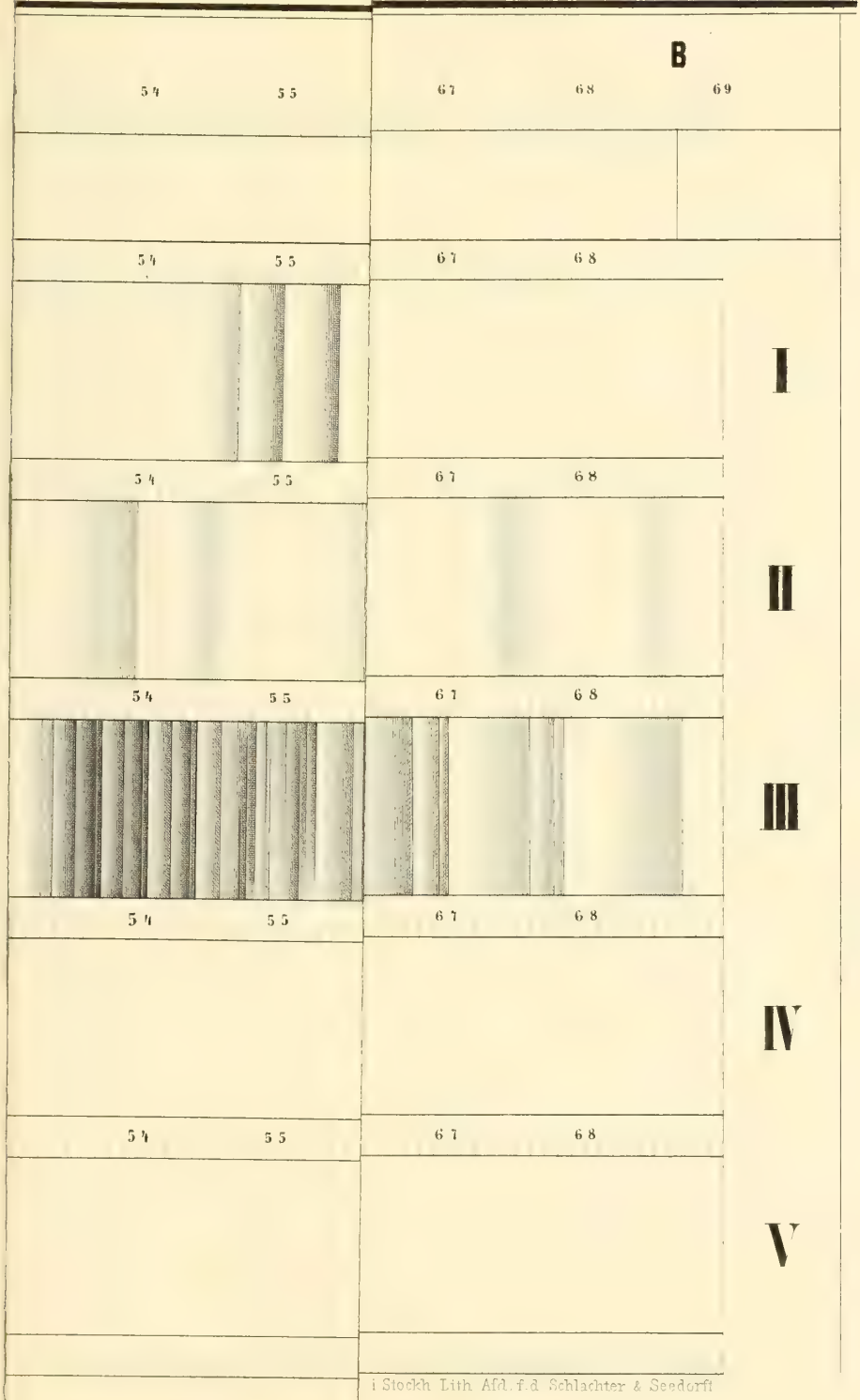
[1]) Il y a une multitude de maxima de lumière, quoique très faibles entre C et l'endroit au-delà de D, dont les situations dans le spectre ont été indiquées par ÅNGSTRÖM sur la planche I, fig. IV.

[2]) Dans le Mémoire d'ÅNGSTRÖM (Pogg. Ann., Jubelband, p. 426; Journal de physique, T. III, p. 212) il y a une faute d'impression, savoir 427,2 au lieu de 428,2.

5. *Carbone* (planche I, fig. V).

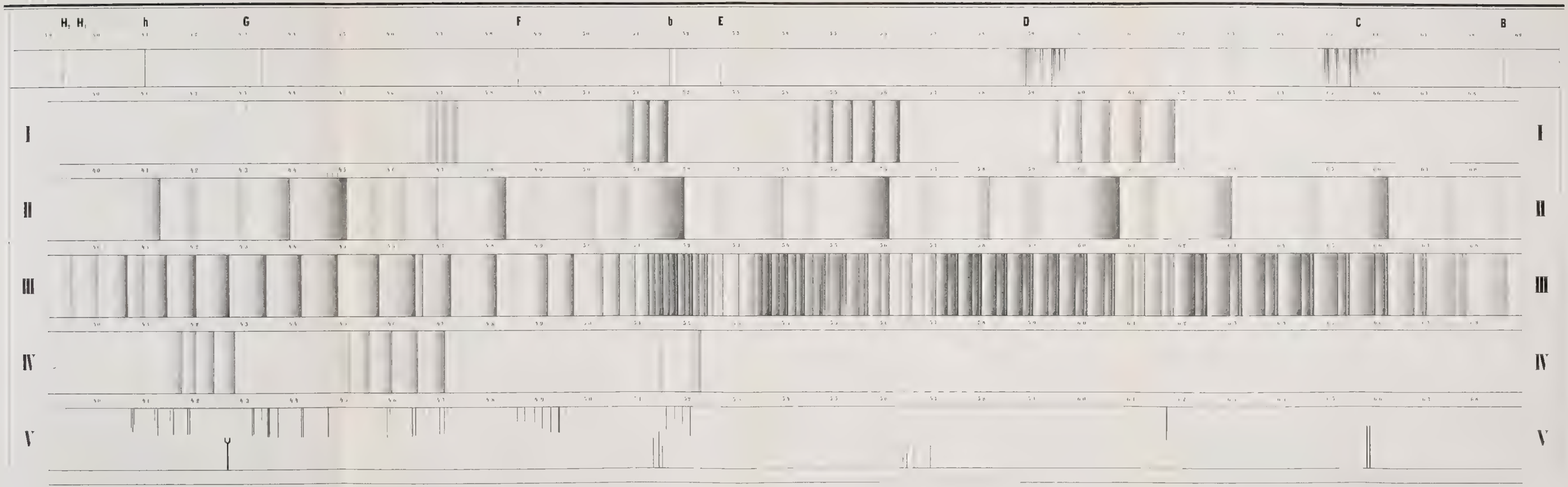
COULEUR des rayons.	LONGUEUR d'onde.	INTENSITÉ.	COULEUR des rayons.	LONGUEUR d'onde.	INTENSITÉ.
Rouge	6583,0	2	Jaune	5379,0	6
	6577,5	1	Vert	5150,5	4
Orangé	5694,1	4		5144,2	3
	5660,9	4		5133,0	5
	5646,5	3	Indigo	4266,0	1 large.
	5638,6	5			

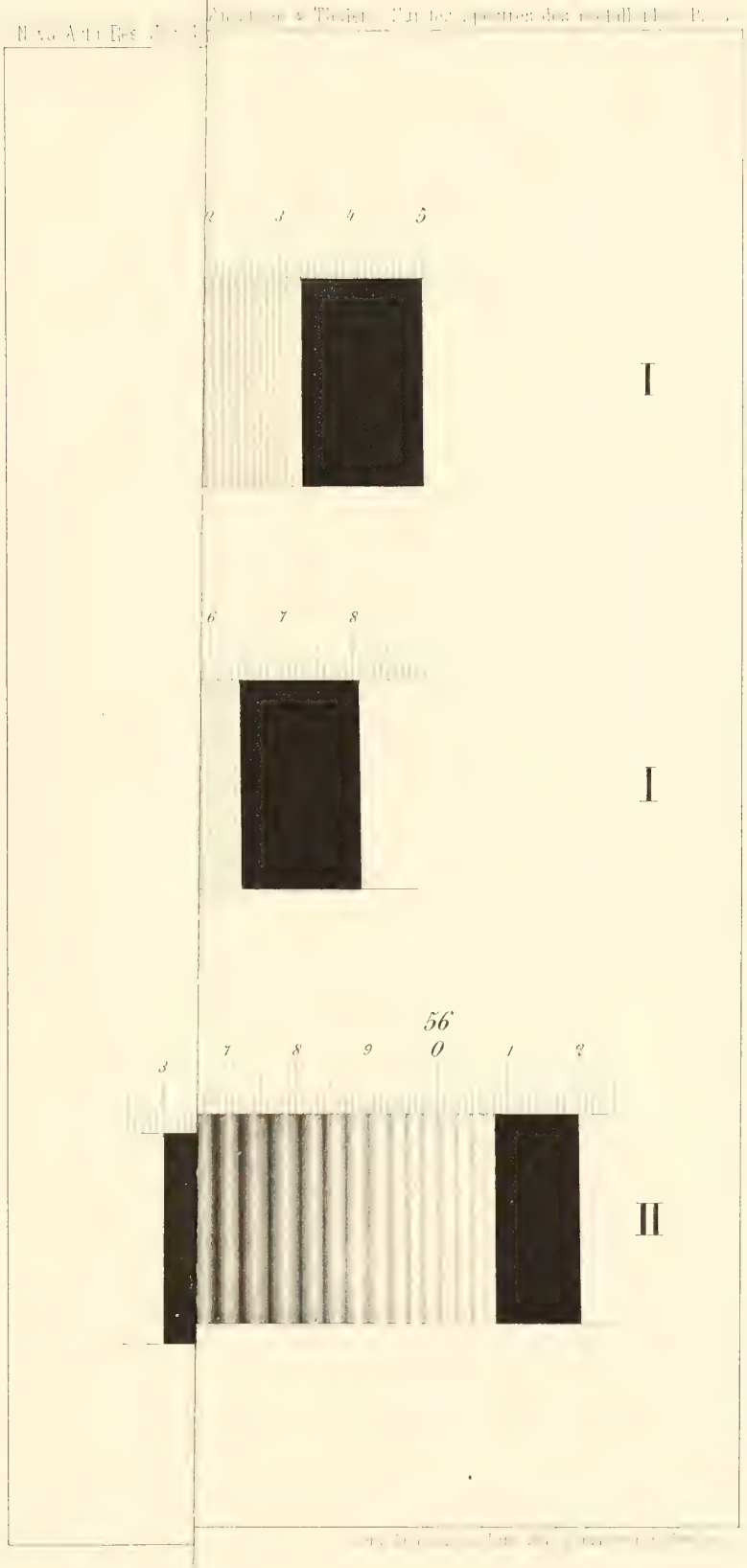
des Metalls

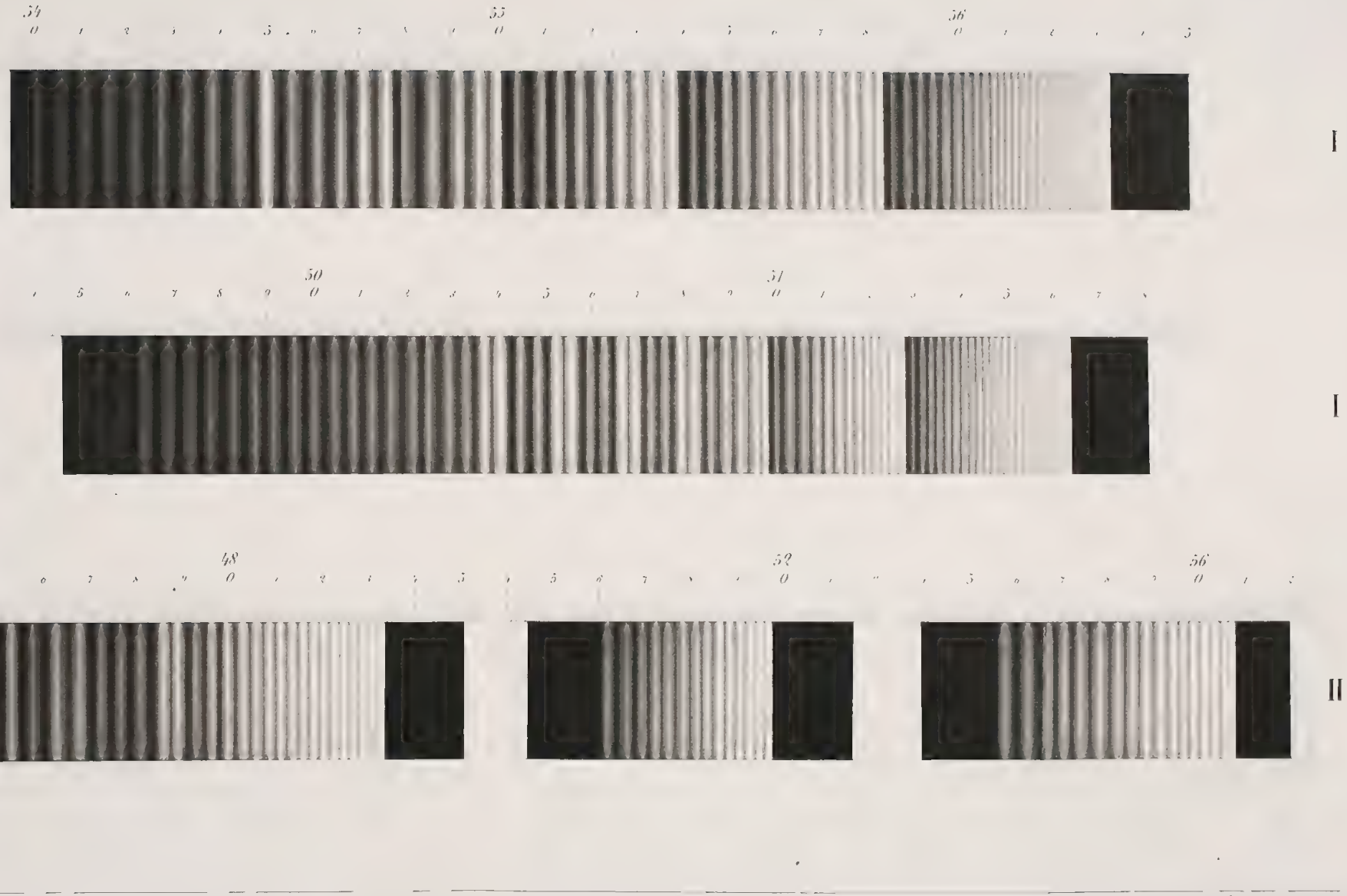


i Stockh. Lith. Afd. f.d. Schlachter & Seedorff

Spectres des Metalloïdes.




2 3 4 5
I
6 7 8
I
56
3 7 8 9 0 1 2
II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